CHOOSING THE GAUSSIAN RESPONSE FUNCTION PARAMETER FOR AN ADAPTIVE MIRROR

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Received July 31, 1989

We determine the optimum ratio of the radius of deformation to the distance between adjustment points in an adaptive mirror with Gaussian response function which will provide the best approximation to various types of wavefronts.

A flexible adaptive mirror is considered the most important element in systems for correcting distortions in wavefronts. One of the major issues which arises in the design of such mirrors involves choosing the types of response function, since this problem is closely related to the issue of the accuracy to which the wavefront can be approximated and, thus, the possibility of choosing the minimum number of independent mirror control channels required to achieve a given correction accuracy.

There have been numerous papers on choosing the form of response function for active mirrors.[1,2,etc.] These studies lead to the conclusion that zone-based control with a Gaussian response function $f(\bar{\rho}) = \exp(-\bar{\rho}^2 / r^2)$ (where $\bar{\rho}$ is the vector of normal coordinates determining the position of a given point relative to the center of an adjustment point and r is the radius of deformation for that section of the surface of the adaptive mirror) minimizes the approximation error for a wide variety of wavefront types. We shall discuss the issue of selecting the optimum ratio of the radius of deformation r to the distance between adjustment points ρ for a given response function.

Due to the fact that distortions in the wavefront are frequently described In terms of a set of known phase distributions (Zernicke polynomials),[3] we shall first study the functional qualities of the response function with respect to statistical correction of the phase distortions. With this aim in view, we solved the problem of obtaining the best approximation in the root-mean -square sense using the first four Zernicke polynomials $Z_j(\bar{\rho})$. The error of approximation was calculated according to the following formula:

$$\varepsilon_j = \left[\frac{1}{S} \int\!\!\int \left[Z_j(\vec{\rho}) - \sum_{i=1}^{N^2} a_i f_i(\vec{\rho}) \right]^2 d^2\rho \right]^{1/2} .$$

where S is the area of the aperture and a_i is the amplitude of the control signal for the ith adjustment point. The adjustment points were assumed to be distributed over the circular aperture in a 2-D square lattice with spacing ρ.

Plots of the approximation error incurred by approximating the Gaussian response function by the first four Zernicke polynomials are shown as a function of the ratio r/ρ in Fig. 1. We see that the minimum approximation error using the Zernicke polynomials occurs for $r/\rho = 0.6$.

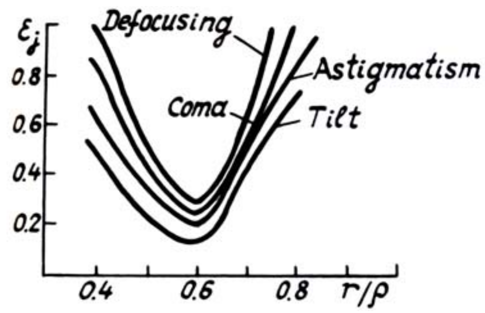

FIG. 1

Under real conditions, the incoming wavefront is a random field, so that it does not always make sense to approximate Zernicke polynomials or any other system of functions using various response functions. It would make more sense to determine how well the response function of the adaptive mirror can compensate for random phase distortions due to a turbulent atmosphere. Since we must deal with an Infinite set of random functions and compensate for distortions of various types, a statistical approach must be used in carrying out the analysis.

Our basic hypothesis for this study is that an adaptive mirror is a spatial-frequency filter.[2,4] In this case, the residual phase error in the correction $\Delta\Phi$ due to the limited bandpass of this filter will then be given by

$$\Delta\Phi(\vec{\rho}) = \Phi(\vec{\rho}) - \frac{\int_{-\infty}^{\infty} \Phi(\vec{\rho}\,')f(\vec{\rho} - \vec{\rho}\,')d^2\rho'}{\int_{-\infty}^{\infty} f(\vec{\rho})d^2\rho} . \quad (1)$$

where $\Phi(\bar{\rho})$ is the phase distribution at the entrance aperture. The variance of the residual phase correc-

0235-6880/90/02 178–03 $02.00

tion error may be determined by integrating over spatial frequency:

$$d = \int_{-\infty}^{\infty} \Phi(\vec{\kappa}) \left| 1 - \frac{(2\pi)^2 f_\kappa(\vec{\kappa})}{\int\limits_{-\infty}^{\infty} f(\vec{\rho}) d^2\rho} \right|^2 d^2\kappa, \tag{2}$$

where $\Phi(\vec{\kappa})$ is the spectral density of the phase distortions, $f_\kappa(\vec{\kappa})$ is the Fourier image of the response function, and $\vec{\kappa}$ is the spatial frequency vector.

Equation (2) was integrated numerically on a computer for the case of phase fluctuations due to atmospheric turbulence, where the spectral density is given by[5]

$$\Phi(\vec{\kappa}) \simeq 0.123\ \rho_0^{-5/3} \kappa^{-11/3},$$

where ρ_0 is the radius of the spatially-coherent region.

The results of the integration indicated that the dispersion of the residual phase error is given by the following relation (to good approximation):

$$d = \alpha (r/\rho_0)^{5/3},$$

where α is a coefficient whose values are given for various values of r/ρ in Table I. A typical family of dispersion curves for the residual phase error as a function of $(\rho/\rho_Q)^{S/3}$ is shown in Fig. 2. We see that the best approximation to the random phase errors with a Gaussian response function occurs for $r/\rho = 0.6$.

TABLE 1

r/ρ	0.4	0.5	0.6	0.7
α	1.78	1.11	0.54	0.70

Our research leads to the conclusion that in designing an adaptive mirror, the designer must take into account the fact that the best approximation to a variety of wavefront distortions is achieved using a Gaussian response function and a ratio of 0.6 between the surface deformation radius and the distance between adjustment points.

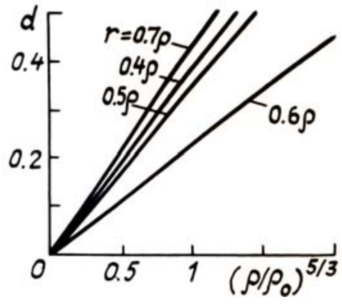

FIG. 2

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