Image reconstruction using preliminary estimates of the point spread function

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We propose an approach to deconvolution of satellite images of the Earth's surface recorded under conditions of atmospheric distortions. The specific feature of this approach is that the point spread function (PSF) used in the linear model of reconstruction is unknown and thus it ought to be estimated first. To do this, the image itself is used together with the information that the observed scene contains objects having certain brightness contrast. We use the Gumbel distribution of extrema as the stochastic model of degraded image fragments with high gradient. Otherwise, the Johnson curves are used in the description. The Bayes decision rule that uses these distributions isolates extremal gradients. The variations of brightness along the directions of gradients in the blurred part of the image serve a basis for reconstructing the PSF. The image itself is recovered using a standard approach. Illustrations to the PSF identification and image reconstruction are also given in the paper.

Introduction

In analysis of the Earth's surface images using algorithms of image recognition and automatic classification, the problem on pre-processing the video data to eliminate distortions caused by the atmospheric aerosols and aerosol formations (haze, fog) is very urgent. The task of video data correction itself is quite a complicated problem, because the transfer operator of the atmosphere, at the time of image recording, is, as a rule, unknown (in a model this operator is determined by the point spread function). The particular optical weather occurring at the time of a satellite overflight can differ significantly from the mean one. The problem arises on the PSF estimation or retrieval from the information just contained in the blurred image itself.

We will consider images of two types. The first type includes images of a sufficiently large number of scenes with surface objects with gradient jumps of radio brightness. The second type incorporates sufficiently homogeneous surfaces with quasistationary areas of roughness.

Depending on the type of an image, we shall consider two approaches to reconstruction of the PSF.[1] Assume that we can isolate, in some way, a fragment of a blurred image, within which the PSF can be considered constant, although being unknown. Let us consider the problem on reconstructing the PSF from the information extracted from the blurred image itself. Such a reconstruction method will be referred to as adaptive, since thus estimated PSF is adequate under particular conditions of observations accurate to errors of image recording. The following *a priori* assumption will be taken as a working hypothesis: boundaries of the physical objects within the observed fragment to be retrieved are sharp. Such boundaries are, for example, forest–ride, forest–road, river–riverside, ploughed land–field, and other surfaces. If the degree or character of these blurred boundaries is estimated in some way, then the PSF can be reconstructed as well. The "blurred" values of image gradients correspond to boundaries of physical objects, which must be clearly seen (without blurring). (At the same time, the image certainly includes objects with smoothly varying intensity, which have the whole spectrum of gradient values.) Thus, in the first approximation the following steps are needed in the algorithm for solution of this problem:

(a) isolation of a stationary area within a blurred image, within which the PSF keeps unchanged;

(b) differentiation of the obtained image and construction of the distribution histogram of the gradients to be estimated;

(c) splitting of the obtained histogram into two distributions with the corresponding weighting factors. One of these distributions (right-hand oriented) describes the distribution of extremal gradients (situation A_0), while the another (left-hand oriented) describes all other non-gradient brightness jumps (situation A_1). That is, the task is to identify the mixed distribution;

(d) construction of the "ayes decision rule for checking two hypotheses: H_1 is a gradient hypothesis, and H_0 is a non-gradient one. This rule reveals the areas in the video data, which correspond to boundaries of sharp but blurred brightness differences of high-contrast surface objects;

(e) scanning within only these areas of the video data for determining the degree of their blurring and thus obtaining "fragments" for reconstruction of the symmetric PSF;

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(f) once having the PSF obtained, one can reconstruct the image using one of the standard methods, for example, the method of inverse filter.

The second approach is based on the method of homomorphic filtration (Ref. 2, p. 254). In this case, the unknown PSF is estimated by making use of the signal and noise covariation functions. In their turn, these covariation functions are estimated using quasistationary areas of the video data.[1] Thus, in this case the information on blurring of the image texture is used. In this paper we do not consider the second method of image correction.

