RETRIEVING THE LASER RADIATION INTENSITY DISTRIBUTION FROM THE SURFACE TEMPERATURE OF A SECTIONED TARGET

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The problem of retrieving the laser radiation intensity from the surface temperature over the target heated upon exposure to a laser beam is solved. Employment of a sectioned target allowed one to reduce the multidimensional inverse problem of heat conduction to a set of one-dimensional problems. Formulas for the temperature inversion have been obtained with different boundary conditions of heating. These formulas/ allow one to retrieve the thermal flux for arbitrary values of thermal and physical parameters. The efficiency of algorithmic implementation is studied in numerical experiments.

In the study of propagation of powerful laser radiation through the atmosphere one faces with a problem of measuring its intensity distribution over the laser beam cross section. A possible approach to the solution of this problem is to retrieve the intensity distribution from neasureraents of the temperature field upon heating a target surface by the laser radiation. The advent of devices designed for remote sensing of the surface temperature fields, in particular, thermal imaging systems[1,2] providing for high spatial and temporal resolution of measurements, Bakes it possible to solve this problem. In general, the problem of retrieving the radiation intensity distribution from the temperature field over a heated surface is reduced to the solution of the multidimensional spatiotemporal inverse problem of heat conduction, which represents the problem of conversion of the boundary conditions. If we use an array of one-dimensional sections (their transverse dimensions should be smaller than the characteristic scale of the Intensity distribution) as a target, the solution of such a spatiotemporal problem is reduced to a set of temporal problems (as шалу problems as there are sections in the target). Prior to formulation of this one-dimensional problem we will consider, using the simplest situations as an example, how the time dependence of a heated surface temperature is related to time dependence of the thermal flux incident on it. We denote the temperature of the target surface as $T(0,\ t)$ and the thermal flux incident on it as $q(t)$. Neglecting the thermal losses we may write $q(t) = (1 - R)\ I(t)$, where R is the reflectance and $I(t)$ is the intensity of the Incident radiation. Using the heat budget equation for the target whose back surface is maintained at a constant temperature $T(0)$, we obtain

$$T(t) = T(0) + \frac{L}{k}\, q(t),\ L \to 0; \qquad (1)$$

for a thin heat—insulated target we have

$$T(t) = T(0) + \frac{a^2}{kL} \int\limits_0^t q(\tau)\ d\tau\ ,\ L \to 0, \qquad (2)$$

where a^2 and k are the thermal diffusivity and the thermal conductivity and L is the thickness of the target.

It follows from Eq. (1) that the temperature of a thin cooled target follows the temporal behavior of the thermal flux. In this case we do not need to solve the inverse problem of heat conduction. Relation is valid for temperature detectors. Note, however, that Eq. (1) is obtained in the limit $L \to 0$ when the absolute values of $T(t)$ are close to the temperature of the target base, so that measurements of $q(t)$ become practically unfeasible against the background noise, which accompanies remote optical measurements of the temperature.[2]

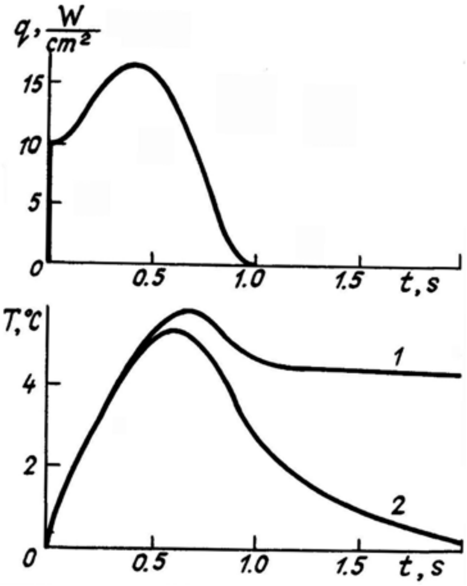

FIG. 1. Time dependence of the surface temperature of heat-insulated, (1) and cooled (2) sections of the target when it is exposed to pulse of laser radiation.

