Formation of shear interferograms in diffusely scattered fields for wave front control at a double-exposure recording of lensless quasi-Fourier hologram

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Received February 22, 2000

Recording of a lensless quasi-Fourier hologram of an opaque screen is considered. The recordings are based on matching speckle fields of two exposures in the hologram plane. The operation of a shear interferometer in this case is analyzed with the use of coherent diffusely scattered fields for a higher than parabolic approximation. It is shown that the sensitivity range of the interferometer is limited because of aberrations of the reference wave, and aberrations in the object channel can lead to errors in control as well.

As was shown in Ref. 1, when recording a double-exposure lensless Fourier hologram of an opaque screen, speckle fields of the two exposures can be matched in the hologram plane by changing the tilt of the wave front under control and displacing a photographic plate before the second exposure. As a result, shear interferograms in infinitely wide bands are formed at the stage of hologram reconstruction. Since the change in the tilt of the wave front under control leads to the displacement of objective speckles in the plane of the photographic plate even in the case that does not correspond to the condition of formation of the Fourier image of an opaque screen in this plane, this method can be extended to the cases of an arbitrary radius of curvature of a convergent or divergent quasispherical wave, including a quasiplane wave, by double-exposure record of the lensless quasi-Fourier hologram of an opaque screen.

This has been demonstrated in Ref. 2 for the case of a quasiplane wave front. Both in Ref. 1 and in Ref. 2, the operation of a holographic interferometer was analyzed with the use of coherent diffusely scattered fields in the Fresnel approximation. This analysis has shown that the interference pattern characterizing the controlled wave front localizes in the plane of formation of the image of the opaque screen, and the interference pattern characterizing phase distortions of the reference wave due to aberrations in the forming optical system localizes in the hologram plane. Spatial filtering of the diffraction field in the corresponding planes allows independent recordings of the phase distortions in both the object and reference channels.

In this paper, we analyze peculiarities in operation of a holographic shear interferometer for wave front control based on double-exposure records of a lensless quasi-Fourier hologram of an opaque screen with the allowance made for an approximation higher than the parabolic one in order to determine the sensitivity range of the interferometer and errors in the control.

As is shown in Fig. 1, the opaque screen *1* placed in the plane (x_1, y_1) is illuminated by a coherent radiation of a convergent quasispherical wave having the radius of curvature R in its plane. The radiation diffusely scattered by the opaque screen is recorded onto a photographic plate *2* at the first exposure by the scheme for recording the lensless quasi-Fourier hologram with the use of the divergent spherical reference wave having the radius of curvature l equal to the distance between the opaque screen and the photographic plate. The spatial filter p_0 (Ref. 3) excludes possible phase distortions in the channel of formation of the reference wave. Before the second exposure, the wavefront tilt of the radiation used for illuminating the opaque screen is changed, for example, in the plane (x, z) by α, and the photographic plate is displaced by b along the direction of the x axis.

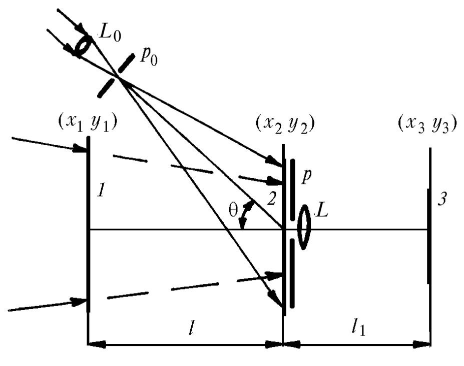


Fig. 1. Scheme of recording and reconstruction of a double-exposure quasi-Fourier hologram: opaque screen *1*, photographic plate-hologram *2*, plane of recording the interference pattern *3*, lenses L_0 and L, spatial filter p_0, and aperture diaphragm p.

The first-exposure distribution of the complex amplitude of the object wave in the hologram plane (x_2, y_2) is written in the form

0235-6880/01/02 111-06 $02.00

$$u_1(x_2, y_2) \sim \iint\limits_{-\infty}^{\infty} t(x_1, y_1) \exp\left\{ik\left[-\frac{1}{2R}(x_1^2 + y_1^2) + \right.\right.$$

$$\left.\left. + \frac{1}{8R^3}(x_1^2 + y_1^2)^2\right]\right\} \exp -i\varphi(x_1, y_1) \exp ik\left\{\frac{1}{2l}[(x_1 - x_2)^2 + \right.$$

$$\left. + (y_1 - y_2)^2] - \frac{1}{8l^3}[(x_1 - x_2)^2 + (y_1 - y_2)^2]^2\right\} dx_1\, dy_1, \tag{1}$$

where k is the wave number; $t(x_1, y_1)$ is the complex amplitude of transmittance of the opaque screen (the amplitude is a random function of coordinates); $\varphi(x_1, y_1)$ is the phase function characterizing distortions of the wave front under control due to aberrations of the forming optical system.