1. Spatial differentiation of images

Let us consider the search for gradient jumps of brightness and the approach to estimation of such differences. To do this, let us introduce the concept of the vicinity of an image point under analysis (Ref. 3, pp. 69–87). Assume, that the image to be analyzed is digitized, and the digital presentation has the form of a 2D matrix $\{z^{ji}\}$, where z^{ji} is the digitized brightness at the point (pixel) with coordinates (i, j) in the $M \times N$ plane of the observed image. The set of elements $\{z^{ji}\}$ of a local image area with the coordinates (i, j) falling within the square of $(2l + 1) \times (2l + 1)$ pixels will be called the fragment with the central element $(i = 0, j = 0)$ and the local coordinate system $-l \le i \le +l$, $-l \le j \le +l$, where l is the size of a window. To describe the behavior of brightness $\{z^{ji}\}$ within the fragment $(2l + 1) \times (2l + 1)$, we use the Haralic–Watson facet model, which was introduced in Ref. 3. In this model local image characteristics are described by the plane sections – facets. The equation of this plane in the Cartesian coordinate system has the form

$$Ax + By + Cz + D = 0 ,$$

and at $D = 0$ the plane goes through the origin of coordinates. The coefficients A, B, and C are equal to the x, y, and z-projections of the vector $\mathbf{A} = (A, B, C)^{\mathrm{T}}$ normal to the plane; $\mathbf{A}$ is the normalized directing vector of the plane; T denotes transposition. From here on we use the following version of the equation:

$$z = -\frac{A}{C}x - \frac{B}{C}y - \frac{B}{C} = \alpha x + \beta y + \mu , \qquad (1)$$

where $C \ne 0$. Let us draw a plane through the set of digitized intensities for the vicinity of some central point, so that the following square-law discrepancy criterion is minimum:

$$J(\alpha, \beta, \mu) = \sum_j \sum_i [\alpha i + \beta j + \mu - z^{ji}]^2 = \min_{(\alpha, \beta, \mu)} , \qquad (2)$$

where z^{ij} are the values of brightness in the vicinity of the central point with the local coordinates $i = 0$ and $j = 0$; the summation limits are $-l$ and $+l$. Upon differentiating $J(\,)$ with respect to the parameters to be estimated and making the partial derivatives to be equal zero, we obtain necessary conditions of the extremum

$$\begin{cases} \dfrac{\partial J(\alpha, \beta, \mu)}{\partial \alpha} = \sum\limits_j \sum\limits_i [\alpha i + \beta j + \mu - z^{ij}]\, i = 0, \\ \dfrac{\partial J(\alpha, \beta, \mu)}{\partial \beta} = \sum\limits_j \sum\limits_i [\alpha i + \beta j + \mu - z^{ij}]\, j = 0, \\ \dfrac{\partial J(\alpha, \beta, \mu)}{\partial \mu} = \sum\limits_j \sum\limits_i [\alpha i + \beta j + \mu - z^{ij}] = 0. \end{cases}$$

Removing the brackets in this equation and summing up, we obtain

$$\begin{cases} \alpha \sum\limits_j \sum\limits_i i^2 = \sum\limits_j \sum\limits_i z^{ji}\, i = \sum\limits_i i \sum\limits_j z^{ji} , \\ \beta \sum\limits_j \sum\limits_i j^2 = \sum\limits_j \sum\limits_i z^{ji}\, j = \sum\limits_j j \sum\limits_i z^{ji} , \\ \sum\limits_j \sum\limits_i \mu = \sum\limits_j \sum\limits_i z^{ji} . \end{cases} \qquad (3)$$

From this we find the estimates of the unknown parameters

$$\hat{\alpha} = \frac{3\sum\limits_i i \sum\limits_j z^{ji}}{l\,(l + 1)(2l + 1)^2} , \quad \hat{\beta} = \frac{3\sum\limits_j j \sum\limits_i z^{ji}}{l\,(l + 1)(2l + 1)^2} ,$$

$$\hat{\mu} = \frac{\sum\limits_j \sum\limits_i z^{ji}}{(2l + 1)^2} . \qquad (4)$$