It follows from Eq. (2) that the heat—insulated target integrates $q(t)$, and thus it is most convenient as the energy meter. In order to determine the temporal dependence of $q(t)$, one needs to differentiate the values of the temperature during heating. However, for high enough density of the laser radiation the target without heat elimination runs hot and is destroyed. Therefore, the above-described simplest target configurations cannot be used for practical measurements of $q(t)$. The target should acquire an elevated surface temperature in order to guarantee the signal-to-noise ratio which is necessary for temperature measurements and should, at the same time, efficiently carry away heat to prevent its own destruction. In order to combine these two conditions the sections must be sufficiently thick. The required specifications for their thermal and physical, and geometric parameters will be formulated below. Figure 1 shows the results of computation of the temperature over an aluminum target (a^2 = 9.28 cm^2/s, and L = 1 cm) when it is exposed to

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pulse of laser light for $q(0) = 10\ W/cm^2$. The distorting effect of heat transfer on the intensity distribution is apparent. The problem of compensating for this effect is treated below. Algorithms have been constructed and numerical simulations have been made to retrieve $q(t)$ for various target types.

Assuming the side surface of the section to be heat-insulated and the intensity distribution over its front surface to be uniform, we may describe the process of heat transfer across the section by a one–dimensional heat conduction equation:

$$\frac{\partial T}{\partial t} = a^2 \frac{\partial^2 T}{\partial z^2},\ t > 0,\ 0 \le z \le L. \tag{3}$$

The boundary and initial conditions

$$q(t) = -k\left.\frac{\partial T}{\partial z}\right|_{z=0} = (1-R)I(t) - \vartheta\left[T(0,t) - T_0\right] - \sigma b\left[T^4(0,t) - T_0^4\right], \tag{4}$$

$$T(z,t)\big|_{z=0} = T(t), \tag{5}$$

$$T(z,t)\big|_{z=L} = T_0, \tag{6}$$

$$-k\left.\frac{\partial T}{\partial z}\right|_{z=L} = 0, \tag{7}$$

$$T(z,0) = T_0 \tag{8}$$

we use in the following combinations: Eqs. (4), (6), and (8) – to solve the problem of heating of the cooled target; Eqs. (4), (7), and (8) – to solve the problem of heating of the heat–insulated target. The combinations of Eqs. (5), (6), and (8) and of Eqs. (5), (7), and (8) will be needed to solve the inverse problems for the cooled target and the heat–insulated target, respectively. Here υ is the heat convection coefficient, σ is the Stefan–Boltzmann constant, b is the emission coefficient.

To start with, we consider the situation in which the temperature of the back surface of the section is maintained at its initial magnitude $T(L,\ t) = T_0$. Solving the heat conduction equation with the boundary conditions (4), (6), and (8) for the target surface temperature we obtain:

$$T(0,t) = T_0 + \frac{2a^2}{kL}\int_0^t q(\tau)\ d\tau q(\tau) \times$$

$$\times \sum_{n=1}^{\infty} \exp\left\{-\frac{a^2\pi^2(2n-1)^2}{4L^2}(t-T)\right\}. \tag{9}$$

To invert relation (9), we solve Eq. (3) with boundary conditions (5), (6), and (8) using the Duhamel principle[4]. In accordance with Eq. (4), calculating the derivative of the temperature with respect to the spatial coordinate at zero point we have

$$q(t) = \frac{k}{L}\int_0^t \frac{dT(\tau)}{d\tau}\left(1 + 2\sum_{n=1}^{\infty}\exp\left\{-\frac{a^2\pi^2 n^2}{L^2}(t-\tau)\right\}\right)d\tau. \tag{10}$$

Expression (10) is valid for arbitrary values of a^2, L, and of a variable t. However it may be simplified for certain relations among these variables. Using a generalized thermal and physical parameter $F_0 = a^2t/L^2$ (the Fourier parameter) and the Laplace method[5] we obtain from Eq. (10) for $Fo \gg 1$

$$q(t) = \frac{k}{L}\left[T(t) - T_0 + \frac{1}{3}\frac{L^2}{a^2}\frac{dT(t)}{dt}\right]. \tag{11}$$

For another limiting case $F_0 \ll 1$ (in the approximation of semi–infinite body) replacing the summation in Eq. (10) by integration we have

$$q(t) = \frac{k}{a\sqrt{\pi}}\int_0^t \frac{dT(\tau)}{d\tau}\frac{d\tau}{\sqrt{t-\tau}}. \tag{12}$$

The latter expression is the solution of Abel's integral equation[6], to which the inverse problem of heat conduction of a semi–infinite body is reduced.