Ignoring the screen roughness, the second-exposure distribution of the complex amplitude of the object wave in the plane (x_2, y_2) because of the small wavefront tilt before the second exposure is determined by the equation

$$u_2(x_2, y_2) \sim \iint\limits_{-\infty}^{\infty} t(x_1, y_1) \exp ik\left\{-\frac{1}{2R}[(x_1 - R\sin\alpha)^2 + \right.$$

$$\left. + y_1^2] + \frac{1}{8R^3}[(x_1 - R\sin\alpha)^2 + y_1^2]^2\right\} \exp -i\varphi(x_1 + a, y_1) \times$$

$$\times \exp ik\left\{\frac{1}{2l}[(x_1 - x_2 - b)^2 + (y_1 - y_2)^2] - \right.$$

$$\left. - \frac{1}{8l^3}[(x_1 - x_2 - b)^2 + (y_1 - y_2)^2]^2\right\} dx_1\, dy_1, \tag{2}$$

where a is the displacement of the wave front of the radiation used for illuminating the opaque screen; this displacement is caused by the change of the tilt before the second exposure.

The first-exposure distribution of the complex amplitude for the spatially limited reference wave in the hologram plane can be written in the form

$$u_{01}(x_2, y_2) \sim \exp ik\left\{\frac{1}{2l}[(x_2 + c)^2 + y_2^2] - \right.$$

$$\left. - \frac{1}{8l^3}[(x_2 + c)^2 + y_2^2]^2\right\}, \tag{3}$$

where $c = l\sin\theta$ is taken for the sake of brevity; θ is the angle between the axis of the spatially limited reference wave and the normal to the plane of the photographic plate.

The second-exposure distribution of the complex amplitude of the reference wave in the plane (x_2, y_2) is described by the equation

$$u_{02}(x_2, y_2) \sim \exp ik\left\{\frac{1}{2l}[(x_2 + c + b)^2 + y_2^2] - \right.$$

$$\left. - \frac{1}{8l^3}[(x_2 + c + b)^2 + y_2^2]^2\right\}. \tag{4}$$

If the double-exposure hologram is recorded within the linear part of the blackening curve of a photographic material and the diffracted waves are spatially separated,[3] then the distribution of the complex amplitude in the plane (x_2, y_2) for the component corresponding to the (−1) diffraction order for $b = l\sin\alpha$ takes the form [based on Eqs. (1)–(4)]

$$u(x_2, y_2) \sim \exp(-ikx_2\sin\theta)\exp\left(-\frac{ikc^2}{2l}\right)\times$$

$$\times \exp ik\left\{\frac{1}{8l^3}[(x_2 + c)^2 + y_2^2]^2 - \frac{1}{8l^3}(x_2^2 + y_2^2)^2\right\} \times$$

$$\times\left\{\iint\limits_{-\infty}^{\infty} t(x_1, y_1)\exp -i\varphi(x_1, y_1)\exp\left\{ik\left[\left(-\frac{1}{2R} + \frac{1}{2l}\right)\times\right.\right.\right.$$

$$\left.\left.\times (x_1^2 + y_1^2) + \left(\frac{1}{8R^3} - \frac{1}{8l^3}\right)(x_1^2 + y_1^2)^2\right]\right\} \times$$

$$\times \exp -i\psi_1(x_1, y_1; x_2, y_2)\exp[-ik(x_1x_2 + y_1y_2)/l]\,dx_1dy_1 +$$

$$+ \exp i\psi_2(x_2, y_2, b)\iint\limits_{-\infty}^{\infty} t(x_1, y_1)\exp -i\varphi(x_1 + a, y_1)\times$$

$$\times \exp\left\{ik\left[\left(-\frac{1}{2R} + \frac{1}{2l}\right)(x_1^2 + y_1^2) + \left(\frac{1}{8R^3} - \frac{1}{8l^3}\right)(x_1^2 + y_1^2)^2\right]\right\}\times$$