Thus, the equation of the plane (1) has the following form:

$$z = \frac{3\sum\limits_i i \sum\limits_j z^{ji}}{l\,(l + 1)(2l + 1)^2} x + \frac{3\sum\limits_j j \sum\limits_i z^{ji}}{l\,(l + 1)(2l + 1)^2} y + \frac{\sum\limits_j \sum\limits_i z^{ji}}{(2l + 1)^2} ,$$

or, introducing the designations:

$$\underbrace{l\,(l + 1)(2l + 1)^2}_{\gamma'} z - \underbrace{3\sum_i i \sum_j z^{ji}}_{\alpha'} x - \underbrace{3 \sum_j j \sum_i z_{ij}}_{\beta'} y - \underbrace{-l\,(l + 1)\sum_j \sum_i z^{ji}}_{D} = 0 ,$$

$$\gamma' = l\,(l + 1)\,(2l + 1)^2, \quad \alpha' = -3 \sum_i i \sum_j z^{ji} ,$$

$$\beta' = -3 \sum_j j \sum_i z^{ji} , \quad D = l\,(l + 1)\sum_j \sum_i z^{ji} ,$$

we have

$$\gamma' z + \alpha' x + \beta' y + D = 0 .$$

The equation of this plane written in terms of direction cosines has the form

$$\cos\theta_z\, z + \cos\theta_x\, x + \cos\theta_y\, y + p = 0, \qquad (5)$$

where

$$\cos\theta_x = \frac{-3\sum_i i \sum_j z^{ji}}{\sqrt{(\gamma')^2 + (\alpha')^2 + (\beta')^2}},$$

$$\cos\theta_y = \frac{-3\sum_j j \sum_i z^{ji}}{\sqrt{(\gamma')^2 + (\alpha')^2 + (\beta')^2}},$$

$$\cos\theta_z = \frac{l\,(l+1)(2l+1)^2}{\sqrt{(\gamma')^2 + (\alpha')^2 + (\beta')^2}},$$

$$p = \frac{l\,(l+1)}{\sqrt{(\gamma')^2 + (\alpha')^2 + (\beta')^2}}.$$

The facet model of a fragment is used for isolation of gradient areas of the image. The value of the image gradient at some point (x_0, y_0) is estimated by the spatial variable, defined as the ratio of the area ds of the slant plane drawn through the set of brightness values of the ensemble of points forming the vicinity of (x_0, y_0) within a $(2l + 1) \times (2l + 1)$ square to the area of the base of this fragment dΔ. If writing the equation of the plane in terms of the direction cosines (5), we have the following estimate of the gradient:

$$\frac{\mathrm{d}s}{\mathrm{d}\Delta} = \frac{1}{|\cos\theta_z|} =$$

$$= \left[\left(\frac{3\sum_i i \sum_j z^{ji}}{l(l+1)(2l+1)^2}\right)^2 + \left(\frac{3\sum_j j \sum_i z^{ji}}{l(l+1)(2l+1)^2}\right)^2 + 1\right]^{1/2}. \qquad (6)$$

Thus obtained estimate of the gradient is assigned to the central point of the fragment with the local coordinates $i = 0$ and $j = 0$. ”y performing similar differentiation in each fragment of the image under analysis and assigning the values of the corresponding gradients to central elements of the sliding window of $(2l + 1) \times (2l + 1)$ pixels, we can pass from the initial image of brightness values to the image of gradients $\{w^{ji}\}$. The procedure of gradient determination introduced above possesses filtering properties, although it results in an additional smoothing of the parameters being estimated.

2. Decision rule for isolation of extremal gradients

To construct the ”ayes decision rule for detection and isolation of extremal values of gradients in the obtained gradient image, we need, first of all, to reconstruct the probability models of the situations that A_1 is a gradient and A_0 is not a gradient and estimate their *a priori* probabilities. Toward this end, we should split the obtained distribution histogram of gradients into two distributions, one of which being the distribution of extremal gradients and the other one the distribution of the rest non-extremal gradients. That is, we should identify the components of the following model:

$$g(x) = P\,f_0(x) + Q\,f_1(x)\,, \qquad (7)$$

where $f_1(x)$ is the distribution of extremal gradients; $f_0(x)$ is the distribution of non-extremal gradients; P and Q are the *a priori* probabilities of the situations A_0 and A_1, respectively; $P + Q = 1$, $x = w$. Then the problem of optimization of the square-law quality criterion arises in the following form:

$$J(\theta) = \frac{1}{m}\sum_{j=1}^{m}\{\tilde{f}(x_j) - P\,f_0(x_j) - Q\,f_1(x_j)\}^2\,, \qquad (8)$$

where $\tilde{f}(x_j)$ is the histogram of image gradients distribution; θ is the vector of unknown parameters consisting of the component P and parameters of the density functions $f_0(x)$ and $f_1(x)$ belonging to the parametric families of functions. It should be noted that the problem of reconstructing components of a mixture has a solution only if those can be identified. This condition can hardly be formalized and checked. From the geometrical point of view it means that $f_0(x)$ and $f_1(x)$ must have pronounced modes. Therefore, the fraction or measure of fragments with the extremal values of gradients should be large enough for the density function $f_1(x)$ to manifest itself.

The problem of splitting the components of a mixed distribution not always has a solution because of a high uncertainty. The *a priori* data on the component distributions should be involved. In this connection, let us refer to the following fact of mathematical statistics or, to be more precise, the theory of extremal values. It is known that the distribution density of maxima of n independent random values in the asymptotics of the growing number of observations $n \to \infty$ of type I [Gumbel distribution of maximum values (Refs. 4 and 5, p. 137)] has the following form:

$$f_1(x) = f_1(x;\mu,\sigma) = \frac{1}{\sigma}\exp\left[-\frac{1}{\sigma}(x-\mu) - \mathrm{e}^{-(x-\mu)/\sigma}\right],$$

$$-\infty < x < +\infty,\ -\infty < \mu < +\infty,\ \sigma > 0, \qquad (9)$$

where μ is the parameter (mode) of the distribution center; σ is the distribution scale; and the estimated mathematical expectation $\hat{\mu}$ and variance $\hat{\sigma}$ are connected with μ and σ as $\hat{\mu} = \mu + 0.577\,\sigma$ and $\hat{\sigma} = 1.283\,\sigma$. As $f_0(x)$ we selected the Johnson distribution S_B with the parameters ε (lower boundary of x), λ (sample size), and the shape parameters η and γ. Since some parameters can be estimated from the data sample,[5] the vector of unknown parameters has actually only three components and $\theta = (P, \eta, \gamma)^T$, where T denotes transposition. To optimize the criterion (8), the adaptive methods of search for extremum are used.[6] As the mixture is identified, the ”ayes decision rule is applied to check two hypotheses: H_1 is a gradient and H_0 is not a gradient. This decision rule reveals all image areas with sharp, but blurred brightness boundaries:

$$u = \arg\max_{\{0,1\}} \{P\, f_0(x),\ Q\, f_1(x)\}\ , \qquad (10)$$

where u is the decision or the number of a hypothesis, $u \in \{0,1\}$.

The "ayes decision rule (10) transforms the image of gradients into the image of contour lines corresponding to the extremal values of gradients.

3. The reconstruction of a PSF and retrieval of the image

In the previous phase of the procedure we have revealed from the blurred image the areas with blurred boundaries, and it is of greatest interest for us linear sections of images. Let us consider the problems associated with reconstruction of the PSF using a blurred boundary.[1,2] The point spread function is assumed normalized and $\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} h(x, y)\, \mathrm{d}x\, \mathrm{d}y = 1$. In a projection to the plane zOx the PSF has the form $h_l(x) = \int\limits_{-\infty}^{\infty} h(x, y)\, \mathrm{d}y$. At the same time, in the initial 3D space we have a line or a slit spread function. For the edge of a half-plane, the intensity distribution along the direction normal to the edge is described by a sum of the line spread functions, so the obtained intensity has the following form[7]:

$$I(x) = \int\limits_{-\infty}^{x} h_l(u)\, \mathrm{d}u\ . \qquad (11)$$

Thus, if we know the edge spread function, then $h_l(x) = \dfrac{\mathrm{d}I(x)}{\mathrm{d}x}$. Assume that the PSF is axisymmetric. In this case the spatial PSF can be reconstructed using only one cross section – projection of the PSF onto a plane. Actually, the spectrum of the PSF projection onto the plane involving the Oz axis and coming at an angle θ to the axis Ox, or (because of the axial symmetry of the PSF) onto the plane zOx, has the form $G_l(u) = \int\limits_{-\infty}^{\infty} h_l(x)\, \mathrm{e}^{-iux}\, \mathrm{d}x$. The 2D Fourier transform of the PSF can be written as follows:

$$F(u, v) = \int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} h(x, y)\, \exp\{-\, i(ux + vy)\}\, \mathrm{d}x\, \mathrm{d}y. \qquad (12)$$

"y comparing F and G, we can see that $G(u) = F(u, v)\big|_{v=0}$. From this it follows that $h_l(x)$ is the cross section of $h(x, y)$ by the plane zOx and, because of the axial symmetry, an arbitrary central cross section. In this case, $h(x, y)$ can be reconstructed from the cross section $h_l(x)$ by setting the radius-vector $r = \sqrt{x^2 + y^2}$, that is, $h_l(r) = h(r)$. Thus, to reconstruct the PSF from the actual blurred image, one should estimate the edge spread function. Then it should be differentiated to obtain the PSF cross section.

Analyzing the image, note that actual physical objects, whose boundaries are step-wise functions of brightness jumps, are characterized by some value a of the brightness difference. So, if blurring of a unit step gives the intensity $I(x)$, then blurring of the step with the contrast a gives $aI(x)$. In this case $\dfrac{\mathrm{d}\{a\, I(x)\}}{\mathrm{d}x} = ah_l(x)$ and the coefficient a for each detected edge can be estimated by normalizing $a = \int\limits_{-\infty}^{\infty} ah_l(x)\, \mathrm{d}x$. The next step is to reveal edge blurring sections $I(x)$ coming through the points of maximum values of gradients and to obtain an average section:

$$I_{\mathrm{av}}(x) = \frac{1}{N} \sum_{i=1}^{N} I_i(x)\ , \qquad (13)$$

where N is the number of sections with high gradients found.

Let us first construct the local coordinate system for estimation of the edge blurring profile, which will be referred to as a section. Assume that $x = 0$ and $y = 0$ in the equation of the plane (5), then $z = -\, D/\gamma'$ is the "center" of the plane. Let us find the projection of the normal vector $\mathbf{r}$ onto the plane yOx, which makes the angle ψ with the Ox axis. This projection cannot be zero, because the isolated plane under consideration has a high gradient:

$$\psi = \arccos \frac{\cos \theta_x}{\sin \theta_z}\ ,$$

where θ_x, θ_y, and θ_z are the angles between the directing vector of the plane and Ox, Oy, and Oz axes, respectively. Thus, in the plane xOy we pass to the "unrolled" coordinate system and, making x' and $\mathbf{r}$ coincident, we obtain

$$x' = x \cos \psi + y \sin \psi,$$
$$y' = -\, x \sin \psi + y \cos \psi\ , \qquad (14)$$

where $x'\ y'\ z'$ is the new coordinate system associated with the section.

To obtain the brightness distribution over the section, we should determine the intensities of the blurred image along the coordinate axis Ox' in the direction of the gradient (14). To decrease the error due to noise, we should take several neighboring paths and average them over the area Δ (size of the PSF carrier). Then we should smooth the data by a spline, determine $aI(x)$, and differentiate $\dfrac{\mathrm{d}aI(x)}{\mathrm{d}x} = ah_l(x)$.

4. Example of a degraded image retrieval

To illustrate the proposed approach, we took the image of the surface recorded from the RESURS satellite with the resolution pixel of 45×45 m^2 size. This image and the fragment isolated for analysis is shown in Fig. 1.

Fig. 1. The initial image with the fragment isolated for correction.

