Optimal beam focusing at the second harmonic generation in an uniaxial crystal. Approximation of a preset field

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The problem of the second harmonic generation (SHG) is considered in the preset-field approximation for the scalar «*ooe*»-synchronism in a KDP crystal for the fundamental radiation at $\lambda \geq 0.53$ μm. It is shown that the maximum of nonlinear conversion efficiency is reached at focusing the incident beam into the crystal using two crossed cylindrical lenses with the focal lengths being the unique optimal pair. In this case, wave detuning should be optimal as well. It is proved that the use of optimal cylindrical focusing increases the efficiency by approximately 25% as compared to the optical spherical focusing. Empirical expressions are proposed for approximate estimates of the optimal values of focal lengths and wave detuning. It has been revealed that the accuracy of these estimates keeps tolerable even if the initial parameters vary in quite a wide range.

Introduction

Within the framework of this study, we consider the steady-state second harmonic generation (SHG) mode in a homogeneous uniaxial crystal with a quadratic nonlinearity. Assume that the laser beam that propagates along the Z-axis of a Cartesian coordinate system is a spatially coherent and monochromatic radiation. Besides, restrict the consideration to the preset-field approximation and the scalar «*ooe*»-interaction. In this case, the slowly varying complex amplitudes $A_1(x, y, z)$ at the fundamental frequency and $A_2(x, y, z)$ at its second harmonic are solutions of the following equations[1,2]:

$$\frac{\partial A_1}{\partial z}+\frac{1}{2ik_{1o}}\left(\frac{\partial^2 A_1}{\partial x^2}+\frac{\partial^2 A_1}{\partial y^2}\right)=0, \qquad (1a)$$

$$\frac{\partial A_2}{\partial z}+\rho\frac{\partial A_2}{\partial x}+\frac{1}{2ik_2^e}\left(\frac{\partial^2 A_2}{\partial x^2}+\frac{\partial^2 A_2}{\partial y^2}\right)=i\sigma A_1^2 e^{-i\Delta_k z}, \qquad (1b)$$

where

$$k_{1o}=kn_o(\omega),\quad k_2^e=2kn^e(2\omega,\theta),\quad k=\omega/c,$$

θ is the angle between the optical axis of the crystal, lying in the plane XZ, and the Z-axis, $\Delta_k=k_2^e-2k_{1o}$ is the wave detuning, ρ is the birefringence angle, σ is the coefficient of nonlinear coupling.

The Green function of a homogeneous uniaxial medium is known (see, for example, Ref. 3), which allows Eq. (1) to be presented in the following form:

$$A_2(x_0,y_0,z,L_0)=iT_2\sigma\int_0^z e^{-i\Delta_k t}\left[-\frac{ik}{\pi t_L}\right]\times$$

$$\times\left[\int_{-\infty}^{+\infty}\int A_1^2(x,y,t)\, e^{ik\frac{(x-x_0+\rho L-\rho t)^2}{t_L}}\, e^{ik\frac{(y-y_0)^2}{t_L}}\,dx\,dy\right]dt, \qquad (2)$$

where T_2 is the Fresnel coefficient for the normal-incidence refraction at the exit facet of a crystal, $t_L = z_0 + (L - t)/n$ under the assumption that $n\approx n_o(\omega)\approx n^e(2\omega,\theta)$; we have taken into account that the field (2) passes the distance $L - z$ in an anisotropic medium and it refracts at the entrance and exit facets of the crystal.[4]

Equation (2) defines the SH field that arises at a distance z from the entrance to a crystal with the length L and then linearly propagates to the observation plane $L_0 = L + z_0$. If the distance z_0 from the crystal to this plane tends to infinity and the field at the fundamental frequency forms a Gaussian beam, then Eq. (2) easily reduces to the so-called Boyd–Kleinman formula.[5]