$$\times \exp -i\psi_1(x_1, y_1, x_2, y_2)\exp i\psi_3(x_1, y_1, x_2, y_2, b)\times$$

$$\left.\times \exp[-ik(x_1x_2 + y_1y_2)/l]\,dx_1\,dy_1\right\}, \tag{5}$$

where

$$\psi_1(x_1, y_1, x_2, y_2) = \frac{k}{8l^3}(6x_1^2x_2^2 + 6y_1^2y_2^2 - 4x_1^3x_2 -$$

$$- 4x_1^2y_1y_2 + 2x_1^2y_2^2 - 4x_1x_2^3 - 4x_1x_2y_1^2 + 8x_1x_2y_1y_2 -$$

$$- 4x_1x_2y_2^2 + 2x_2^2y_1^2 - 4x_2^2y_1y_2 - 4y_1^3y_2 - 4y_1y_2^3);$$

$$\psi_2(x_2, y_2, b) = \frac{k}{8l^3}(12x_2^2bc + 4y_2^2bc +$$

$$+ 12x_2bc^2 + 12x_2b^2c);$$

$$\psi_3(x_1, y_1, x_2, y_2, b) = k\left\{\left[-\frac{1}{8R^3}\left(\frac{R}{l}\right) + \frac{1}{8l^3}\right]4x_1^3b + \right.$$

$$+\left[\frac{1}{8R^3}\left(\frac{R}{l}\right)^2 - \frac{1}{8l^3}\right]6x_1^2b^2 + \left[-\frac{1}{8R^3}\left(\frac{R}{l}\right) + \frac{1}{8l^3}\right]4x_1y_1^2b +$$

$$\left. + \left(\frac{1}{8l^3}\right)4x_1b^3 + \left[\frac{1}{8R^3}\left(\frac{R}{l}\right)^2 - \frac{1}{8l^3}\right]2y_1^2b^2\right\} -$$

$$- \frac{k}{8l^3}(12x_1^2x_2b - 12x_2^2x_1b - 12x_1x_2b^2 -$$

$$- 4x_1y_2^2b + 4y_1^2x_2b - 8y_1y_2x_2b + 8x_1y_1y_2b - 4y_1y_2b^2).$$

In the equation for the function $\psi_2(x_2, y_2; b)$, the constant component is rejected, because it is insignificant for further consideration.

Let the double-exposure hologram be reconstructed by a copy of the reference wave corresponding, for example, to the first exposure. For this wave the distribution of the complex amplitude $u'_{01}(x_2, y_2)$ is described by Eq. (3). Assume that the diffraction field is spatially filtered in the hologram plane by the opaque screen p (see Fig. 1) with a round aperture and the lens L with the focal length f is in the plane (x_2, y_2). Then the distribution of the complex amplitude of the diffraction field in the plane (x_3, y_3) is

$$u(x_3, y_3) \sim \iint_{-\infty}^{\infty} p(x_2 + x_{02}, y_2 + y_{02}) u'_{01}(x_2, y_2) \times$$
$$\times\, u(x_2, y_2) \exp ik \left[-\frac{1}{2f}(x_2^2 + y_2^2) + \frac{1}{8f^3}(x_2^2 + y_2^2)^2 \right] \times$$
$$\times \exp ik \left\{ \frac{1}{2l_1}[(x_2 - x_3)^2 + (y_2 - y_3)^2] - \right.$$
$$\left. - \frac{1}{8l_1^3}[(x_2 - x_3)^2 + (y_2 - y_3)^2]^2 \right\} \mathrm{d}x_2 \mathrm{d}y_2, \qquad (6)$$

where $p(x_2 + x_{02}, y_2 + y_{02})$ is the function of transmittance of the opaque screen with a round hole,[4] whose center is at the point with the coordinates x_{02} and y_{02}; l_1 is the distance between the planes (x_2, y_2) and (x_3, y_3).

