MINIMIZATION OF PHASE DISTORTIONS FOR A LASER BEAM PROPAGATING THROUGH CONVECTIVE DRAFTS

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A closed model of laser beam self-action is constructed for conditions of self-induced convection in the penetrated medium. A numerical analysis is carried out of the structure of the radiation upon exit from the gas cell for a wide range of heat release parameter values. A quite simple algorithm is suggested for dynamic control of the beam wavefront at the cell entrance based on the use of the current distribution of the uncontrolled output phase.

Propagation of laser radiation through closed volumes is accompanied by various nonlinear effects, the most important among them being refraction at self-induced fluctuations of the medium density and temperature. Temperature gradients produced by absorption of radiation energy in such volumes lead to the development of convective drafts in the medium. Conversely, such convective movements alter the temperature- dependent refractive index distribution and thus affect the conditions of the beam propagation, so that the problem of thermal self-action becomes one of self-consistency.

The analysis of thermal defocusing under the conditions of self-induced convection presents quite a difficult mathematical problem; indeed, the convective velocity distribution cam only be retrieved by integrating the equations of hydrodynamics for the considered volume. Authors of studies in radiation-induced convection[1–3] have mostly analyzed the structure of gas or fluid flow in flat closed volumes, assuming these to be illuminated by a flat homogeneous light beam (in connection with questions of "mixing" and heat transfer). Studies 4 and 5 regard the case of convection induced by a limited laser pulse. The inverse effect of convection upon the propagating radiation was treated in Refs. 6 and 7 within the thin-lens approximation; more detailed calculations may be found in Ref. 8. Note that the last work considered the case of stationary convection only. However, it is non-stationary perturbations in particular which are important for propagation of light pulses of duration comparable to the characteristic setting-up time of convective drafts.

The present study is dedicated to a theoretical analysis of light beam propagation in the regime of nonstationary self-induced convection. A closed computational model was constructed to describe such self-action, including the Navier-Stokes equations and the equation for the complex amplitude of the light field. To control the incident radiation phase front a flexible mirror deformed by a system of lateral forces and torque moments was employed. The quality of such control was estimated by expanding the exit radiation phase into lower–order optical aberrations. A simple algorithm of dynamic compensation for nonstationary thermal self-action of the beam in convective drafts is studied.

1. MATHEMATICAL MODEL OF BEAM PROPAGATION

Let us consider the process of laser beam interaction with the medium. A light beam of initial radius a_0 is propagating along the axis of a horizontal gas-containing cell of length z_0 and lateral dimension l.

The longitudinal scale of the light field variations is assumed to substantially exceed the corresponding lateral scale. The same assumption is assumed to be valid for the velocity field and the temperature field of the medium, with the exception of boundary zones at the front ($z = 0$) and rear ($z = z_0$) walls of the cell. To quantitatively estimate the thickness of such boundary layers, we refer to the results of a dimensional analysis,[9] according to which the principal parameter for photoabsorptive convection is the dimensionless thermal complex:

$$q = \frac{\alpha I_0 a_0^5 \beta g}{\nu^3 \rho_0 C_p}, \qquad (1)$$

where α, β, ρ_0, and C_p are the molecular absorption and thermal expansion coefficients, the volume specific heat of the medium; v is the kinematic viscosity; g is the acceleration due to gravity; I_0 is the characteristic radiant power density at the cell entrance. As a rule, $q \gg 1$ in practice, so that the regime of developed convection is realized in the gaseous medium, with characteristic flux velocities

$$V_c = \nu q^{1/3}/a_0. \qquad (2)$$

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We find from (2) that the boundary layer thickness, defined according to Ref. 10 as $\delta \simeq \sqrt{Lv / V_k}$, does not, in most cases, exceed the beam radius a_0, thus satisfying the inequality $\delta \ll z_0$. Hence one may neglect boundary layer effects at both ends of the cell upon the velocity and temperature fields within the cell volume, and reduce the problem to its two-dimensional hydrodynamic analog. As is well known,[10] the motion of a viscous heat-conducting gas is described by the system of Navier-Stokes equations in Bussinesq's approximation, which, neglecting compressibility, may be written as

$$\frac{\partial \omega}{\partial t} + (\vec{V}\nabla)\omega = \frac{1}{\mathrm{Re}} \Delta\omega + q \frac{1}{\mathrm{Re}^3} \frac{\partial T}{\partial x}, \quad (3)$$

$$\frac{\partial T}{\partial t} + (\vec{V}\nabla)T = \frac{1}{\mathrm{PrRe}} \Delta T + f(x, y, z, t), \quad (4)$$

$$\Delta\psi = -\omega, \quad (5)$$

The differential operators ∇ and Δ are here taken to operate along the transverse coordinates x, y; the gas velocity $\vec{V}$ has two components: $\vec{V} = (V_x, V_y, 0)$; and the medium vorticity ω and current function ψ, satisfy the relations

$$\omega = \partial V_y/\partial x - \partial V_x/\partial y, \quad V_x = \partial\psi/\partial y, \quad V_y = -\partial\psi/\partial x.$$

