

# On the experience of using the Kalman filtering algorithm for ultra-short-term prediction of mean wind components based on lidar data

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The principles and algorithms for realizing the method of ultra-short-term prediction of atmospheric parameters using the Kalman filtering are considered, and results of their qualitative estimation based on the data of lidar wind measurements are presented.

## Introduction

Investigations related to super-short-term (with forestalling up to 12 hours) forecast of wind in the near-ground layer of the atmosphere fill a highly important place among the basic and applied investigations into the problem of atmospheric ecological monitoring over a limited territory. It is caused by the fact that air circulation in this layer significantly determines the state and evolution of the level of pollution of a limited air basin.

One can judge on the role of wind in the formation and evolution of air pollution field, in particular, using the equation of budget (transfer) of an atmospheric admixture. This equation for a particular admixture in a turbulent atmosphere can be written in the following form<sup>1</sup>:

$$\frac{\partial S_a}{\partial t} + u \frac{\partial S_a}{\partial x} + v \frac{\partial S_a}{\partial y} + w \frac{\partial S_a}{\partial z} + \frac{\partial w_a S_a}{\partial z} - k_1 \Delta S_a - \frac{\partial}{\partial z} k \frac{\partial S_a}{\partial z} = \varepsilon_a, \quad (1)$$

where  $S_a$  is the volume concentration of the admixture  $a$ ;  $u$ ,  $v$ , and  $w$  are the components of the wind velocity in the coordinate system  $x$ ,  $y$ ,  $z$ ;  $w_a$  is the vertical velocity of the admixture ( $w_a < 0$ );  $k(z)$  and  $k_1$  are the turbulence coefficients at motion of particles along the vertical and horizontal directions;  $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$  is the

two-dimensional Laplacian,  $\varepsilon_a = \varepsilon_a(x, y, z, t)$  is the source (sink) of the admixture, i.e., the rate of its appearance (disappearance) in a unit volume.

It is well seen from Eq. (1) that horizontal components of the wind velocity play a significant role causing the advective influx of the contaminating admixture (second and third terms in the left-hand part of the transfer equation). One should emphasize that these components of the wind velocity are related to the input parameters of the transfer model. So usually they are calculated using a mesoscale hydrodynamic

model (see, for example, Ref. 2). However, in using the hydrodynamic model, we meet quite a cumbersome algorithm for its realization and quite large contribution of the errors in the initial data to the error of the hydrodynamic scheme of forecasting.<sup>3</sup> Besides, significant bulk of the observation data collected over a territory are necessary in this case.

Taking into account all the aforementioned, as well as the necessity of solving the problem of forecast for a limited territory, we propose a simplified dynamic-stochastic model based on the Kalman filtering algorithm and stochastic differential equation of the first order describing the dynamics of temporal variations of a random process. The peculiarity of such an approach is that the procedure of solving a complicated system of hydrodynamic equations is avoided. Besides, time variation of a parameter of the atmospheric state (wind in our case) is a stochastic process with known correlation properties. As a result, at least, according to the approach proposed, the forecast can be realized using the data of observations at a single station. The dimension of the vector of state is restricted, realization of the filtering algorithm is simplified, and its stability increases noticeably.

The methodology and algorithms for solving the problem of super-short-term forecast based on Kalman filtering are considered in this paper. The results of their qualitative estimation carried out using the data of measurements of wind by means of three-path correlation lidar are presented.

It should be emphasized first that investigations into the problem of time forecasting of the parameters of the atmospheric state (in particular, wind) realized using Kalman filtering continue our previous investigation,<sup>4</sup> in which the algorithms were based on the complex approach that used the procedure of complication of the Modified Group Method of Data Handling (MGMDH) and the method of optimal extrapolation of a random process. In spite of its noticeable advantages against traditional methods of

regression analysis (the advantages include the possibility of realizing the algorithm using the data of a restricted sample, multi-criteria selection of the best model, orientation toward obtaining the forecasting model of optimal complication, etc.), the complex algorithm has certain restrictions. They are mainly related to the necessity of forming a preliminary sample of expeditious data array of the total bulk of the order of  $N = k + 1$  (here  $k$  is the number of considered levels or the layers of the atmosphere) and to the requirement of equality of the period of forecast with the period of observations.