When the back surface of the target is heat–insulated, then using Eqs. (1), (4), (7), and (8) and (1), (5), (7), and (8), we obtain in analogy with the case of the cooled target[7,8]

$$T(t) = T_0 + \frac{a^2}{kL}\int_0^t q(\tau)\left(1 + 2\sum_{n=1}^{\infty}\exp\left\{-\frac{a^2\pi^2 n^2}{L^2}(t-\tau)\right\}\right)d\tau. \tag{13}$$

$$q(t) = \frac{2k}{L}\int_0^t \frac{dT(\tau)}{d\tau}\sum_{n=1}^{\infty}\exp\left\{-\frac{a^2\pi^2(2n-1)^2}{4L^2}(t-\tau)\right\}d\tau. \tag{14}$$

If $F_0 \gg 1$, expression (14) yields

$$q(t) = \frac{kL}{a^2}\frac{dT(\tau)}{d\tau} \tag{15}$$

and if $F_0 \ll 1$, Eq. (14) is transformed into the solution of Abel's equation (12), which is the particular case of representations (10) and (14). A significant part of the error in retrieval of $q(t)$ with the help of the algorithms being constructed in accordance with formulas (10), (12), and (14) is associated with the accumulated error in calculating the derivative $\frac{\partial T}{\partial \tau}$ on the interval of integration $[0,\ t]$. For example, we have the following majorant estimate for the solution of Abel's equation:

$$\left|q(t) - q_S(t)\right| \le \sup_{0\le\tau\le t}\left|\frac{\partial T(\tau)}{\partial\tau} - \frac{\partial S(\tau)}{\partial\tau}\right| 2\sqrt{t}, \tag{16}$$

where $S(t)$ is a smooth approximation of the temperature. It follows from the above unequality that the error in retrieval depends on the accuracy of computation of the derivative $\frac{\partial T}{\partial \tau}$ and that it increases for extended interval of integration $[0,\ t]$. This circumstance calls for designing the algorithms of retrieval based on the formulas that do not involve the operation of differentiation of the measured temperature.

Assuming that $T(t)$ satisfies the Hölder boundary condition[9], after integration by parts we obtain from Eqs. (10), (12), and (14)

$$q(t)=\frac{k}{L}\left\{T(t)+2T(t)\sum_{n=1}^{\infty}\exp\left\{-\frac{a^2\pi^2n^2}{L^2}t\right\}+2\int_0^t d\tau\left[T(t)-T(\tau)\right]\right\}\times$$

$$\times\sum_{n=1}^{\infty}\exp\left\{-\frac{a^2\pi^2n^2}{L^2}(t-\tau)\right\}\frac{a^2\pi^2n^2}{L^2}\Bigg\},\qquad(17)$$

$$q(t)=\frac{k}{a\sqrt{\pi}}\left\{\frac{T(\tau)}{a\sqrt{\pi}}+\frac{1}{2}\int_0^t\frac{T(t)-T(\tau)}{(t-\tau)^{3/2}}d\tau\right\},\qquad(18)$$

$$q(t)=\frac{2k}{L}\left\{T(t)\sum_{n=1}^{\infty}\exp\left\{-\frac{a^2\pi^2(2n-1)^2}{4L^2}t\right\}+\int_0^t d\tau\left[T(t)-T(\tau)\right]\right\}\times$$

$$\times\sum_{n=1}^{\infty}\exp\left\{-\frac{a^2\pi^2(2n-1)^2}{4L^2}(t-\tau)\right\}\frac{a^2\pi^2(2n-1)^2}{4L^2}.\qquad(19)$$

The structure of integrand expressions in Eqs. (10), (12), (14), and (17)–(19) is such that integration does not lead to smoothing the noise component contained in the measured values of the temperature. Therefore direct computation of $q(t)$ according to these formulas results in the inversion instability.[6,10] That is why in algorithmic implementation the function $T(t)$ was approximated by smoothing cubic splines[6,11] which take into account the measurement errors.

We simulated numerically the retrieval of the intensity for thermal and physical situations described by the boundary conditions (5)–(7). The initial temperature was then set to be equal to zero: $T(0) = T_0 = 0$. We chose aluminum for the target material. Experimental temperature values were recorded at points τ_1, where $0=\tau_1 < \tau_2 < \tau_3 \dots < \tau_n = 1$ s with the resolution time Δt equal to that of the thermal imaging system $\Delta t = 1/24$ s. The starting data included a random measurement error ξ_i such that

$$\tilde{T}=T(\tau_1)+\xi_1,\ \overline{1,n}.$$

It was further assumed that ξ_i obeys the normal distribution with a zero mean and variance σ^2. The function $T(t)$ was approximated by a smoothing cubic spline $S_{n,\alpha}(t)$ with boundary conditions $S''_{n,\alpha}(0) = 0$, $S''_{n,\alpha}(t = 1\text{ s}) = 0$ (see Ref. 6) taking in account of the peculiarities of the solution of ill–posed problems. After substitution, the integrals in the right sides of Eqs. (10), (12), (14), and (17)–(19) could be taken analytically. Infinite summation in Eqs. (10), (14), (17), and (19) was limited by the given error. In any case it did not exceed 0.001%.