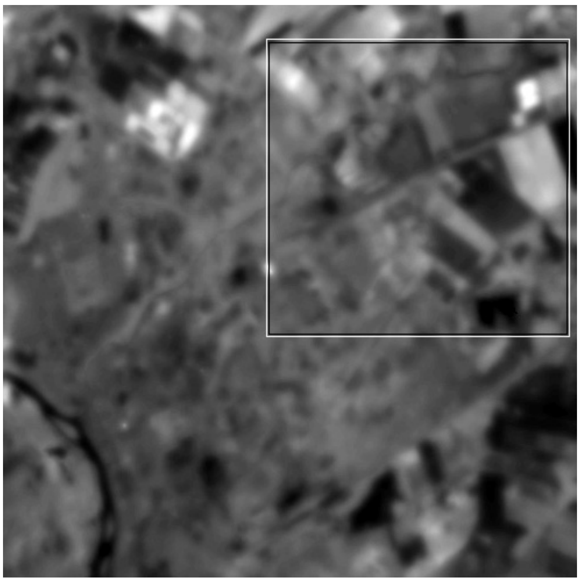


Fig. 2. Simulated degradation with the PSF in the form of weighted sum of two Gaussian curves.

We did not use data on the scale and geometry of the image and did not assign the image to any particular atmospheric conditions. Our purpose was only to illustrate the possibility of simultaneously estimating the PSF and reconstructing the blurred image. So all dimensions for simplicity are in pixels or digitized readouts. To simulate fog, the linear model of image convolution was used with the PSF in the form of a sum of two Gaussian density functions with the weights $p = 0.1$ and $C = 0.9$ and the rms deviations $\sigma_p = 1.0$ and $\sigma_q = 4.0$, respectively. The degraded version of the initial image is shown in Fig. 2, and its projection onto the plane of the model PSF is shown in Fig. 3 (curve *1*).

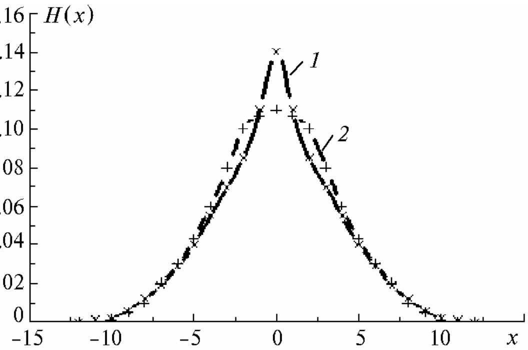


Fig. 3. Projection on the plane of the PSF simulating degradation (*1*) and reconstructed using sections (*2*).

The result of processing of the degraded image by the algorithm of spatial differentiation and isolated gradient differences of radio brightness is shown in Fig. 4. The histogram of the obtained gradient differences is shown in Fig. 5*a*. Then the observed spectrum of gradient differences was presented by a sum of two weighted density functions, the left-hand one being the model of non-gradient class (situation A_0) and the right-hand one is the model of a gradient class (situation A_1).

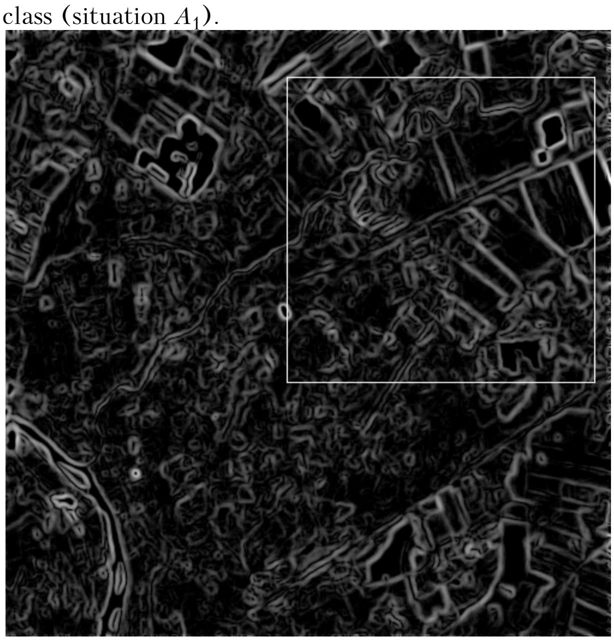
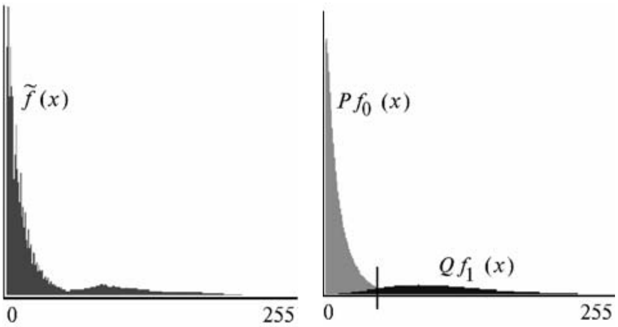