The necessity appeared in this connection of the development of new methods for time forecast, which do not have such restrictions.

## 1. Statement of the problem and forecasting algorithms in terms of the Kalman filter

In synthesizing the algorithms for estimating and forecasting in terms of the Kalman filtering it is a general requirement that a possibility should exist of representing a mathematical model of evolution of the sought parameters of a dynamic system in the form of differential stochastic equations of the first order. In our case the dynamic system is the atmosphere, the parameters of which (for example, wind) are random. Let us synthesize the algorithm for estimating and forecasting supposing that the correlation properties of the meteorological parameters interesting for us are known.

By virtue of the temporal randomness of the values of meteorological parameters, their statistical properties can be set by corresponding correlation functions  $\mu(\tau)$ . Passage from correlation functions to differential equations describing the dynamics of variability of the random processes is realized by the known operator methods using the Laplace transform.<sup>5</sup> Depending on the meteorological parameter and, hence, on  $\mu(\tau)$ , differential equations describing the space of states can take different forms.<sup>6</sup> Let us give an example of passage from elementary correlation function to a differential stochastic equation.

The normalized autocorrelation function of some meteorological parameters, and, in particular, orthogonal components of the wind velocity, can be represented by the exponential function of the following form:

$$\mu(\tau) = \exp(-\alpha\tau). \quad (2)$$

The Laplace transform of this function has the form<sup>6</sup>:

$$L\{\mu(\tau)\} = L\{\exp(-\alpha\tau)\} = 1/(s + \alpha), \quad (3)$$

where  $\alpha = 1/\tau_0$  is the coefficient inversely proportional to the correlation interval  $\tau_0$ ,  $s$  is the parameter of the Laplace transform.

Assuming that  $L\{\mu(\tau)\}$  represents the transmission function of a linear system with the "white noise" at the input, one can write

$$\frac{L\{\xi(\tau)\}}{L\{W(\tau)\}} = \frac{1}{s + \alpha} \quad (4)$$

or, correspondingly,

$$L\{\xi(\tau)\}(s + \alpha) = L\{W(\tau)\}, \quad (5)$$

where  $\xi(\tau)$  is the response of the linear system (variability of the meteorological parameter) to the input impact  $W(\tau)$  ("white noise").

After application of the inverse Laplace transform to the left- and right-hand parts of Eq. (5) and replacing the operator  $s \rightarrow \frac{d}{dt}$  we obtain the following differential equation:

$$\frac{d\xi(t)}{dt} + \alpha\xi(t) = W(t),$$

from which

$$\frac{d\xi(t)}{dt} = -\alpha\xi(t) + W(t). \quad (6)$$

Passage from differential equation to difference can be done by means of the Euler method

$$\frac{d\xi(t)}{dt} \rightarrow \frac{\Delta\xi(t)}{\Delta t},$$

where  $\Delta\xi = [\xi(t_2) - \xi(t_1)]$ , at  $\Delta t = (t_2 - t_1) \rightarrow 0$ ,

from which the formula follows

$$\xi(t_2) = \xi(t_1) - \alpha \xi(t_1) \Delta t + W(t_1). \quad (7)$$

Equation (7) shows that the value of the process  $\xi(t_2)$  at the moment  $t_2$  is determined by the value  $\xi(t_1)$  at the moment  $t_1$ , time interval  $\Delta t$ , the value of the coefficient  $\alpha = 1/\tau_0$ , and the value of the engendering noise  $W(t_1)$ .

The difference equation (7) describing evolution of the random process can be presented in discrete time in the following form:

$$\xi(k + 1) = \xi(k) - \alpha \xi(k)\Delta t + W(k) \quad (8)$$

or

$$\xi(k + 1) = \xi(k)(1 - \alpha\Delta t) + W(k), \quad (9)$$

where  $k = 0, \dots, K$  is the discrete current time with the discretization step  $\Delta t$  ( $t_k = k \Delta t$ );  $\xi(k)$  is the response of the linear system (temporal behavior);  $W(k)$  is the input impact ("discrete white noise").

Formula (9) can be used as a model of the space of states at synthesis of the algorithms for estimating and time forecast of the meteorological parameter  $\xi(k)$  within the frameworks of the Kalman filtering theory.