Since scaling does not change significantly the results, we assume for brevity that the lens L (see Fig. 1) forms the image of the opaque screen in the plane (x_3, y_3) with the magnification factor equal to unity, i.e., $f = l/2$. It follows from Eq. (5) that the interference pattern caused by off-axis aberrations of the reference wave localizes in the plane of the double-exposure hologram.[5] Therefore, assume that the width of its fringe does not exceed the diameter of the filtering aperture. Then, assuming that the functions $\psi_1(x_1, y_1; x_2, y_2)$ and $\psi_3(x_2, y_1; x_2, y_2; b)$ take, respectively, the values $\psi_1(x_1, y_1; x_{02}, y_{02})$ and $\psi_3(x_1, y_1; x_{02}, y_{02}; b)$ because of the small diameter of the filtering aperture and substituting Eqs. (3) and (5) into Eq. (6), we obtain

$$u(x_3, y_3) \sim \exp ik \left[\frac{1}{2l}(x_3^2 + y_3^2) - \frac{1}{8l^3}(x_3^2 + y_3^2)^2 \right] \times$$
$$\times \left\{ \left\{ t(-x_3, -y_3) \exp -i\varphi(-x_3, -y_3) \exp \left\{ ik \left[\left(-\frac{1}{2R} + \frac{1}{2l} \right) \times \right.\right.\right.\right.$$
$$\left.\left. \times\, (x_3^2 + y_3^2) + \left(\frac{1}{8R^3} - \frac{1}{8l^3} \right)(x_3^2 + y_3^2)^2 \right] \right\} \times$$
$$\times \exp -i\psi_1(-x_3, -y_3; x_{02}, y_{02}) +$$
$$+\, t(-x_3, -y_3) \exp -i\varphi(-x_3 + a, -y_3) \times$$
$$\times \exp \left\{ ik \left[\left(-\frac{1}{2R} + \frac{1}{2l} \right)(x_3^2 + y_3^2) + \left(\frac{1}{8R^3} - \frac{1}{8l^3} \right)(x_3^2 + y_3^2)^2 \right] \right\} \times$$
$$\times \exp -i\psi_1(-x_3, -y_3; x_{02}, y_{02}) \times$$
$$\left. \times \exp i\psi_3(-x_3, -y_3, x_{02}, y_{02}, b) \right\} \otimes P(x_3, y_3) \otimes \Phi(x_3, y_3) \right\}, \qquad (7)$$

where symbol $\otimes$ denotes convolution;

$$P(x_3, y_3) = \iint_{-\infty}^{\infty} p(x_2 + x_{02}, y_2 + y_{02}) \times$$
$$\times \exp[-ik(x_2 x_3 + y_2 y_3)/l]\, \mathrm{d}x_2 \mathrm{d}y_2 ;$$
$$\Phi(x_3, y_3) = \iint_{-\infty}^{\infty} \exp -i\psi_4(x_2, y_2, x_3, y_3) \times$$
$$\times \exp[-ik(x_2 x_3 + y_2 y_3)/l]\, \mathrm{d}x_2 \mathrm{d}y_2$$

are Fourier transforms of the corresponding functions;

$$\psi_4(x_2, y_2; x_3, y_3) = \frac{k}{8l^3}(-6x_2^4 - 6y_2^4 - 12x_2^2 y_2^2 +$$
$$+ 6x_2^2 x_3^2 + 6y_2^2 y_3^2 - 4x_2^3 x_3 - 4x_2^2 y_2 y_3 + 2x_2^2 y_3^2 -$$
$$- 4x_2 x_3^3 - 4x_2 x_3 y_2^2 + 8x_2 x_3 y_2 y_3 - 4x_2 x_3 y_3^2 +$$
$$+ 2x_3^2 y_2^2 - 4x_3^2 y_2 y_3 - 4y_2^3 y_3 - 4y_2 y_3^3).$$

If in Eq. (7) the period of the function

$$\exp -i\varphi(-x_3, -y_3) + \exp -i\varphi(-x_3 + a, -y_3) \times$$
$$\times \exp i\psi_3(-x_3, -y_3, x_{02}, y_{02}, b)$$

exceeds by at least an order of magnitude[6] the width of the function $P(x_3, y_3) \otimes \Phi(x_3, y_3)$, which determines the size of a subjective speckle in the plane (x_3, y_3), then it can be removed out from the convolution integral. Then the distribution of illumination in the recording plane *3* (see Fig. 1) is determined by the equation

$$I(x_3, y_3) \sim \{1 + \cos[\varphi(-x_3 + a, -y_3) - \varphi(-x_3, -y_3) -$$
$$- \psi_3(-x_3, -y_3, x_{02}, y_{02}, b)]\} \,|\, t(-x_3, -y_3) \times$$
$$\times \exp \left\{ ik \left[\left(-\frac{1}{2R} + \frac{1}{2l} \right)(x_3^2 + y_3^2) + \left(\frac{1}{8R^3} - \frac{1}{8l^3} \right)(x_3^2 + y_3^2)^2 \right] \right\} \times$$
$$\times \exp -i\psi_1(-x_3, -y_3; x_{02}, y_{02}) \otimes P(x_3, y_3) \otimes \Phi(x_3, y_3)\,|^2. \qquad (8)$$

