TRANSFORMATIONS OF SHAPE AND SPECTRUM OF A SUBPICOSECOND LASER PULSE FOCUSED ONTO A HOMOGENEOUS KERR MEDIUM

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Influence of the beam self–compression when it is focused onto a homogeneous Kerr medium on time– dependent phase self–modulation and frequency spectrum of the pulse is studied analytically. It is shown that at a strong nonlinearity the frequency spectrum can take a specific feature, that is, the position of the main spectral maximum of the beam intensity shifts towards the carrier frequency of the pulse.

Recently created high–power femtosecond laser systems make it possible to carry out the experiments on observing broadening and deformation of the pulse frequency spectrum during the laser radiation focusing in air.[1,2] Such experiments have revealed that prior to the appearance of superbroadening the principal mechanism of the pulse deformation is the phase self–modulation (PSM). A salient feature of subpicosecond pulses is that the avalanche processes of breakdown in the beam channel, in spite of high intensities ($I \approx 10^{13}$–10^{15} W/cm^2), are incapable of developing and there may appear a cubic nonlinearity of the Kerr type[3] with fast relaxation ($\tau_n \approx 10^{-14}$ s). It can easily be shown that in a homogeneous medium (in contrast, e.g., to optical fiber waveguides) the intrinsic power of the PSM coincides, by the order of magnitude, with the critical power of self–focusing and therefore the time–dependent effects are developed simultaneously with the spatial self–compression of the beam. Although the spectral characteristics of the pulse have been studied sufficiently well for the case of the ordinary PSM (for a plane wave or transverse mode of a fiber waveguide),[4] their behavior in a homogeneous medium is not quite clear.

This paper concerns the study of the behavior of variations in the pulse shape and spectrum during the beam focusing in a homogeneous medium with rapid response cubic nonlinearity.

FORMULATION OF THE PROBLEM

To calculate spectral characteristics of a pulse we employ a parabolic equation of quasioptics for a medium with the cubic nonlinearity written for an axisymmetric beam[5]

$$2ik\,\frac{\partial E}{\partial z} = \left(\frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial}{\partial \rho}\right)E + \frac{2n_2}{n_0}\,\kappa^2|E|^2E\ . \qquad (1)$$

Let us consider the propagation of the focused Gaussian beams

$$E(\rho,\ z = 0) = \sqrt{I_0(t)}\,\exp(-\,\rho^2/2a_0^2 + i\kappa\rho^2/2R_f)\ . \qquad (2)$$

The pulse shape upon entrance into a nonlinear medium $I_0(t)$ can be an arbitrary function of time, however, when making numerical estimates we assumed the Gaussian envelope:

$$I_0(t) = \exp(-\,t^2/\tau_0^2)\ . \qquad (3)$$

Equation (1) for a nonlinear diffraction of the beam was written without taking into account the pulse variance. A simple estimate shows that in air when no resonances occur the dispersion length for an 85–fs pulse is about $L_d \approx 200$ m at a wavelength of 628 μm. Under conditions of strong focusing the dispersion length exceeds, as a rule, the distance to the beam caustic, where the basic nonlinear effects occur.

To obtain the spectrum of output radiation it is necessary to find the solution of Eq. (1) under boundary condition (2) at different times. Using then the Fourier transform it is possible to obtain a transformed time spectrum of the pulse.

SOLUTION OF THE THREE–DIMENSIONAL PROBLEM

To solve Eq. (1) we employ a nonaberrational approximation modified in comparison with that given in Ref. 6. Let us now transfer to dimensionless variables

$$E' = E/E_0\ , \qquad z' = z/\kappa a_0^2\ , \qquad \rho' = \rho/a_0\ , \qquad t' = t/\tau_0 \qquad (4)$$

and introduce a normalized radius of focusing $R'_f = R_f/\kappa a_0^2$. In these variables (the primes are omitted) Eq. (1) under boundary condition (2) has the form

$$2i\,\frac{\partial E}{\partial z} = \left(\frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial}{\partial \rho}\right)E + R(t)\,|E|^2E\ ; \qquad (5)$$

$$E(\rho,\ z = 0) = \sqrt{I_0(t)}\,\exp(-\,\rho^2/2 + i\rho^2/2R_f)\ . \qquad (6)$$