To model the initial thermal flux in the numerical experiment, we chose two functions, namely, "pulse"

$$q(t) = I_0\,\theta(\tau)\left\{17\,\tau^4 - 32\,\tau^3 + 14\,\tau^2 + 1\right\},$$

$$\theta(\tau)=\begin{cases}1, & \tau\ge 0\\ 0, & \tau<0,\ \tau\le 1,\end{cases}\qquad(20)$$

where $I = 10$ W/cm^2, $\tau = t/t_0$, and $t_0 = 1$ s and a "cap"

$$q(t)=\begin{cases}I_0\exp\left\{-\dfrac{(\tau-0.5)^2}{(0.5)^2-(\tau-0.5)^2}\right\}, & |\tau-0.5|\le 0.5\\ 0, & |\tau-0.5|>0.5.\end{cases}\qquad(21)$$

Dependence (20) is shown in Figs. 2 and 3 (curve 1). In the absence of the errors in the measurement of the temperature the error in computation of $q(t)$ is caused by the substitution of the function $T(t)$ by a cubic spline $S_n(t)$ and by finite summation instead of infinite. The numerical experiment showed that such an approximation ensures high enough accuracy of computation of corresponding integrals. Even in the case of "pulsed" dependence (20) we found that for $\Delta t = 1/24$ the error in retrieval of $q(t)$ did not exceed 2.5%. Random oscillations appear in the solution obtained with the use of interpolating splines when there is measurement noise (curves 2 in Figs. 2, and 3). They intensify for extended interval of integration, so that further analysis was conducted using the smoothing splines $S_{n,\alpha}(t)$. The smoothing parameter a was determined based on the discrepancy technique[6,10].

Figures 2 and 3 show the results of retrieving $q(t)$ from model dependence (20) for 3% error in the initial data. Model dependence (21) resulted in more accurate retrieval of $q\{t)$. Figure 2 corresponds to cooled and Fig. 3 – to heat–insulated target. Here points indicated the results of retrieval after Eqs. (10) and (14) and curves – such results after Eqs. (17) and (19). As follows from Fig. 2 the quality of retrieval of $q(t)$ depends weakly on the type of the boundary problem considered. The dependences obtained from Eqs. (17) and (19) that do not involve the operation of differentiation under the sign of integral, are more exact. For adequate comparison of the results, we computed the solution error "variance" for a given t from a sequence of the length $N_s (N_s = 10)$

$$\Delta^2(t)=\frac{1}{N_s}\sum_{l=1}^{N_s}\left(q_{n,\alpha}^{(l)}(t)-q(t)\right)^2.\qquad(22)$$

Here $q(t)$ is the exact solution, $q_{n,\alpha}^{(1)}(t)$ is the solution constructed from the lth noisy temperature realization based on the use of the splines with the smoothing parameter α.

The results of such a comparison for $\sigma = 0.03$ are presented below in Table I ("pulsed" dependence (20) was considered), where

$$D(q_{n,\alpha})=\left[\frac{1}{n_t}\sum_{j=1}^{n_t}\Delta^2(t_j)\right]^{1/2}$$

is the rms error of the solution ($n_t = 25$). It follows from the table that a value of d^2t yielded by the algorithms that do not involve the differentiation turns to be much smaller than the corresponding values obtained from the algorithms that involve the differentiation. Comparison of the results of computations for "pulse" and "cap" functions shows that such a difference is typical of the initial segment of "pulsed" dependence (20).

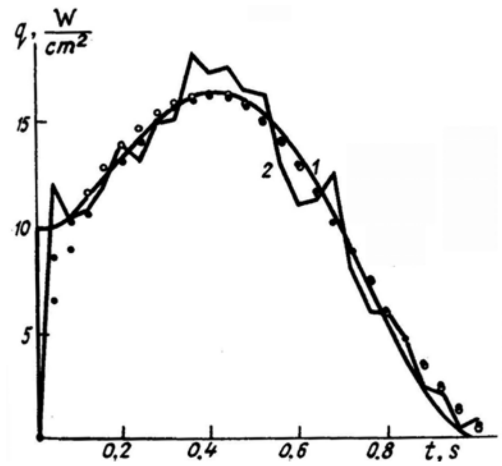

FIG. 2. Retrieving model dependence (20) of q(t) from the surface temperature of cooled target: 1) exact solution; 2) use of interpolating splines. Dots indicate the smoothed solution on the basis of Eq. (10) and circles – of Eq. (17).