Other notations are as follows: χ is the thermal diffusion coefficient; f is the laser beam intensity profile; $\mathrm{Re} = a_0 V_k/\nu$ is the Reynolds number; $\mathrm{Pr} = \nu/\chi$ is Prandt number. The independent variables in the dimensionless system of Eqs. (3)–(5) are normalized by the following scale factors: the coordinates x, y – by the laser beam radius a_0; the velocity V – by the characteristic velocity of developed convection V_c (2); the time t – by $t_0 = a_0/V_c$; the temperature T – by the scale $T_0 = a I_0 a_0/(\rho_0 C_p V_c)$; the vorticity ω – by $1/t_0$; the current function ψ – by $a_0 V_c$. The system is completed by the boundary conditions of zero gas velocity and constant temperature at the cell walls.

To integrate system (3)–(4) a two-step explicit Lax-Wendroff scheme[11] was employed and the Poisson equation (5) was solved using Hawkney's technique.[12]

Laser beam propagation through a weakly absorbing medium is described by the quasi-optical equation written for the complex slowly varying amplitude of the light wave $E(x, y, z, t)$. Using the above scales for the transverse coordinates and the temperature perturbations, and also introducing a longitudinal scale $z_d = ka_0^2$ (here k is the wave number), and an electric field intensity scale $A_0 = \sqrt{8\pi I_0 / (cn_0)}$, we may write the equation of beam propagation in its dimensionless form

$$2i \frac{\partial E}{\partial z} = \Delta_\perp E + RTE. \quad (6)$$

The nonlinearity parameter entering in Eq. 6

$$R = \frac{2k^2 a_0^3 \alpha I_0 (\partial n/\partial T)}{\rho_0 C_p V_k n_0} \quad (7)$$

is proportional to the radiant power $P = \pi a_0^2 I_0$ and the radiation-medium interaction time $t_0 = a_0/V_c$ in the transient convection regime. Thus the self-consistent problem of radiation-medium interaction is characterized by four parameters: R, Re, Pr, and q.

In addition, in the treatment of the problem of gravitational convection it is convenient to use such dimensionless parameters as the Grasshoff number $Gr = \beta g a_0^3 \alpha I_0 t_0 / (\rho_0 C_p v^2)$ and the Rayleigh number $\mathrm{Ra} = \mathrm{Gr} \cdot \mathrm{Pr}$. At the entrance to the cell ($z = 0$) the boundary conditions for the complex amplitude are given

$$E(x, y, 0, t) = E_0(x, y) F(t) \exp(iU(x, y, t)), \quad (8)$$

where $E_0(x, y)$ is the fixed amplitude profile of the beam; $F(t)$ is the temporal envelope of the light pulse; $U(x, y, t)$ is the controlled phase profile. In numerical simulations of self-action the amplitude profile is taken to be Gaussian:

$$E_0(x, y) = \exp\left(-\left(x^2 + y^2\right)/2\right), \quad (9)$$

and the following expression is used for the pulse envelope:

$$F(t) = \begin{cases} 0 \text{ at } t < 0, \\ 1 \text{ at } t \ge 0. \end{cases} \quad (10)$$

Equation (6) was solved by decomposing it into to its physical factors,[13] using the Fourier fast transform.[4]

2. EXIT PHASE ANALYSIS AND WAVEFRONT CONTROL

To analyze the spatial-temporal structure of the phase distortions in the beam exiting from the cell, we expand the outgoing wavefront $\varphi(x, y, z_0, t)$, minus its constant component, into a system of primitive optical aberrations

$$\varphi\left(x, y, z_0, t\right) = \sum_{k=1}^{5} a_k(t)\, Z_k(x, y), \quad (11)$$

where

$Z_1 = R_1^1(\rho) \sin v = y$ is the distortion,
$Z_2 = R_2^0(\rho) = 2\rho^2 - 1$ is the defocusing,
$Z_3 = R_2^2(\rho) \sin 2v = x^2 - y^2$ is the astigmatism,
$Z_4 = R_3^1(\rho) \sin v = (3\rho^2 - 2)y$ is the coma, and
$Z_5 = R_4^0(\rho) = 6p^4 - 6p^2 + 1$ is the spherical aberration.