From Eq. (8) it follows that the shear interferogram in the infinitely wide bands is formed in the screen image plane. This interferogram characterizes the controlled wave front and modulates the subjective speckle structure. In the general case, the interference pattern is distorted due to aberrations in the object channel, and this gives rise to errors in the control. If the diffraction field is spatially filtered on the optical axis in the hologram plane at the stage of hologram reconstruction, then the phase function ψ_3 takes the form

$$\psi_3(-x_3, -y_3; 0, 0; b) = k\left\{\left[\frac{1}{8R^3}\left(\frac{R}{l}\right) - \frac{1}{8l^3}\right]4x_3^3 b + \right.$$

$$+\left[\frac{1}{8R^3}\left(\frac{R}{l}\right)^2 - \frac{1}{8l^3}\right]6x_3^2 b^2 - \left(\frac{R}{l}\,4x_3 b^3\right) +$$

$$+\left[\frac{1}{8R^3}\left(\frac{R}{l}\right) - \frac{1}{8l^3}\right]4y_3^2 x_3 b +$$

$$\left. +\left[\frac{1}{8R^3}\left(\frac{R}{l}\right)^2 - \frac{1}{8l^3}\right]2y_3^2\, b^2\right\}.$$

The shape of interference fringes determined from the condition $\psi_3(-x_3, -y_3, 0, 0, b) = 2n\pi$, where $n = 0, 1, 2, \ldots$, corresponds to a spherical aberration,[7] which arises in the object channel in the case under consideration. For $R = l$ the spherical aberration is zero. If the diffraction field is filtered out on the optical axis, then the additional term

$$\frac{k}{8l^3}\,(12x_3^2 x_{02} b + 12x_3 x_{02}^2 b + 4y_3^2 x_{02} b + 4x_3 y_{02}^2 b +$$

$$+ 8x_3 y_3 y_{02} b + 8y_3 x_{02} y_{02} b + 12x_3 x_{02} b^2 + 4y_3 y_{02} b^2)$$

of the phase function $\psi_3(-x_3, -y_3; x_{02}, y_{02}; b)$ characterizes off-axis aberrations in the object channel, that take place regardless of whether $R = l$ or $R \neq l$.

In the case that the hologram under consideration is reconstructed by a small-aperture laser beam, whose direction forms the angle θ with the normal to the hologram plane, to make the image of the opaque screen brighter, let us use Eq. (5) and assume that for the lens L (see Fig. 1) the focal length f is equal to l. Then for the diameter of the laser beam reconstructing the hologram (this diameter is equal to the diameter of the filtering aperture of the diaphragm p) the distribution of illumination in the focal plane (x_3, y_3) of the lens L in the (−1) diffraction order is described by Eq. (8) neglecting changes in the distribution of speckles.

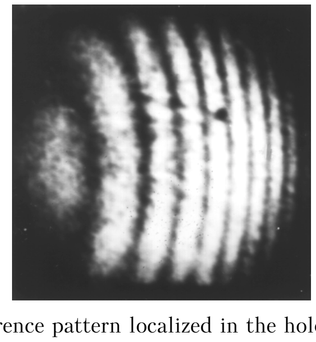
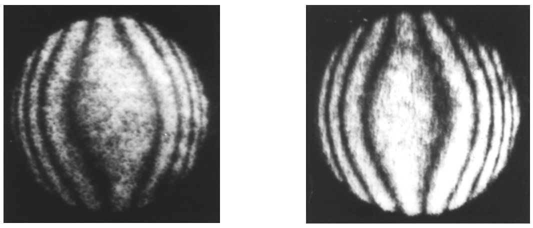


Fig. 2. Interference pattern localized in the hologram plane.