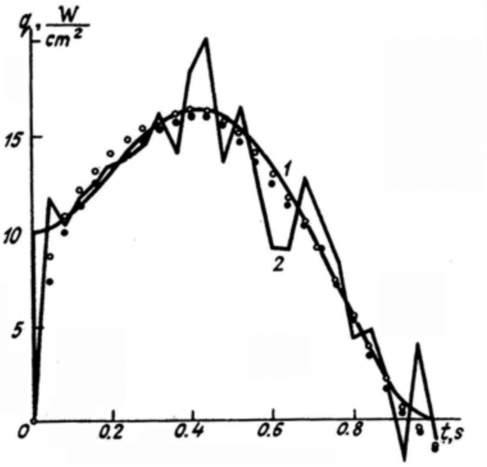

FIG. 3. Retrieving model dependence (20) of q(t) from the surface temperature of heat-insulated target: 1) exact solution; 2) use of interpolating splines. Dots indicate the smoother solution based on Eq. (14) and circles – on Eq. (19).

TABLE I.

Time	Solution for σ = 0.03 with boundary conditions			
	$\frac{dT(0,t)}{dz} = -\frac{1}{k}q(t)$; $T(L,t) = 0$		$\frac{dT(0,t)}{dz} = -\frac{1}{k}q(t)$; $\frac{dT(L,t)}{dz} = 0$	
	Error "variance" for the solution on the basis of the algorithms			
	(10)	(17)	(14)	(19)
Δt	0.141	0.021	0.1657	0.031
$2\Delta t$	0.038	0.003	0.05	0.004
$3\Delta t$	0.011	0.003	0.017	0.002
$4\Delta t$	0.004	0.003	0.007	0.002
$5\Delta t$	0.002	0.002	0.004	0.002
$D^2(q_n,\alpha)$	0.0006	0.0002	0.0008	0.0002

For smoother initial function (21) such a difference becomes less noticeable. Numerical simulations were conducted for targets of L = 1 cm. The Fourier parameter F_o was equal to 0.86 for observation time t = 1 s. It turns out that if the condition $F_o \le 0.2$ is satisfied one may use the algorithm constructed on the basis of Abel's inverse transformation (12) and (18).

We have solved the problem of retrieving the laser beam intensity distribution from the surface temperature of the heated target. The solution yields the formulas for temperature inversion that make it possible to reconstruct the thermal flux passing through a one–dimensional target surface from the surface temperature of the target for various thermal and physical boundary conditions. The corresponding algorithms have been constructed and numerical simulations conducted for the thermal fluxes from the temperature distributions under "noisy" conditions. The errors of the above algorithms have been estimated on the basis of typical models of the time dependence of the radiation intensity. Analysis shows that smoothing splines and algorithms that do not involve the operation of differentiation of the values of temperature led to more accurate thermal heat flux retrieval.

REFERENCES

1. J.M. Lloid, *Thermal Imaging Systems* (Plenum, New York, 1975).
2. A.V. Kuzikovskii, V.A. Pogodaev, and A.E. Rozhdestvenskii, in: *Abstract of Reports at the Second Conference on Atmospheric Optics,* Tomsk (1980), Vol. 2, p. 177.
3. O.M. Alifanov, *Identification of the Heat Exchange Processes for Aircrafts* (Mashinostroenie, Moscow, 1979).
4. A.V. Lykov, *Theory of Thermal Conduction* (Vysshaya Shkola, Moscow, 1967).
5. M.V. Fedoryuk, *Steepest Descent Method* (Nauka, Moscow, 1977).
6. Yu.E. Voskoboinikov, N.G. Preobrazhenskii, and A.I. Sidel'nikov, *Mathematical Processing of Experiment in Molecular Gas Dynamics* (Nauka, Novosibirsk, 1984).
7. A.V. Grigor'ev, *Pulsed Regime of Heating Beams* (Nauka, Moscow, 1979), Vol. 2.
8. V.A. Zhuk and A.S. Golosov, Prikl. Fiz. Zh. **29**, No. 1, 45–50 (1975).
9. F.D. Gakhov, *Boundary Problems* (Nauka, Moscow, 1977).
10. A.N. Tikhonov and V.Y. Arsenin, *Method of Solving the Ill–Posed Problems* (Nauka, Moscow, 1979).
11. Yu.E. Voskoboinikov, Avtometriya, No. 1, **38** (1986).