Let us determine the general ideas, which will be put to the foundation of the method for the forecast considered below. Let us suppose that the meteorological parameters are measured at some point of the space, at a discrete moments in time, and with the known error. In this case the model of current measurements of the meteorological parameter can be set as simple additive mixture of the useful data (true value of the meteorological parameter) and some addition (error)

$$\tilde{Y}(k) = \xi(k) + E(k), \quad (10)$$

where  $\tilde{Y}(k)$  are the current values of the meteorological parameters at the discrete time moment  $k$ ;  $\xi(k)$  is the true value of the meteorological parameter,  $E(k)$  is the error in measuring.

Formula (9) determines the time behavior of the true value of the meteorological parameter  $\xi(k)$ . Let us write the equation of states (9) and observations (10) according to the generally accepted notations of the Kalman filtering theory

$$X(k+1) = X(k) (1 - \alpha\Delta t_k) + W(k); \quad (11)$$

$$\tilde{Y}(k) = X(k) + \mathbf{E}(k), \quad (12)$$

where  $X(k)$  is the variable of state to be estimated and forecasting (in our case it corresponds to  $\xi(k)$ ).

For a convenience of notations in terms of the Kalman filter theory, let us present Eqs. (11) and (12) in the matrix form and define the components entering the formula:

$$\mathbf{X}(k+1) = \mathbf{F}(k) \cdot \mathbf{X}(k) + \mathbf{W}(k); \quad (13)$$

$$\tilde{\mathbf{Y}}(k) = \mathbf{H}(k) \cdot \mathbf{X}(k) + \mathbf{E}(k) \quad (14)$$

where  $\mathbf{X}(k)$  and  $\mathbf{X}(k+1)$  is the vector of state of the dimension  $(1 \times 1)$  at the time moments  $k$  and  $(k+1)$ , respectively. Here  $\tilde{\mathbf{Y}}(k)$  is the vector of observations of the dimension  $(1 \times 1)$ ;  $\mathbf{F}(k) = [1 - \alpha\Delta t_k]$  is the transition matrix of the dimension  $(1 \times 1)$ ;  $\mathbf{H}(k) = [1]$  is the matrix of observations of the dimension  $(1 \times 1)$ ;  $\mathbf{W}(k) = [W(k)]$  is the vector of noise of state of the dimension  $(1 \times 1)$ ;  $\mathbf{E}(k) = [E(k)]$  is the vector of noise of observations of the dimension  $(1 \times 1)$ .

Formalization of the matrices entering formulas (13) and (14) makes it possible to synthesize the structure of the linear Kalman filter providing for an optimal estimate of the vector of state  $\mathbf{X}(k)$ . The optimal estimate  $\hat{\mathbf{X}}(k)$  of the vector  $\mathbf{X}(k)$  is considered to average the value providing for the minimum of the mean square of the error  $\tilde{\mathbf{X}}(k) = \mathbf{X}(k) - \hat{\mathbf{X}}(k)$  at any moment in time  $k$ .

In this case the formulas for optimal estimating of the vector of state  $\mathbf{X}(k)$  have the following form:

$$\hat{\mathbf{X}}(k+1) = \hat{\mathbf{X}}(k+1|k) + \mathbf{G}(k+1) \cdot [\tilde{\mathbf{Y}}(k+1) - \mathbf{H}(k) \cdot \hat{\mathbf{X}}(k+1|k)], \quad (15)$$

where  $\hat{\mathbf{X}}(k+1|k)$  is the vector of forecasting estimates at the time moment  $(k+1)$  from the data at the step  $k$ ,  $\mathbf{G}(k+1)$  is the matrix of the weighting coefficients of the dimension  $(1 \times 1)$ .