The method of more simple and easy-to-use presentation of the considered solution has been discussed in Ref. 6. If tending z_0 to infinity, as in Ref. 5, solution to Eq. (1) is proposed to be sought in the following form

$$A_1(x,y,L_0)=U_1(x,y,L_0)\,e^{ik\frac{x^2+y^2}{2R}}, \qquad (3a)$$

$$A_2(x,y,z,L_0)=U_2(x,y,z,L_0)\,e^{i2k\frac{(x-\rho L)^2+y^2}{2R}}, \qquad (3b)$$

where

$$R=\frac{1}{n}(\Delta_f+L/2)+z_0\approx z_0$$

is the radius of curvature of the wave front. In this case, Eq. (2) is replaced by

$$U_2(x_0,y_0,z,L_0)=i\sigma\left(\frac{ik}{T_2\pi z_0}\right)\int_0^z e^{-iQ_0t}\times$$
$$\times\left[\int\int_{-\infty}^{+\infty} e^{-ikV_t(x^2+y^2)}U_1(x_0-x,y_0-y,L_0)\times\right.$$
$$\left.\times U_1(x_0+x,y_0+y,L_0)\mathrm{d}x\mathrm{d}y\right]\mathrm{d}t=i\sigma z\left(\frac{ik}{T_2\pi z_0}\right)e^{-iQ_0z/2}\times$$
$$\times\int\int_{-\infty}^{+\infty} e^{-ikV_z(x^2+y^2)}U_1(x_0-x,y_0-y,L_0)\times$$
$$\times U_1(x_0+x,y_0+y,L_0)\mathrm{sinc}\left(\frac{Qz}{2}\right)\mathrm{d}x\mathrm{d}y, \quad (4)$$

where

$$Q_0=\Delta_k-2k\frac{x_0\rho}{z_0};\quad V_t=(\Delta_f-L/2+t)/nz_0^2,$$
$$V_z=(\Delta_f-L/2+z/2)/nz_0^2;\quad \mathrm{sinc}(x)\equiv\sin(x)/x,$$
$$Q=Q_0+k(x^2+y^2)/nz_0^2,$$

Δ_f is the distance between the waist plane and the crystal center ($\Delta_f > 0$ if the beam is focused before the crystal center. In the given case we don't go beyond the variant that $|\Delta_f| \le L/2$, which is most interesting in practice).

The main advantage of this result, against that given by Eq. (2), is that for its use the solution (1a), [i.e., the form of Eq. (3a)], has to be known only in the L_0-plane but not in all the crystal volume, as in the case with Eq. (2). This is not very important for Gaussian fields, while in the case of beams with more complicated amplitude forms the transition to Eq. (4) allows one to obtain the analytical result in simpler quadratures and correspondingly to shorten the time for its numerical computation.

In contrast to Eq. (2), Eq. (4) is obviously an approximate solution of Eq. (1). Therefore, finding the limits of its applicability is the first goal of this study. The second goal is to formulate the conditions providing maximum efficiency of the SHG process. All computations will be conducted for the KDP crystal described in Ref. 7. In this study, we restrict ourselves to the spectral range from 0.530 to 1.06 μm where the optimization problem of a nonlinear process is most easily solvable. The peculiarities of the SHG at angles of synchronism close to 90° (fundamental radiation wavelength close to 0.5174433... μm) will be considered in a different paper.

Assume that the laser radiation is focused to a nonlinear crystal by two crossed cylindrical lenses L_x and L_y. The L_x lens (focal length f_x) focuses the beam in the principle optical plane (XZ coordinate plane), while the L_y lens (focal length f_y) in the plane YZ. From Eq. (3a) it follows that in the most general case, the amplitude $U_1(x,y,L_0)$ can be presented, accurate to a constant phase shift, by the following approximation[6]:

$$U_1(x,y,L_0)=A_0\exp\left[-\left(\frac{x^2}{a_x{}^2}\right)^{m_x}\right]\exp\left[-\left(\frac{y^2}{a_y{}^2}\right)^{m_y}\right], \quad (5)$$

where $A_0=\sqrt{8\pi P/cI}$, P is the laser radiation power measured in the plane L_0;

$$I(L_0)=\int\int_{-\infty}^{+\infty}(U_1(x,y,L_0)/A_0)^2\mathrm{d}x\mathrm{d}y.$$

For all the following computations the power $P = 1$ kW. If $f_x = f_y$ ($a_x = a_y$) then the focusing is considered as spherical. In the general case ($f_x \ne f_y$), lenses L_x and L_y are to be shifted along the Z-axis so that their waists are in the same plane. It is for this reason that the radii of curvature of the wave front, in Eq. (3), turn out to be coincident in the planes XZ and YZ. It is assumed that m_x and m_y can take arbitrary positive values. However, one should keep in mind that simultaneous increase of both the coefficients m_x and m_y and the focal lengths f_x and f_y can result in an essential deviation of the approximation (5) from the exact representation of the field amplitude at the fundamental frequency. In case of such a situation, one should still use Eq. (3a) (Eq. (4) remains valid) and replace Eq. (5) by the exact solution (1a) in the observation plane L_0. This will not add any difficulty more, but the amplitude $U_1(x, y, L_0)$ in this case can of course be set only numerically.