In the experiments, both Fourier and quasi-Fourier lensless hologram were recorded onto Mikrat VRL photographic plates using He–Ne laser radiation at the wavelength of 0.63 μm. As an example, Figure 2 shows the interference pattern localized in the hologram plane. It characterizes the off-axis aberrations in the reference channel.[5] This follows from the form of the phase function $\psi_2(x_2, y_2; b)$ for the spatially limited spherical wave with the tilt angle $\theta = 10°$. The distance between the opaque screen and the photographic plate was $l = 325$ mm. The diameter D of the controlled convergent wave front with the radius of curvature $R = 325$ mm was equal to 25 mm. Before the second exposure of the photographic plate, the tilt α of the controlled wave front was changed by $10'35'' \pm 3''$, and the photographic plate was displaced by $b = (1 \pm 0.002)$ mm. The interference pattern shown in Fig. 2 with the size $d = 60$ mm was recorded, according to Ref. 1, by spatially filtering out the diffraction field in the (−1) order on the optical axis in the screen image plane.

If the lateral displacement is further increased before the second exposure at the stage of double-exposure recording of the hologram in order to increase the sensitivity of the interferometer, then the spatial frequency of interference fringes in Fig. 2 increases too. This limits the capability of performing the spatial filtering of the diffraction field at different points in the hologram plane, because as the width of an interference fringe decreases for the interference pattern localized in the hologram plane, the diameter of the laser beam reconstructing the hologram should be decreased as well when recording the shear interferogram characterizing the wave front controlled. In its turn, the decrease of the beam diameter leads to an increase in the size of the subjective speckle in the screen image plane,[8] where the interference pattern localizes. As the speckle size increases, visibility of the interference pattern decreases, because the width of an interference fringe becomes comparable with the speckle size.



a *b*

Fig. 3. Shear interferograms recorded at spatial filtering in the hologram plane: on (*a*) and off (*b*) the optical axis.

The shear interferogram shown in Fig. 3*a* was recorded in the focal plane of the objective with $f = 50$ mm in the case that the diffraction field was filtered out in the plane of the double-exposure hologram on the optical axis by reconstructing the hologram with a small-aperture (≈2 mm) laser beam. It characterizes the spherical aberration in the paraxial focus of the controlled convergent quasispherical wave front with the radius of curvature $R = l$. In the case that the diffraction field is filtered at the point with the coordinates $x_{02} = 28.2$ mm and $y_{02} = 0$, the interference pattern to be filtered is shown in Fig. 3*b*. As is seen from Fig. 3*b*, the number of interference fringes increases by one to the right from the optical axis as compared to the number of fringes in Fig. 3*a*. The left part of the interference pattern changes less significantly. Such asymmetric character of distortion of the interference pattern seen in Fig. 3*b* is explained by the presence of aberrations like coma in the object channel.

Actually, at $R = l$ we can omit in the phase function $\psi_3(-x_3, -y_3; x_{02}, y_{02}; b)$ in Eq. (8) the terms $12x_3x_{02}b^2$ and $4y_3y_{02}b^2$ because of their higher order of smallness. Then

$$\psi_3(-x_3, -y_3; x_{02}, y_{02}; b) = -\frac{kb}{8l^3}(12x_3^2x_{02} + 12x_3x_{02}^2 + 8x_3y_3y_{02} + 8y_3x_{02}y_{02} + 4x_3y_{02}^2 + 4y_3^2x_{02})$$

is caused by the above-mentioned type of aberration. In this case, the phase function $\psi_3(-x_3, -y_3; x_{02}, y_{02}; b)$ increases asymmetrically with the distance from the optical axis, and relatively large changes of $\psi_3(-x_3, -y_3; x_{02}, y_{02}; b)$ occur, when the diffraction field is filtered in the hologram plane on the displacement axis. Therefore, to estimate distortions of the controlled wave front due to coma aberration in the object channel, assume, for example, that the phase function $\psi_3(-x_3, -y_3; x_{02}, 0; b)$ changes by 2π at the edge of the controlled wave front on the displacement axis. Then solving the equation

$$(D/2)x_{02}^2 + (D/2)^2x_{02} - 2\lambda l^3/3b = 0,$$

where λ is the wavelength of the coherent radiation used for recording and reconstruction of the hologram, we can find the coordinate of a hologram point, at which the above condition is fulfilled at spatial filtering of the diffraction field. For the above-listed parameters of the recording scheme $x'_{02} = 28.2$ mm and $x''_{02} = -40.7$ mm. The coordinate x''_{02} is beyond the hologram and of the spatial domain meeting the used order of approximation for $D = 25$ mm and $d = 60$ mm.