The following matrix equation is used for calculation of the vector of forecast

$$\hat{\mathbf{X}}(k+1|k) = \mathbf{F}(k) \cdot \hat{\mathbf{X}}(k). \quad (16)$$

The weighting coefficients in the linear Kalman filter are calculated using the recurrent matrix equations of the following form:

$$\mathbf{G}(k+1) = \mathbf{P}(k+1|k) \cdot \mathbf{H}^T(k) \times \\ \times [\mathbf{H}(k) \cdot \mathbf{P}(k+1|k) \cdot \mathbf{H}^T(k) + \mathbf{R}_E(k+1)]^{-1}, \quad (17)$$

$$\mathbf{P}(k+1|k) = \mathbf{F}(k) \cdot \mathbf{P}(k|k) \cdot \mathbf{F}^T(k) + \mathbf{R}_w(k), \quad (18)$$

$$\mathbf{P}(k+1|k+1) = [\mathbf{I} - \mathbf{G}(k+1) \cdot \mathbf{H}(k)] \cdot \mathbf{P}(k+1|k), \quad (19)$$

where  $\mathbf{P}(k+1|k)$  is the *a posteriori* correlation matrix of the errors in forecasting of the dimension  $(1 \times 1)$ ,  $\mathbf{P}(k+1|k+1)$  is the *a priori* correlation matrix of the estimation errors of the dimension  $(1 \times 1)$ ,  $\mathbf{R}_E(k+1)$  is the diagonal correlation matrix of the noise of observations of the dimension  $(1 \times 1)$ ,  $\mathbf{R}_w(k)$  is the diagonal correlation matrix of the noise of state of the dimension  $(1 \times 1)$ ,  $\mathbf{I}$  is the unit matrix of the dimension  $(1 \times 1)$ ,  $\mathbf{T}$  denotes the transposition.

To start running the filtering algorithm (15)–(19) at the moment  $k=0$  (initiation moment), it is necessary to set the following initial conditions:  $\hat{\mathbf{X}}(0) = \mathbf{M}\{\mathbf{X}(0)\}$  is the initial vector of estimates,  $\mathbf{P}(0|0) = \mathbf{M}\{[\mathbf{X}(0) - \mathbf{M}\{\mathbf{X}(0)\}][\mathbf{X}(0) - \mathbf{M}\{\mathbf{X}(0)\}]^T\}$  is the initial correlation matrix of the estimation errors, as well as the values of the coefficients of correlation matrices of noises  $\mathbf{R}_E(0)$  and  $\mathbf{R}_w(0)$ . In practice the values  $\hat{\mathbf{X}}(0)$  and  $\mathbf{P}(0|0)$  can be set based on the minimum bulk of data on the real properties of the system, and in the case of the complete absence of useful data the values are set to be  $\hat{\mathbf{X}}(0) = 0$ , and  $\mathbf{P}(0|0) = \mathbf{I}$ .

In our case  $\hat{\mathbf{X}}(0) = 0$ , and  $\mathbf{P}(0|0) = \sigma^2$  (here  $\sigma = 3$  m/s is the rms deviation of the wind velocity components, which characterizes their variability).

It should be noted that Eq. (16) determines the algorithm for forecasting the meteorological parameter  $\hat{\mathbf{X}}(k+1|k)$  in between measurements. Taking into account Eq. (11), the solution (16) can be realized recurrently with an arbitrary discretization interval  $\Delta t$ , that makes it possible to provide for the forecast for any time period until next measurement. The forecast estimates are corrected at the moment of next measurement by means of the filtering algorithm (15), and then the forecast equation (16) continues to be solved with revising the initial conditions. Thus, the problem of time forecast is solved together with the problem of estimating.

## 2. Results of numerical experiments on estimation of the quality of forecast by means of the Kalman filter algorithm

The Kalman filter algorithm considered above was subject to qualitative examination at its application to the problem of super-short-term (with forecast of 4 and 8 hours) forecast of wind in the boundary layer of the atmosphere, where the principal transfer of pollutants of the industrial origin is usually observed.<sup>7</sup> To do it, the data of lidar observations of wind were used. The data were obtained by means of the three-path correlation lidar (its diagram and the operational principles are described in Ref. 8) near Tomsk (56°N, 85°E) since June 10 till August 12, 1994. Total set of

90 timed (2, 6, and 10 a.m., 2, 6, and 10 p.m.) observations of the vertical distribution of the wind velocity and direction in the layer 140 to 1140 m with the vertical resolution of 100 m were used for numerical experiments on estimation of the quality of the Kalman filter algorithm. The data on the wind velocity and direction served as the basis for calculation of its zonal  $V_x$  and meridional  $V_y$  components.