Here ρ and v are the polar coordinates in the $z = z_0$ plane, and $R_n^m(\rho)$ are the radial Zernike polynomials.[5]

The expansion coefficients in the given basis (11) are given by the formulas

$$a_k(t) = \frac{1}{\|Z_k\|} \iint \varphi(x, y, z_0, t)\, Z_k(x, y)\, dx\, dy \qquad (12)$$

Here $\|Z_k\|$ is the norm of the kth mode. The extent of the beam phase distortions is also characterized by an integral criterion

$$J_{ph}(t) = \left\{ \frac{1}{\Omega} \int\int_{\Omega} \varphi^2(x, y, z_0, t)\, dx dy \right\}^{1/2}, \qquad (13)$$

recorded within an aperture of area Ω. In our calculations Ω was modeled as a circle with radius close to that of the beam.

In certain problems an estimate of the contribution of aberrations of the third and higher orders to J_{ph} is of separate interest. Such an estimate may be obtained by representing the phase φ as $\varphi = \sum_{k=1}^{3} a_k Z_k + \tilde{\varphi}$ and calculating the value of the criterion

$$\tilde{J}_{ph}(t) = \left\{ \frac{1}{\Omega} \int\int_{\Omega} \tilde{\varphi}^2(x, y, z_0, t)\, dx\, dy \right\}^{1/2}, \qquad (14)$$

To minimize the nonlinear distortions accumulated by the beam in the cell, we propose to introduce dynamical control of the input wavefront. One may use a flexible mirror, gimballed at its center and deformed by a system of lateral forces and torque moments (Fig. 1).

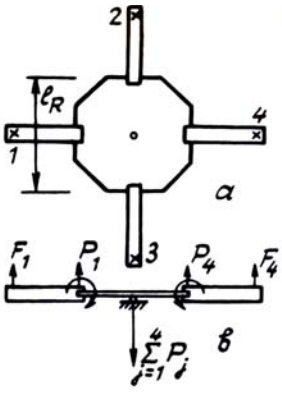

FIG. 1. Flexible mirror: a) top view; b) side view.

When the beam is almost normally incident upon the mirror, whose flexure is given by $\omega(x, y, t)$, the reflected beam accumulates a phase run-on

$$U(x, y, t) = 2k\, w(x, y, t). \qquad (15)$$

It is known from the theory of thin plates[16] that the flexure of the central area of such a plate, approximately coinciding with the flexure of the reflecting mirror surface. Is described by biharmonic equation

$$D\left[\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right] = Q(x, y), \qquad (16)$$

where D is the cylindrical rigidity of the plate; $Q(x, y)$ is its distributed lateral load. For the chosen mode of mirror deformation we have $Q = 0$, and the boundary conditions at its perimeter L may be written as follows:

$$D\left(\frac{\partial^2 w}{\partial n^2} + \nu \frac{\partial^2 w}{\partial s^2} \right)\Bigg|_L = \sum_{j=1}^{4} \frac{1}{L_j}\, \delta(\vec{r} - \vec{r}_j)\, M_{sj},$$

$$\frac{\partial}{\partial n}\left[D\left(\frac{\partial^2 w}{\partial n^2} + \nu \frac{\partial^2 w}{\partial s^2} \right)\Bigg|_L \right] = \sum_{j=1}^{4} \frac{1}{L_j}\, \delta(\vec{r} - \vec{r}_j)\, P_j. \qquad (17)$$

Here n and s are the normal and the tangent to the respective part of the mirror perimeter L_j; $\vec{r}_j$ is the radius vector of the application point of the lateral force P_j and the torque moment M_{sj} ($j = 1, 2, 3, 4$). At the central, gimballed point we have

$$w\Big|_{0,0} = \frac{\partial^2 w}{\partial x^2}\Big|_{0,0} = \frac{\partial^2 w}{\partial y^2}\Big|_{0,0} = 0. \qquad (18)$$

Calculations of the mirror flexure under the boundary conditions (17)–(18) were performed using the finite elements technique.[7]

3. SPATIAL-TEMPORAL STRUCTURE OF THE PULSE DISTORTIONS

Numerical experiments with collimated beams ($U = 0$), propagating through cells of $0.001 \le z_0 \le 0.01$ length, were performed with the heat release parameter q varying within the range $10^5 \le q \le 10^7$. The corresponding values of the parameter R were chosen from within the range $90 \le |R| \le 1800$.

The calculations demonstrated that under such conditions the temporal behavior of the beam exit phase profiles is quite conformative, so that they differ from each other mainly in the amplitude of their distortions as a function of the values of z_0, q, Re, and R. Figure 2 presents, as an example, a typical dependence of the phase expansion coefficients in the basis (11). It can be seen that the wavefront slope in the vertical plane a_1 and its defocusing a_2 are nonmonotonic, the maximum in a_1 being reached at $t \simeq 3t_0$, and the maximum in $|a_2|$ – at $t \simeq 2t_0$; following these maxima such aberrations start to decrease, reaching their stationary values at $t \simeq (4–5)\, t_0$. The astigmatism $|a_3|$ increases almost monotonically up to $t \simeq 4t_0$, following which it stabilizes. As a result the beam divergence in the vertical plane is markedly less than in the horizontal. Joint analysis of Figs. 2 and 3, where the solid lines represent the changes in the integral phase criteria J_{ph} and $\tilde{J}_{ph}$, shows that the aberrations of the orders higher than the second are quite insignificant for collimated beams.