The cylindrical laser radiation focusing to a nonlinear unaxial crystal (i.e., the conditions when the beam divergences in two mutually orthogonal planes differ essentially) has been successfully used for a long time. The most popular arrangement of cylindrical and combined focusing systems can be found in Ref. 1, as well as the detailed bibliography on this problem. Nevertheless, in these investigations (mostly experimental) we have found no recommendations on which focusing system is to be used to provide maximum SHG efficiency of a laser radiation with arbitrary specified parameters. Development of a technique for easy-to-make estimates of this type is the principal result of the research presented in this paper. The general theoretical aspects of the technique have been considered in Ref. 6, while the practical recommendations formulated following Ref. 6 are given in this paper.

1. Check of the approximate solution

To check the solution (4), consider the following task. Assume that the SHG is realized in a crystal with the length L = 4 cm. The laser beam has the wavelength λ = 0.5782 μm (one of the copper vapor lasing lines). Its amplitude distribution in the observation plane $L_0 = L + z_0$ is characterized by the coefficients $m_x = 4$ and $m_y = 3$. The radius a_0 of a laser beam incident on the focusing system taken to be equal to 1 cm. The laser radiation is focused to a nonlinear crystal by use of two crossed cylindrical lenses with the focal lengths $f_x = 460$ cm and $f_y = 150$ cm, Δ_f is chosen to be equal to 1.99 cm. The crystal is assumed to be oriented so that the wave detuning $\Delta_k = 3.98\ \text{cm}^{-1}$.

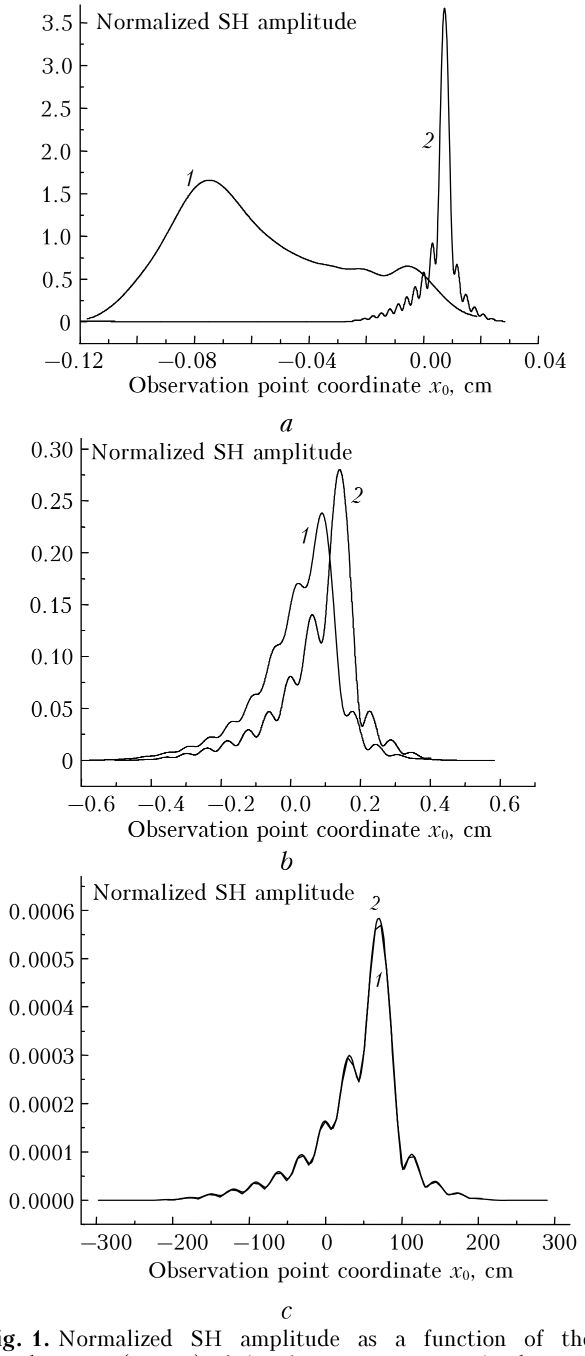

Fig. 1. Normalized SH amplitude as a function of the coordinate x_0 ($y_0 = 0$) of the observation point. The distance z_0 from the crystal to the observation point is 20 (*a*), 100 (*b*), and 100 000 cm (*c*); curves *1* present the exact solutions, curves *2* the approximate ones.