The parameter $R(t) = 2\kappa_0^2\,a_0^2\,I_0(t)\,n_2/n_0$ is proportional to the current pulse power. A solution of Eq. (5) is sought after in the form

$$E(\rho,\ z,\ t) = \frac{\sqrt{I_0(t)}}{\omega(z,\ R(t))}\,\exp \times$$

$$\times \left(\frac{-\,\rho^2}{2\omega^2(z,\ R(t))} + \frac{i\rho^2}{2}\,f(z,\ R(t)) + i\varphi(z,\ R(t))\right), \qquad (7)$$

in this case $\varphi(z, R(t))$ describes the pulse PSM taking into account the beam self–compression. For a dimensionless radius of the beam, and using the method of moments,[7] we obtain

$$\omega^2(z,\ R(t)) = 1 - \frac{2z}{R_f} + \left(1 + \frac{1}{R_f^2} - R(t)\right)z^2\ , \qquad (8)$$

where $\tilde{R}(t) = R(t)/4 = P(t)/P_c$ is the ratio of a current power to the critical one at which (within the

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nonaberrational approximation) the beam is collapsed at the geometric focus.

The solution for $\varphi(z, R(t))$ in the subcritical and supercritical regimes has the forms

$$\omega(z, \tilde{R}(t) < 1) = \frac{1 - 2\tilde{R}(t)}{\sqrt{1 - \tilde{R}(t)}} \times$$

$$\times \left[\arctan\left(\frac{(1 + R_f^{-2} - \tilde{R}(t))z - R_f^{-1}}{\sqrt{1 - \tilde{R}(t)}}\right) + \arctan\left(\frac{R_f^{-1}}{\sqrt{1 - \tilde{R}(t)}}\right)\right], \quad (9)$$

$$\omega(z, \tilde{R}(t) > 1) = \frac{1 - 2\tilde{R}(t)}{2\sqrt{\tilde{R}(t) - 1}} \times$$

$$\times \ln\left(\frac{\sqrt{\tilde{R}(t) - 1} - (1 - \tilde{R}(t))z}{\sqrt{\tilde{R}(t) - 1} + (1 - \tilde{R}(t))z}\right). \quad (10)$$

DISCUSSION OF THE RESULTS

Subcritical regime. Let us consider the propagation of pulses with limited intensity when for any time moment the inequality $0 \le \tilde{R}(t) < 1$ is valid and the phase self–modulation is described by formula (3).

The caustic of a focused beam, in this case, can be located within the medium. Let us now analyze the pulse parameters on the beam axis for two limiting cases: $R_f^2 \ll 1$ (i.e., a focused beam) and $R_f = \infty$ (a collimated beam).

Pulse duration. As can be seen from Fig. 1, the pulse shape of a collimated beam varies at distances $z > 1$ if the parameter $\tilde{R}$ $(t = 0)$ is not too small. It can also be seen that in this case the total length of the pulse remains constant while its half–width decreases (see Fig. 1*a*) due to "underlining" the central portion of the pulse.

In a focused beam the pulse shape the variations occur within the region of linear caustic (Fig. 1*b*). In this case it decreases before the focus and then it takes its initial value.