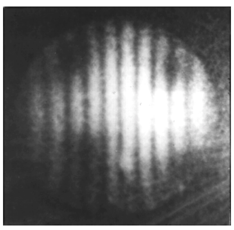


Fig. 4. Shear interferogram characterizing spherical aberration in the object channel.

To study the spherical aberration in the object channel, the opaque screen at the stage of double-exposure record of the quasi-Fourier hologram was illuminated by a divergent spherical wave with the radius of curvature in the screen plane $R = 165$ mm. For this purpose, spatial filtering[3] was performed as in the reference channel, i.e., $\varphi(-x_3, -y_3) = 0$ in Eq. (8). The distance between the opaque screen and the photographic plate remained the same as in the previous case ($l = 325$ mm), but the diameter of the controlled wave front was $D = 50$ mm. Besides, the change in the wavefront tilt before the second exposure was $10'35'' \pm 3''$, and the photographic plate was displaced by (0.96 ± 0.002) mm.

The interference pattern shown in Fig. 4 localizes in the opaque screen image plane. It was recorded as the diffraction field was filtered in the hologram plane on the optical axis. This pattern characterizes spherical aberration with hyperfocal focusing in the object channel. The periodic component of the interference pattern in Fig. 4 is caused by the displacement of the photographic plate before the second exposure by 0.04 mm from its position, at which the shear interferogram in infinitely wide bands is formed in the screen image plane. The spherical aberration arising in the object channel can be estimated using the equation for the phase function $\psi_3(-x_3, -y_3; x_{02}, y_{02}; b)$ with allowance made for the sign of the radius of curvature of the controlled wave front and the fact that the hologram is reconstructed at the point with coordinates $x_{02} = 0$ and $y_{02} = 0$. Then

$$\psi_3(-x_3, -y_3; 0, 0; b) = k\left\{\left[\frac{1}{8R^3}\left(\frac{R}{l}\right) - \frac{1}{8l^3}\right]4x_3^3b + \left[\frac{1}{8R^3}\left(\frac{R}{l}\right)^2 - \frac{1}{8l^3}\right]6x_3^2b^2 - \left(\frac{1}{8l^3}\right)4x_3b^3 + \left[\frac{1}{8R^3}\left(\frac{R}{l}\right) - \frac{1}{8l^3}\right]4y_3^2x_3b + \left[\frac{1}{8R^3}\left(\frac{R}{l}\right)^2 - \frac{1}{8l^3}\right]2y_3^2b^2\right\}$$

and for the given values of λ, b, R, l, and D we obtain that the interference pattern in Fig. 4 characterizes deviation from the spherical surface by a wavelength on the edge of the screen image on the displacement axis. This corresponds to the results of interpretation of the shear interferogram. As the diffraction field is filtered in the plane of the considered double-exposure quasi-Fourier hologram at a point off of the optical axis, the filtered interference pattern is changed due to coma aberration in the object channel.

To estimate the acceptable diameter D_{max} of the controlled wave front in the case that spatial filtering is performed on the optical axis within its small part at the stage of hologram reconstruction, we assume that the deviation from the spherical wave surface in the area of the diameter D_{max} must not exceed one tenths of wavelength. Then D_{max} must satisfy the condition

$$\left\{\left[\frac{1}{8R^3}\left(\frac{R}{l}\right) - \frac{1}{8l^3}\right]4(D_{max}/2)^3b + \left[\frac{1}{8R^3}\left(\frac{R}{l}\right)^2 - \frac{1}{8l^3}\right] \times 6(D_{max}/2)^2b^2 - \left(\frac{1}{8l^3}\right)(D_{max}/2)4b^3\right\} \le 0.1\lambda$$

(for a convergent wave front the sign of the radius of curvature should be altered). In this case, we should keep in mind that for the approximation used $(D_{max}/2) \le \sqrt[6]{4.8\lambda l^5}$.

Thus, the results of the theoretical and experimental studies showed that in the case of double-exposure records of the lensless quasi-Fourier hologram of an opaque screen for wave front control the sensitivity range of the shear interferometer is limited because of the off-axis aberrations of the reference spherical wave. Besides, due to aberrations in the object channel, control errors are possible. To exclude these errors, the diffraction field should be spatially filtered out at the stage of hologram reconstruction in the hologram plane on the optical axis with the allowance for the restrictions imposed on the diameter of the controlled wave front.

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