The above task has been solved by two different methods, namely by the method of direct integration of Eq. (1) using the method of splitting physical factors and applying fast Fourier transform[8,9] (exact solution) and with the use of Eq. (4) (approximate solution). The calculated results are shown in Fig. 1 as SH amplitude dependences on the coordinate x_0 ($y_0 = 0$) of a point of observations in the plane L_0 (curve *1* corresponds to the exact solution and curve *2* – to the approximate one). The value $U_2(x_0, y_0, z, L_0)$ was normalized to the amplitude A_{00} of the fundamental radiation at the entrance of the focusing system. The computations have been performed for $z_0 = 10$, 200, and 100000 cm.

From the results shown, it follows that the approximate solution (4) steadily tends to the exact one while the observation plane moves away from the exit crystal facet. In this case, the results of these two methods become indistinguishable (for chosen m_x, m_y and f_x, f_y) beginning from $z_0 \approx 10^5$ cm. Taking this into account, all calculations that follow were made using $z_0 = 10^5$ cm assuming that possible errors can be neglected.

2. Optimizing the SHG process

Let us formulate rigorous statement of the problem in the following way. Consider the parameters of a laser radiation ($\lambda, P, m_x, m_y, a_0$) and of a nonlinear crystal (L, $n(\lambda, \theta)$, $\sigma(\lambda, \theta)$) to be known *a priori*. The aim of our study is to find such values of

$$\Delta_f = \Delta_{fop},\ \Delta_k = \Delta_{kop},\ f_x = f_{xop},\ \text{and}\ f_y = f_{yop},$$

at which efficiency of SHG reaches its maximum. Ideally, the result should be both explicit dependences among the optimized parameters and their dependences on all the initial parameters.

Different researchers have studied various special cases of the general problem stated. In our opinion, the most complete but not comprehensive are the results presented in Ref. 5, where the SHG was optimized at the spherical focusing of a Gaussian beam into an uniaxial crystal. The principal, for practical purposes, result is formulated as follows. The maximum of SHG efficiency is achieved at the so-called 90°-synchronism ($\rho = 0$) if the following conditions are fulfilled:

$$\Delta_{fop} = 0; \quad \frac{\Delta_{kop} L}{2} = -1.6; \quad \xi_{op} = \frac{kL}{4n}\left(\frac{a_0}{f_{op}}\right)^2 = 2.84. \quad (6)$$

In this paper, we describe the results of the study, which is somewhat similar to those in Ref. 5, but for a more general problem. In general, a laser beam may not be Gaussian and a focusing system may consist of two cylindrical lenses. Besides, we restrict ourselves to the case when the angles θ_c of synchronous interaction are essentially less than 90° (fundamental radiation wavelength is longer than 0.53 μm). Though the problem, characterized by these three features, is of certain practical interest, its detailed optimization [i.e., derivation of the equations similar to Eqs. (6)] has not been carried out, as far as we know.

In the course of the numerical experiments, we have concluded that optimal values of the parameters of our interest for the above-mentioned general case can be sought with a reasonable accuracy using the following empirical relationships:

$$\Delta_{\text{fop}} = 0, \tag{7a}$$

$$L\Delta_{\text{kop}}/2 = -1.5. \tag{7b}$$

Optimal focal lengths are determined in the following way:

$$\xi(f_j) = \frac{kL}{4n}\left(\frac{a_0}{f_j}\right)^2.$$

1. Spherical focusing ($f_x = f_y \equiv f_c$):

$$\xi_{\text{cop}} \approx 1.44. \tag{7c}$$

2. Focusing by two crossed cylindrical lenses:

$$\xi_{x\text{op}} \approx 0.4, \quad \xi_{y\text{op}} \approx 3.2. \tag{7d}$$

The beam focusing by one cylindrical lens is of less interest and we do not consider it in this paper.