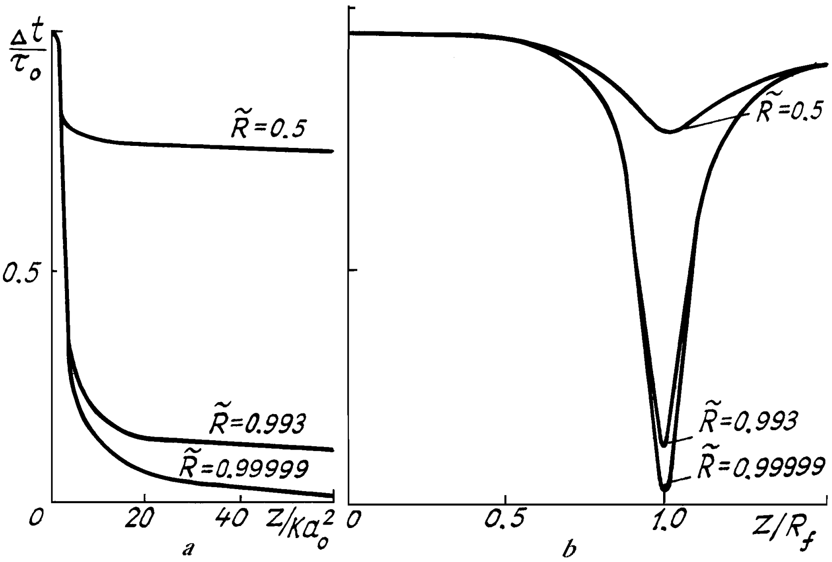

FIG. 1. Pulse duration Δt at e^{-1} level in the initially collimated (a) and focused (b, $R_f = 0.2$) beams as a function of the propagation distance.

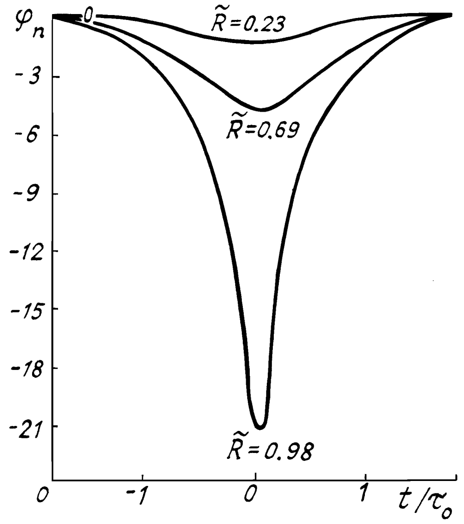

FIG. 2. Nonlinear phase shift φ_n on the axis of the initially focused beam ($R_f = 0.333$, $z \gg R_f$).

Phase. Phase modulation at large distances for a collimated ($z \gg (1 - \tilde{R})^{-1/2}$) and focused ($z - R_f \gg R_f^2$) beams is the same. However, in a focused beam (with a constant pulse shape) the phase modulation index is two times larger than that in a collimated beam. A nonlinear phase shift $\varphi_n(t) = \varphi(t) - \varphi(t \to -\infty)$, for not very large nonlinearity parameters follows the input pulse shape. Time dependence of the phase becomes stronger with increasing nonlinearity: in the focused beam the width of the function $\varphi_n(t)$ becomes smaller than the pulse duration, and its shape differs from the Gaussian (Fig. 2).

Frequency spectrum. The frequency spectrum can be conveniently analyzed by comparing it with the conventional PSM (the beam compression can be neglected) in a nondisperse regime (Fig. 3). It is well known[4] that in this case for large nonlinear phase shifts the global maximum of spectral intensity is located at the edges of the spectrum.

Two specific regions of the frequency spectrum transformations can be isolated within the pulse top φ_{max} depending on a nonlinear phase shift. In the region of a moderate nonlinearity (to say conventionally at $\varphi_{max} < 4\pi$) the frequency spectrum can experience strong broadening, and a modulation structure can be seen in it. However, such a transformation of the spectrum fully corresponds to the spectrum modification occurring at the ordinary PSM.