It should be noted here that not the data of wind measurements at fixed levels are usually used for practical calculations of propagation of the cloud of a polluting admixture of industrial origin, but their values averaged over vertical layers.<sup>9</sup> So the procedure of averaging over layers was applied for forming the initial sets of the data on the zonal  $V_x$  and meridional  $V_y$  components of wind velocity. Averaging was carried out by the formulas

$$\langle V_x \rangle_{h_0,h} = \frac{1}{h-h_0} \int_{h_0}^h V_x(z) dz, \quad (20)$$

$$\langle V_y \rangle_{h_0,h} = \frac{1}{h-h_0} \int_{h_0}^h V_y(z) dz, \quad (21)$$

where the symbol  $\langle \bullet \rangle$  denotes the procedure of averaging over the vertical layer of the atmosphere  $h-h_0$  (in our case  $h_0=140$  m, and  $h=240, 340, \dots, 1140$  m is the height of the upper boundary of the

considered layer). The values  $\langle V_x \rangle_{h_0,h}$  and  $\langle V_y \rangle_{h_0,h}$  are usually called the layer-average (or simply average) values of the zonal and meridional wind.

As for estimation of the quality of the Kalman filter algorithm at its use in the problem of super-short-term forecast of the layer-average values of zonal and meridional wind, it was carried out by means of rms error of such a forecast  $\delta_\xi$  determined by the following formula:

$$\delta_\xi = \left[ \frac{1}{n} \sum_{i=1}^n (\hat{\xi}_i - \xi_i)^2 \right]^{1/2}, \quad (22)$$

here  $\hat{\xi}_i$  and  $\xi_i$  are the forecasted and measured values of the meteorological parameter, respectively, and  $n$  is the number of realizations processed), as well as the probabilities of the errors in forecasting  $\Delta_i = \hat{\xi}_i - \xi_i$ , which are smaller or larger than certain preset value (in our case smaller than  $\pm 1, \dots, \pm 4$  m/s and larger than 4 m/s). Besides, the relative forecasting error  $\theta = \delta_\xi / \sigma_\xi$ , where  $\sigma_\xi$  is the rms deviation of the considered meteorological parameter, was also used for the same estimate.

The rms errors  $\delta_\xi$  and the probabilities  $P$  of the errors in super-short-term (with forestall of 4 and 8 hours) forecast of the layer-average values of the velocity of zonal and meridional wind below  $\pm 1, \dots, \pm 4$  m/s and higher than 4 m/s by means of the Kalman filtering method are presented in Table 1.

**Table 1. Rms errors  $\delta_\xi$  and probabilities  $P$  of the errors in super-short-term forecast of the layer-average values of the velocities of zonal and meridional wind below  $\pm 1, \dots, \pm 4$  m/s and higher than 4 m/s obtained by means of the Kalman filter algorithm and using data of lidar measurements with forestall of 4 (1) and 8 (2) hours**

Layer, m	Probability, $P \times 10^2$										$\delta_\xi$	
	$\Delta V \leq 1 \text{ m/s}$		$\Delta V \leq 2 \text{ m/s}$		$\Delta V \leq 3 \text{ m/s}$		$\Delta V \leq 4 \text{ m/s}$		$\Delta V > 4 \text{ m/s}$			
	1	2	1	2	1	2	1	2	1	2	1	2
<i>Zonal wind (<math>V_x</math>)</i>												
140-240	100	100	100	100	100	100	100	100	00	00	0.1	0.1
140-340	97	97	100	100	100	100	100	100	00	00	0.1	0.1
140-440	93	89	100	100	100	100	100	100	00	00	0.3	0.3
140-540	92	89	97	97	100	100	100	100	00	00	0.5	0.5
140-640	89	88	94	97	97	100	100	100	00	00	0.7	0.7
140-740	86	86	94	94	97	97	100	100	00	00	0.9	0.9
140-840	83	81	94	92	97	94	100	97	00	0.3	1.2	1.3
140-940	80	78	94	89	97	92	100	94	00	0.6	1.3	1.9
140-1040	76	72	89	83	97	89	100	94	00	0.6	1.5	2.3
140-1140	67	61	87	81	97	86	100	92	00	0.8	1.8	2.7
<i>Meridional wind (<math>V_y</math>)</i>												
140-240	100	100	100	100	100	100	100	100	00	00	0.1	0.1
140-340	94	94	100	100	200	100	100	100	00	00	0.2	0.2
140-440	89	88	100	97	100	100	100	100	00	00	0.3	0.4
140-540	84	83	97	94	100	97	100	100	00	00	0.6	0.8
140-640	78	77	92	90	100	97	100	99	00	0.1	1.0	1.2
140-740	68	67	90	88	100	97	100	98	00	0.2	1.2	1.5
140-840	66	64	89	87	100	95	100	97	00	0.3	1.5	1.9
140-940	64	62	89	87	97	94	100	97	00	0.3	1.7	2.1
140-1040	62	60	86	85	97	94	100	97	00	0.3	1.9	2.2
140-1140	61	60	84	83	97	94	100	97	00	0.3	2.0	2.3