Below we shall try to prove the validity of the expressions proposed by varying the initial parameters within quite a wide range. For this, compare the SHG efficiency calculated at accurate optimization of the process and with the use of approximate Eqs. (7).

Note, first, that validity of the Eq. (7a) follows from Eq. (4). Indeed, the increase of the modulus of Δ_{f} increases oscillations of the integrand and, hence, results in a decrease of both the SH amplitude at every observation point (x_0, y_0) and the efficiency of nonlinear conversion. This fact is known quite well (see, e.g., Refs. 1 and 5), so hereinafter we set $\Delta_{\text{f}} = 0$.

Figure 2 shows the dependences of the efficiency of nonlinear conversion η on the wave detuning Δ_{k} for three values of the length L of a nonlinear crystal. The beam ($\lambda = 0.5782$ μm) with $m_x = 4$ and $m_y = 3$ was focused to the crystal by two crossed cylindrical lenses. The solid lines show the exact solutions (optimization by two parameters f_x and f_y) while the dotted lines show the approximate ones obtained with the use of Eq. (7d).

It follows from the results presented that, first, the positions of the efficiency maxima are quite well defined by Eq. (7b) and, second, in the case of near optimal wave detuning the use of Eq. (7d) does not result in any essential errors. Finally, note one interesting feature. Though the value of the optimal wave detuning is essentially large (the shorter the crystal length the larger the value), a practical gain of such optimization (in comparison with the case of precise fulfillment of the synchronism condition) is minimum being on the order of 1%. Hence, in the considered wavelength range of the fundamental radiation, the efficiency maxima can be estimated with the use of Eq. (7b) or setting $\Delta_{\text{k}} = 0$.

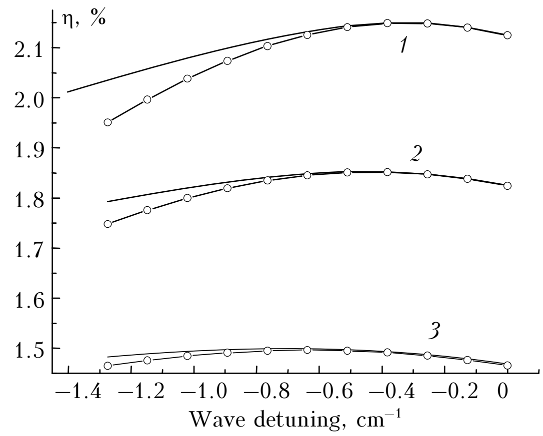

Fig. 2. The SHG efficiency (η) as a function of wave detuning. The crystal length equals to 8 (*1*), 6 (*2*), and 4 cm (*3*). Solid lines show the exact solutions while the dotted lines present the approximate ones.

Figure 3 demonstrates the use of Eqs. (7) at varying parameters m_x and m_y. Here the functions $\eta(m_x, m_y = 1)$ and $\eta(m_x = 1, m_y)$ are shown for the radiation with the wavelength 0.5782 μm focused to a 4-cm long crystal with a spherical lens.

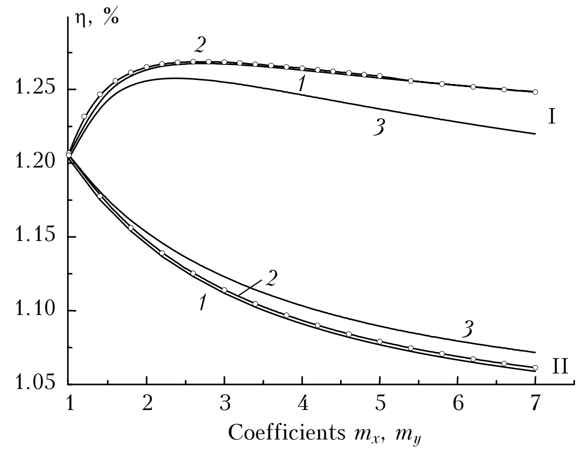

Fig. 3. SHG efficiency η as a function of m_x and m_y at spherical focusing: $\eta(m_x, m_y = 1)$ (curves I) and $\eta(m_x = 1, m_y)$ (curves II). Curves *1* show the optimization over Δ_{k} and f, curves *2* – optimization over f [Δ_{k} is calculated by Eq. (7b)], curves *3* present approximate solutions; Δ_{k} and f are calculated by Eqs. (7).