Fig. 4. Gradients of the degraded image.



a *b*

Fig. 5. Histogram of the gradient values distribution (*a*) and reconstructed Johnson and Gumbel distributions with the weights *P* and *Q* (*b*).

Figure 5*b* shows the gradient distribution under condition A_0, which was reconstructed using Johnson approximation S_B, and the gradient distribution under the condition A_1, which was reconstructed using Gumbel distribution of extremal values. The ” ayes decision rule applied to check the hypotheses m_0 and m_1 was constructed assuming these distributions. It isolates the extreme gradients of brightness, and thus the problem of revealing objects with distinct boundaries, which look blurred in the image, is solved. ” y scanning the blurred image in sections normal to the revealed contour lines of extremal brightness differences (Fig. 6), we can obtain separate realizations of the edge spread function (11) and (13), using which one can readily pass to the PSF.

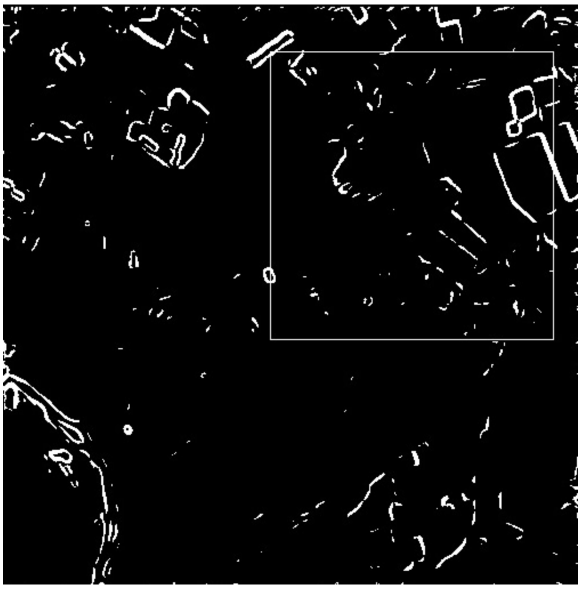


Fig. 6. Extremal gradients revealed by the Bayes decision rule.

Figure 3 shows the projection of thus reconstructed PSF onto the plane (curve *2*) and, for a comparison, the model PSF (curve *1*). These curves differ by 5% according to the square-law quality criterion. Then the image fragment was reconstructed with the synthesized PSF by the method of inverse filtration. This fragment is shown in the figures within the square frame. Figure 7 shows the reconstructed fragment of the blurred image (*a*) and, for a comparison, the initial image (*b*). The blurred and initial images differed by 10% according to the square-law quality criterion, whereas the recovered and the initial images differ by 4%. This demonstrates the efficiency of the approach proposed.

a *b*

Fig. 7. Fragment of the image retrieved by inversion of the PSF (*a*) and the corresponding fragment of the initial image (*b*).

Conclusion

The approach proposed for retrieval of degraded images is worth being used only in combination the *a priori* information of the following character. First of all, an area of homogeneously degraded video data characterized by the constant PSF must be found. ” esides, this area must contain objects with sharp, but observed as blurred, boundaries. In principle, using this technique it is easy to reconstruct the PSF with characteristics smoothly varying in space, but this strongly complicates image reconstruction.

Acknowledgments

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