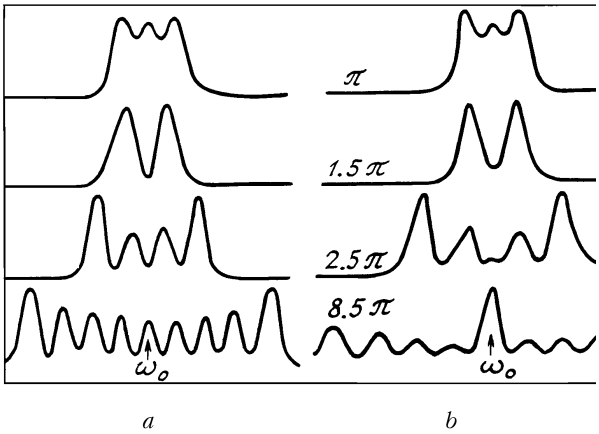

FIG. 3. Frequency spectrum of a pulse caused by the ordinary PSM (a) and after beam–focusing (R_f = 0.333) in a nonlinear medium (b). The parameter at the curves is a nonlinear phase shift at the pulse top, ω_0 is the carrier frequency of the pulse.

In the region of a strong nonlinearity ($\varphi_{max} > 4\pi$) the nature of spectrum transformations is essentially different. For example, the maximum of spectral intensity displaces to the central frequency (Fig. 4) and simultaneously a broadening of the spectrum takes place. In a collimated beam the modulation structure of the spectrum is strongly broadened due to the pulse duration decrease.

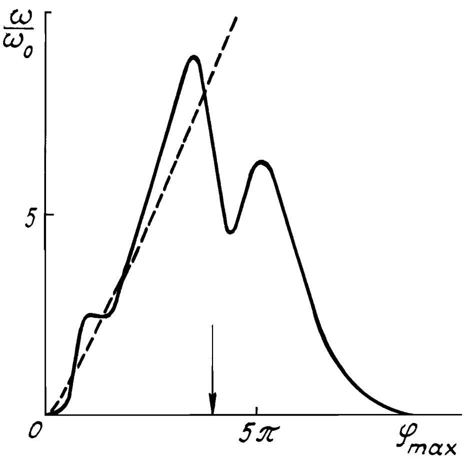

FIG. 4. Location of the global maximum in the frequency spectrum of the pulse after beam focusing as a function of the nonlinear phase shift at the pulse top φ_{max}. The dashed line is the same with the ordinary PSM. An arrow denotes a conditional boundary between the regions of moderate and strong nonlinearities.

Supercritical regime. Let us define the supercritical regime of the pulse propagation as a regime when the inequality $\tilde{R}(t) > 1$ is valid during some period. During this time the PSM is described by formula (10) whereas at the leading and trailing fronts by formula (9). Consider now the propagation of initially collimated beams to the point of the nearest nonlinear focus z_f. To start with, let the PSM be analyzed in broad bands: $a_0^2 \to \infty$, i.e., $R(t) \to \infty$ for $z \to 0$. Relation (10), in this limiting case, is reduced to a well known relation for the PSM of a plane wave in the nondisperse regime $= -\frac{1}{2} Rz)$ and the frequency spectrum of radiation is related to an ordinary phase self–modulation.

A nonlinear phase shift can also occur at an insignificant excess of the nonlinearity parameter over its critical value at the pulse top. In this case large values of φ_{max} are achieved in the vicinity of the nonlinear focus z_f. The calculations reveal that for $\tilde{R}(0) = 1.025$ at $z/z_f = 0.99999$ a nonlinear phase shift at the pulse top φ_{max} is equal to 7.6 π. The frequency spectrum strongly broadened due to the pulse duration decrease possesses all spectrum features occurring in the case of the ordinary PSM. Thus, in particular, the global maximum of the spectral intensity is located at an edge of the spectrum.

CONCLUSION

The calculations done in this paper revealed that when a pulse is focused in a homogeneous medium with a cubic nonlinearity the frequency spectrum, under condition of the strong PSM, in a subcritical regime can acquire a specific feature, i.e., its global maximum appears in the vicinity of the carrier frequency of radiation. This result obtained using a nonaberrational approximation ought to be verified numerically. Direct numerical experiments have been carried out for the case of a moderate nonlinearity (see Ref. 8). The results obtained in Ref. 8 well agree with the data obtained in this paper.

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