As is seen from the comparison of curves *1* and *2*, the use of approximate Eq. (7b) does not reduce the accuracy of calculations. Less successful is the approximation (7c) and it's quality deteriorates with the increase of m_x and m_y. But even in the least favorable cases the use of Eq. (7c) results in errors no larger than 1%, i.e., the accuracy of calculation remains quite acceptable.

The behavior of the efficiency of nonlinear conversion as a function of the coefficients m_x and m_y is also seen from Fig. 3. To check this up, we have repeated the above computations (the results are not presented here) by making use of direct integration of equations (1) as was done in Sec. 1 of this paper. As expected, the results obtained by both methods are identical. Monotonic decrease of the efficiency with the increase of m_y can well be explained by the corresponding decrease of the amplitude of fundamental radiation in the waist plane. As to the efficiency increase with increasing m_x and, particularly, the presence of maximum at $m_x \approx 3$, this requires a separate study which is beyond the scope of this work.

Figure 4 shows the efficiency of nonlinear conversion η as a function of the fundamental radiation wavelength λ.

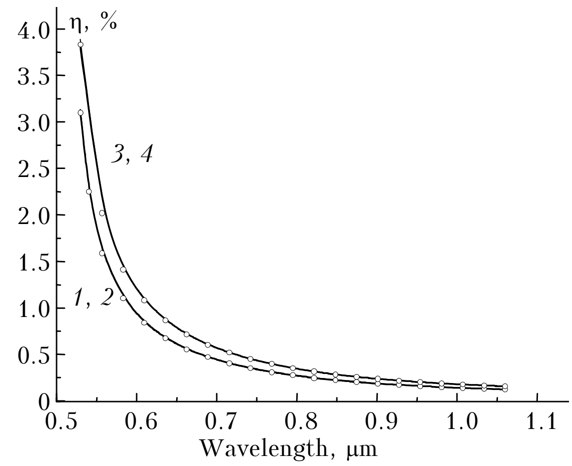

Fig. 4. The SHG efficiency η as a function of the fundamental radiation wavelength at optimal spherical focusing (curves *1* and *2*) and optimal cylindrical focusing (*3* and *4*). Curves *1* and *3* present the exact solutions (solid lines), while curves *2* and *4* show the approximate ones obtained with the use of Eqs. (7) (circles).

Curves *1* and *2* present the case of a spherical beam focusing ($m_x = 4$, $m_y = 3$) to a nonlinear crystal of 4-cm length, while curves *3* and *4* the case with focusing the same beam using two cylindrical lenses. Curve *1* (solid line) is the result of SHG optimization over the parameters Δ_k and $f = f_x = f_y$. Curve *3* (solid line) present the optimization over f_x and f_y. In so doing, the value Δ_k has been chosen in accordance with Eq. (7b). Strictly speaking, the optimization in the last case is to be carried out over three parameters, i.e., f_x, f_y, and Δ_k, however the correction to the result is minimum in this case while the computing time becomes unacceptably long. In the previous case (related to Fig. 3), we considered the spherical focusing but not cylindrical one for this same reason, i.e., to restrict ourselves to two-parameter optimization.

Perfect coincidence of the curves *1*, *2* and *3*, *4* allows one to conclude that the use of approximate optimal conditions (7b)–(7d) for the considered wavelength range of fundamental radiation does not introduce any significant error.

Besides, the results shown in Fig. 4 allow the conclusion to be drawn that focusing by two crossed cylindrical lenses provides better results than those at spherical focusing. For practical estimations, one can suppose the gain due to the cylindrical focusing to be independent of a fundamental radiation wavelength and is about 25%.

The above gain remains at the same level at different nonlinear interaction distances, which is demonstrated by the results shown in Fig. 5.

