WENTZEL–KRAMERS–BRILLOUIN (WKB) APPROXIMATION IMPROVED FOR LARGE SCATTERING ANGLES

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A modification of the WKB approximation is proposed, which allows its correct application to optically “softB particles at large scattering angles (larger than 90 – 100°). Capabilities of the modified WKB approximation are demonstrated as applied to calculation of the scattering phase function (or its element f_{11}) for a cylindrical particle in comparison with the standard WKB and Rayleigh–Gans–Debye approximations. Expressions for calculation of backscattering cross section of the cylindrical particle are derived in the standard and modified WKB approximations.

1. INTRODUCTION

Light scattering methods have long been successfully used in research on atmospheric and oceanic optics, medicine, and biology. Interpretation of results of such research requires some models of objects under study to be applied. Besides, because strict calculations of light scattering characteristics for particles of random shape and structure are very complicated and sometimes even impossible, numerous approximations are used.[1]

For so-called optically soft scattering particles ($|n-1| \ll 1$, where n is the relative refractive index of a particle) the sufficiently strict WKB approximation can be used. This approximation, as proved in Refs. 2 and 3, includes both the Rayleigh–Gans–Debye (RGD) approximation and the anomalous diffraction (AD) approximation, as well as the Fraunhofer diffraction approximation.

However, the WKB approximation adequately describes differential light scattering characteristics mostly for small scattering angles $\beta < 40$–$50°$. Therefore, in this paper an attempt is undertaken to use the slightly modified WKB approximation for cylindrical particles at large scattering angles. The grounds for this modification and its consequences for spherical particles have been considered in Ref. 4.

2. LIGHT SCATTERING AMPLITUDE IN THE STANDARD AND MODIFIED WKB APPROXIMATIONS

Using the integral form of the light scattering amplitude, upon some regrouping, in the WKB approximation we have[5] (for wave incidence along the z-axis):

$$f(\mathbf{s}, \mathbf{i}) = \frac{k^2}{4\pi} [-\mathbf{s} \times (\mathbf{s} \times \mathbf{e}_i)] \int (n^2 - 1) \times$$

$$\times \exp\left[ik \int_{z_1}^{z} (n-1)\,\mathrm{d}z - ik\mathbf{r}'(\mathbf{s} - \mathbf{i}) \right] \mathrm{d}V', \qquad (1)$$

where $\mathbf{s}$ and $\mathbf{i}$ are the unit vectors along the scattering and propagation directions, respectively; z_1 is the entrance coordinate of a point on the particle surface for the wave passing through the point $\mathbf{r}$; $k = 2\pi/\lambda$ is the wave number; λ is the wavelength in a dispersion medium; $\mathbf{e}_i$ is the unit vector in the direction of incident wave polarization.

As shown in Ref. 4, the expression for the light scattering amplitude in the modified WKB (MWKB) approximation has the doubled exponential factor in the integrand, which is responsible for the beam phase shift, that is

$$f^{\mathrm{M}}(\mathbf{s}, \mathbf{i}) = \frac{k^2}{4\pi} [-\mathbf{s} \times (\mathbf{s} \times \mathbf{e}_i)] \int (n^2 - 1) \times$$

$$\times \exp\left[2ik \int_{z_1}^{z} (n-1)\,\mathrm{d}z - ik\mathbf{r}'(\mathbf{s} - \mathbf{i}) \right] \mathrm{d}V'. \qquad (2)$$

The MWKB approximation is applicable provided that the following conditions are met[4]:

1) $|n-1| \ll 1/2$, that is, the condition of optical softness becomes stronger;

2) large scattering angles, that is, small angles $\beta \ll 1$ rad are excluded;

3) small phase shift $\Delta \le 4$–5, where $\Delta = 2\rho\,(n-1)$.

The MWKB approximation[4] gives more reliable results for spherical particles at large scattering angles than the RGD approximation does.

Consequences of application of the MWKB approximation to cylindrical particles are discussed below.

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3. SCATTERING AMPLITUDE AND PHASE FUNCTION FOR CYLINDRICAL PARTICLES

The expression for the scattering amplitude of a cylindrical particle (the symmetry axis along the incident beam) in the standard WKB approximation has been earlier obtained in Ref. 6:

$$f(\beta) = \frac{(kR)^2 H (n^2 - 1) \exp (ikH (n - 1)/2)}{2} \times$$
$$\times \frac{J_1 (kR \sin(\beta))}{kR \sin(\beta)} \frac{\sin(kH (n - \cos(\beta))/2)}{kH (n - \cos(\beta))/2}, \quad (3)$$

where R is the cylinder radius; H is its height.

Similarly, the MWKB approximation gives from Eq. (2)

$$f^{\mathrm{M}}(\beta) = \frac{(kR)^2 H (n^2 - 1) \exp (ikH (n - 1))}{2} \times$$
$$\times \frac{J_1 (kR \sin(\beta))}{kR \sin(\beta)} \frac{\sin(kH (2n - 1 - \cos(\beta))/2)}{kH (2n - 1 - \cos(\beta))/2}. \quad (4)$$

The scattering phase function [or its element f_{11} for the natural light (random polarization)] was calculated by the expression

$$f_{11}(\beta) = \frac{1 + \cos^2(\beta)}{2} k^2 |f(\beta)|^2, \quad (5)$$

where $|f(\beta)|^2$ is the square of the absolute value of the scattering amplitude from Eq. (3) or (4).

Using Eqs. (3) and (4) and taking into account Eq. (5), as well as the scattering amplitude in the RGD approximation,[5] we have calculated the scattering phase function in the RGD, WKB, and MWKB approximations for the cylindrical particle oriented along the sensing direction (Fig. 1). Values have not been scaled to the forward scattering direction. It is seen from Fig. 1 that the values differ widely for large scattering angles. In the contrary case of a disk-like particle, all the three approximations give close results.