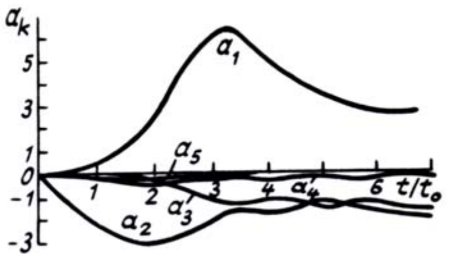

FIG. 2. Dynamics of lower-order aberrations of the wavefront upon exit from the cell. Propagation conditions: $z_0 = 0.05$; $R = -380$; $Re = 100$; $Gr = 10^4$; $Ra = 6 \cdot 10^3$.

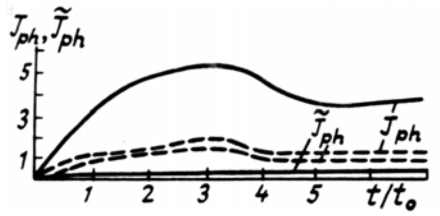

FIG. 3. Integral phase criteria J_{ph} *and* $\tilde{J}_{ph}$ *as functions of time. For propagation conditions see legend to Fig. 2. Solid curves — no control; dashed curves — controlled.*

4. EFFICIENCY OF PROCRAMMED CORRECTION OF PHASE DISTORTIONS

A very simple algorithm of programmed phase front correction may be described by the relation

$$U(x, y, t) = -\gamma\varphi(0, y, z_0, t), \tag{19}$$

where γ is an empirically selected coefficient. When realizing this algorithm numerically the problem of beam propagation through the cell is solved twice: first, to calculate and store the exit phase values of the uncontrolled (collimated) beam at every time step and every grid-point; and second, to model the dynamical control (19) and to analyze the residual phase distortions φ_{res}. To approximate the phase profile formed at the flexible mirror surface the least squares technique is employed.

Numerical experiments demonstrate that such programmed control (19) cam considerably reduce the phase distortions accumulated by the beam in the cell along beam paths not exceeding $z_0 = 0.05$ for $\gamma = 1$. As a result of such control the wavefront curvature and slope are reduced 3–6 times. The value of the integral phase front criterion J_{ph} is reduced by a factor of 2–3, the latter being less sensitive to local deviations of the beam front from a flat. However, the relative contribution of higher-order aberrations (cf. Fig. 3, dashed curves) increases during such control; the latter fact can apparently be explained by changes in the characteristic size of the thermal lens generated along the beam path.

On longer paths ($z_0 \geq 0.1$), when the diffractional and nonlinear phase run-ons become significantly non-additive, the efficiency of correction (19) at $\gamma = 1$ diminishes. In this case it would be advisable to optimize the correction amplitude γ; it would also be convenient to choose as the optimization criterion the temporally averaged value of the integral phase criterion:

$$\langle J_{ph} \rangle = \frac{1}{t_p} \int_0^{t_p} J_{ph}(t)\, dt. \tag{20}$$

An example of such an optimization is illustrated by Fig. 4, which presents the dependence of $\langle J_{ph} \rangle$ on the parameter γ for beam propagation through the cell at $z_0 = 0.1$. A distinct minimum can be observed, attained by this phase criterion in the range $\gamma \simeq 1.2$.

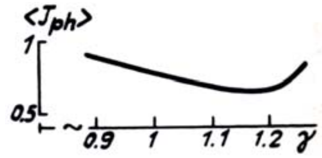

FIG. 4. Optimization of the phase correction amplitude δ. *Propagation conditions:* $z_0 = 0.1$; $R = -83$; $Re = 50$; $Gr = 2 \cdot 10^3$; $Ra = 1.2 \cdot 10^3$

Such an optimization is possible within a wide range of the variables z_0 and R; however, its capabilities are reduced for longer beam paths.

On the whole the above analysis convincingly demonstrates that simple programmed control of the beam wavefront using a flexible mirror with four servodrives efficiently compensates for nonlinear phase distortions in a horizontal cell without producing any significant aperture effects. As a rule, within the ranges of the variables considered, the relative deviations from the undisturbed (vacuum) values of such beam characteristics as energy center shift and exit energy radius of the beam are found to change sign after control is switched on, but do not exceed the respective uncontrolled values in their moduli.

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