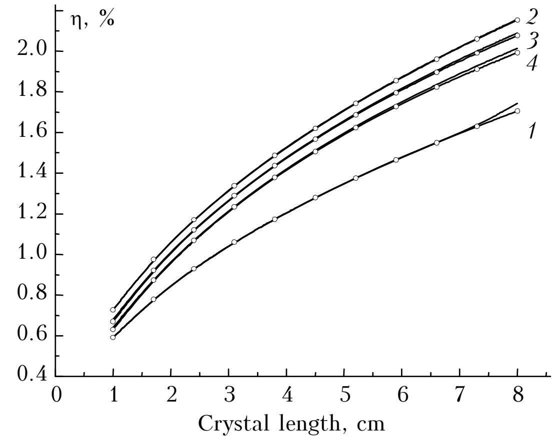

Fig. 5. The SHG efficiency η as a function of nonlinear crystal length. Curves *1* and *2* present the case with a Gaussian beam at optimal spherical and cylindrical focusing, respectively; curves *3* and *4* present the case with optimal cylindrical beam focusing at $m_x = m_y = 3$ and $m_x = m_y = 7$. Solid lines show exact solutions and circles show the approximate ones.

The wavelength of fundamental radiation was chosen at 0.5782 μm. Calculations were carried out by two methods similar to the previous case (see comments to Fig. 4). Solid lines show the case of exact matching of optimal parameters, while circles show the results obtained using approximate conditions (7). Both methods give indistinguishable (at such variations of the efficiency) results thus demonstrating sufficiently high quality of approximations (7) once again.

The dependences of the nonlinear conversion efficiency on m_x and m_y are shown in Fig. 3. Now return to the question on the influence of the amplitude form on the SHG efficiency and show the result obtained at simultaneous increase of the parameters m_x and m_y. It follows from the comparison of curves *3* and *4* with the curve *2*, that the conversion efficiencies of beams, less "fuzzy" in the cross sections, are a little bit lower.

The result of simultaneous, but not obligatory the same, increase of m_x and m_y can be quite precisely predicted if the coefficients

$$c_x(m_x) = \eta(m_x, m_y = 1) / \eta(m_x = m_y = 1), \quad (8a)$$

$$c_y(m_y) = \eta(m_x = m_y = 1) / \eta(m_x = 1, m_y) \quad (8b)$$

are known. For example, the above coefficients can be easily found using the results from Fig. 3. In this case, according to our estimates, the following equation is fulfilled with good accuracy:

$$\eta(m_x,m_y) = \eta(m_x = m_y = 1)c_x(m_x)/c_y(m_y). \quad (9)$$

In other words, the mechanisms (which are unknown so far) determining the influence of the parameters m_x and m_y on the SHG efficiency turn out to be additive.

In addition to the significant increase of the efficiency, the change from spherical to cylindrical focusing provides for another one advantage. To comment this, let us turn to Fig. 6.

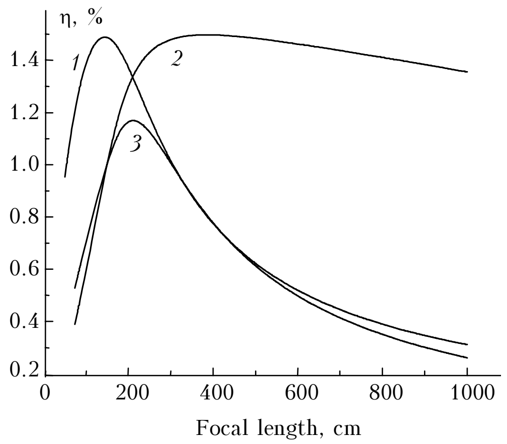

Fig. 6. The SHG efficiency η as a function of the focal length f: $\eta(f_x = f_{x\text{op}}, f_y = f)$ (*1*), $\eta(f_x = f, f_y = f_{y\text{op}})$ (*2*), and $\eta(f_x = f_y = f)$ (*3*).

The considered case is characterized by the following initial data: $\lambda = 0.5782$ μm, $m_x = 4$, $m_y = 3$, and $L = 4$ cm. The first two curves show the case with cylindrical focusing, while the last one presents the case with spherical focusing. The variable f determines the focal length of the lens L_y, in the first case; lens L_x in the second case, and the spherical lens in the third one. The focal lengths $f_{x\text{op}}$ and $f_{y\text{op}}$ for curves *1* and *2* were chosen based on Eq. (7d).

The above-mentioned advantage of the cylindrical focusing is a weak dependence of the efficiency on the focal length f_x, which follows from the view of the curve *2* at $f > f_{x\text{op}}$. This fact is of principal importance for the SHG of powerful laser radiation. In this case, the maximum efficiency of nonlinear conversion is to be limited by the strength of a nonlinear medium to high-power irradiation which is about 1 GW/cm^2 (critical power density) for the majority of known crystals.