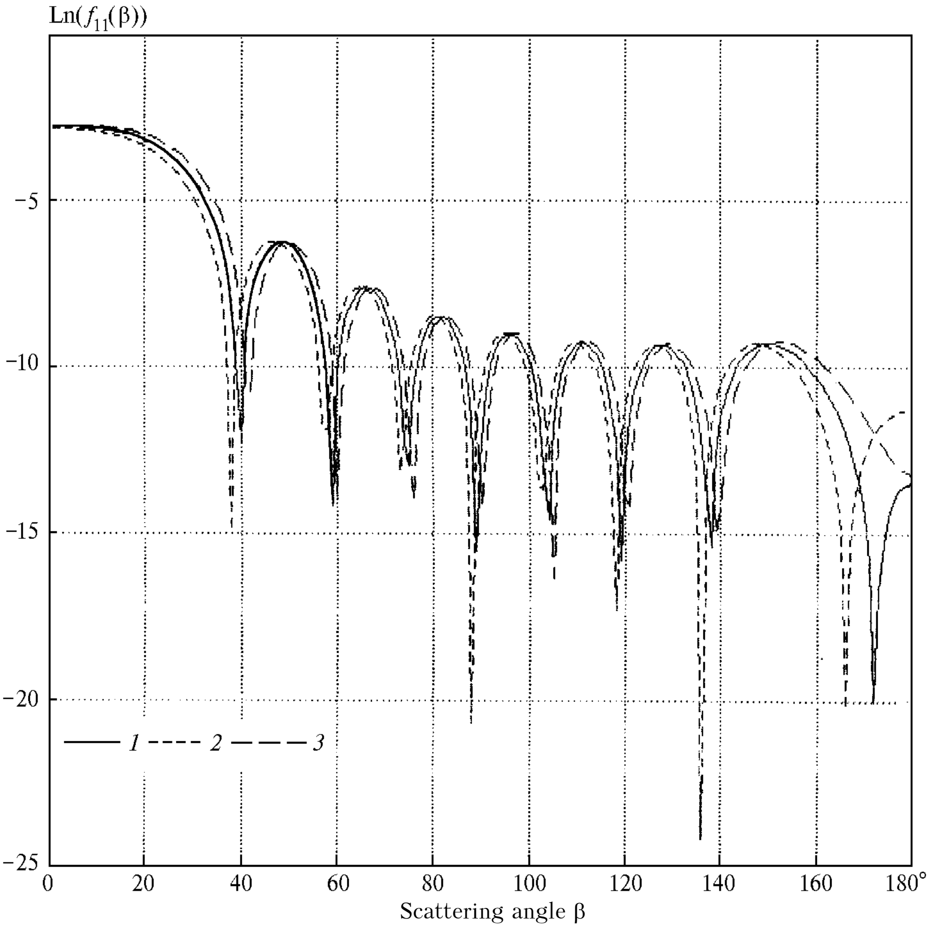

FIG. 1. Logarithm of the scattering phase function of the cylindrical particle Ln($f_{11}(\beta)$) *versus the scattering angle* β *in the WKB (1), RGD (3), and MWKB (2) approximations for* $kR = 1$ *and* $kH = 25$ *at* $n = 1.02$.

4. BACKSCATTERING CROSS SECTION OF CYLINDRICAL PARTICLES

Using the known general expression for the backscattering cross section[5] (scaled to the cross section area):

$$\frac{\sigma_{\mathrm{b}}}{\pi a^2} = \frac{4}{a^2} \, |f(-\mathbf{i}, \mathbf{i})|^2 \tag{6}$$

and Eq. (3), in the WKB approximation we have

$$\frac{\sigma_{\mathrm{b}}^{\mathrm{WKB}}}{\pi R^2} = \frac{(kR)^2 \, (kH)^2 \, (n^2 - 1)^2}{4} \times$$

$$\times \left(\frac{\sin(kH(n+1)/2)}{kH(n+1)/2}\right)^2, \tag{7}$$

while in the MWKB approximation, with use of Eq. (4):

$$\frac{\sigma_{\mathrm{b}}^{\mathrm{MWKB}}}{\pi R^2} = \frac{(kR)^2 \, (kH)^2 \, (n^2 - 1)^2}{4} \left(\frac{\sin(kHn)}{kHn}\right)^2, \tag{8}$$

and in the RGD approximation[5]:

$$\frac{\sigma_{\mathrm{b}}^{\mathrm{RGD}}}{\pi R^2} = \frac{(kR)^2 \, (kH)^2 \, (n^2 - 1)^2}{4} \left(\frac{\sin(kH)}{kH}\right)^2. \tag{9}$$

Note that under the standard condition $|n - 1| \ll 1$ (optical softness of a particle) with small error $n + 1 \approx 2$. Upon some transformations, it follows herefrom that the backscattering cross section in the WKB approximation (7) can be reduced to that in the RGD approximation (9).

5. CONCLUSION

The capabilities of the MWKB approximation are demonstrated as applied to calculation of the scattering phase function of the cylindrical particle (the symmetry axis along the incident light) in comparison with the standard WKB and Rayleigh–Gans–Debye approximations. The expressions for calculation of the backscattering cross section of the cylindrical particle in the modified and standard WKB approximations are derived. Based on the earlier calculations for spherical particles, we can expect that the expressions for backscattering cross section of the cylindrical particle derived in the MWKB approximation will yield more reliable results than those derived in the RGD and WKB approximations do.

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