Analysis of the data given in Table 1 shows that:

– First, the algorithm for super-short-term forecast based on the Kalman filtering method provides for quite good results. Indeed, the probability  $P$  of the errors less than 1 m/s for time forecast  $\tau = 4$  and 8 hours is 67–100% for average zonal wind and 61–100% for average meridional wind, and probability of the errors less than 2 m/s is already 84–100% for both components.

– Second, this algorithm provides for the best results for forecast of  $\tau = 4$  hours, when the rms errors do not exceed 2 m/s in the entire considered layer of the atmosphere.

Let us consider now the results of qualitative estimation of the Kalman filtering method in comparison to the complex algorithm used earlier<sup>4</sup> based on the procedure of combination of MGMDH with the method of optimal extrapolation of the random process. To do this, let us consider Table 2, which contains the values of rms  $\delta_\xi$  and relative errors  $\theta$  in the forecast of the parameters  $\langle V_x \rangle_{h_0, h}$  and  $\langle V_y \rangle_{h_0, h}$ , carried out with the forecast of 4 hours on the basis of two alternative methods: the Kalman filtering method and the complex method based on the algorithm of MGMDH. The data on rms errors in super-short-term forecast of wind carried out by means of the complex method are taken from Ref. 4.

**Table 2. Rms ( $\delta_\xi$ ) and relative ( $\theta$ , %) errors in super-short-term forecast (with the forecast of 4 hours) of the layer-average values of the velocity of zonal and meridional wind carried out by means of the Kalman filter algorithm (1) and the complex method (2) from the data of lidar measurements**

Layer, m	Zonal wind, m/s				Meridional wind, m/s			
	$\delta_\xi$		$\theta$		$\delta_\xi$		$\theta$	
	1	2	1	2	1	2	1	2
140–240	0.1	0.6	0.6	38	0.1	0.6	0.6	35
140–340	0.1	0.8	0.6	44	0.2	0.8	10	38
140–440	0.3	1.0	15	50	0.3	1.0	12	42
140–540	0.5	1.2	20	57	0.5	1.1	20	44
140–640	0.7	1.4	32	63	0.9	1.2	33	44
140–740	0.9	1.5	39	65	1.2	1.3	41	45
140–840	1.2	1.6	48	64	1.5	1.4	50	47
140–940	1.3	1.6	50	62	1.6	1.5	50	47
140–1040	1.5	1.7	53	61	1.7	1.6	50	47
140–1140	1.8	2.0	58	64	1.9	1.8	54	51

It follows from the analysis of the data collected in Table 2 that:

– The Kalman filtering method (in comparison to the complex method) provides for essentially higher-quality results. Indeed, the least values of rms errors of time forecast are characteristic of the Kalman filtering method in the entire considered layer (up to 1140 m) (compare: 0.1–1.8 m/s for the Kalman filter algorithm and 0.6–2.0 m/s for the complex method);

– The Kalman filtering method provides for the highest gain in accuracy under condition that the height  $h \leq 640$  m (for the zonal component) and  $h \leq 540$  m (for the meridional component). In these cases the magnitudes of the rms errors  $\delta_\xi$  are two and more times less than the same error obtained using the complex method.

Thus, the numerical experiments on estimation of the quality of the Kalman filter algorithm at its use in the problem of super-short-term forecast of the average wind components have shown that this algorithm is quite effective, it is superior over the complex method in accuracy and can be successively used in practice of the local atmospheric-ecological monitoring.

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