In case of nanosecond (or shorter) laser pulses, optimal focusing (spherical or cylindrical) produces power densities commensurable or exceeding the above critical value in the waist plane (i.e., inside the crystal) even at a relatively low power. In this situation, to protect the crystal, it is necessary to avoid optimal focusing by increasing the focal lengths of the lenses used. In that case, use of spherical focusing would noticeably decrease the nonlinear conversion efficiency (see curve *3* in Fig. 6). Similar result is obtained in the case of cylindrical focusing, if one decreases the power density and increases the focal length f_y (curve *1*). Different situation is observed when focal length f_x increased. In this case, (see curve *2* in Fig. 6) the power density can be decreased by several times while the SHG efficiency keeps at the level practically indistinguishable from the maximum (optimum) value.

Such a property of the cylindrical focusing, useful for practical purposes, can be explained in the following way. On the one hand, the influence of diaphragm and angular aperture effects (see, e.g., Refs. 1, 2, and 5) decreases with the decrease of fundamental radiation divergence in the principal optical plane (in our case, in the coordinate plane XZ) thus resulting in an increase of the SH efficiency and power. On the other hand, the increase of the focal length f_x is accompanied by a decrease of the power density of the fundamental radiation and, hence, the proportional (at least in the preset-field approximation) decrease of the conversion efficiency. The behavior of curve *2* in Fig. 6 evidences that the influences of these "positive" and "negative" mechanisms, in the region $f_x > f_{x\text{op}}$, is almost completely compensated. In this paper, we restrict ourselves to this qualitative reasoning having in mind that there should be no difficulties in detailed qualitative analysis of mechanisms determining the dependence of the SHG efficiency on the parameters of a focusing system.

Conclusion

Strictly speaking, the main result, i.e., solution (4) enabling us to study of the problems of our interest, does not introduce noticeable errors only when the efficiency of nonlinear conversion does not exceed 5%. This is caused by the fact that solution (4) has been obtained in the preset-field approximation. Nevertheless, we have a good reason to believe (strict demonstration is planned soon) that the optimal conditions (providing the maximum conversion efficiency), defined within the developed model, will change insignificantly even at an essential increase of the laser radiation power. However, in that case the value of the maximum efficiency can be estimated only with an error (overestimation is possible), which increases with the increase of the laser radiation power.

In this study, conditions optimal for SHG have been studied in a KDP crystal at the scalar «*ooe*»-type of synchronism. Unfortunately, at present, it is impossible to conclude about the applicability of the obtained results to other nonlinear processes, in other nonlinear crystals, and for other types of interactions. This question requires an additional study (following to the universal scheme proposed in this paper).

Here the problem of optimizing the SHG process has been considered for the most simple (in our opinion) but practically useful case, when the synchronous interaction angles are far from 90°. In the case of a KDP crystal, this means that the wavelength of fundamental radiation is to be longer than 0.53 μm. The upper limit of 1.06 μm of the spectral range under study has been chosen quite arbitrarily.

We have considered the focusing system of two thin cylindrical lenses crossed in the beam. Such a system is universal for computations since it is the simplest for simulating the use of any focusing system consisting of an arbitrary number of optical elements at a proper choice of focal lengths and distances between the lenses (of course assuming the case of aberration-free optics).

Within the above restrictions, the obtained results allow us to formulate the following principal conclusions.

1. Maximum SHG efficiency is provided when beam focusing to the crystal is being done by two cylindrical lenses with uniquely chosen optimal focal lengths. The mentioned optimal focal lengths can be approximately estimated with the use of empirical Eqs. (7d), which introduces an error no more than 1%, in computing the maximum efficiency, as compared to exact result.

2. As compared to the optimal spherical focusing [focal length agrees with Eq. (7c)], the optimal cylindrical focusing provides the efficiency gain of about 25% and the gain value is practically invariable even if the initial parameters vary in a wide range.

3. The maximum of SHG efficiency is achieved at the wave detuning $\Delta_k = \Delta_{kop}$ obeying Eq. (7b). The efficiency varies insignificantly (no more that by 1.5%) at the wave detuning Δ_k varying from 0 to Δ_{kop}. For this reason, we conclude that such an optimization mechanism is of no practical use for the class of problems under study, hence, for simplifying computations, one may set $\Delta_k = \Delta_{kop} = 0$ for all the cases.

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