# Energy characteristics of a light beam having passed through an ensemble of particles with a strongly forward peaked scattering phase function

**V.I. Bukatyi and T.K. Kronberg**

*Altai State University, Barnaul*

Received November 4, 2002

Propagation of a beam with the Gaussian statistics of photons was investigated by the Monte Carlo method for a medium with anisotropic scatterers. The energy distribution in the beam cross section and the dependence of brightness on the polar angle were obtained for different depths of radiation penetration.

Many problems in optics of scattering media (optical detection and ranging, vision, optical diagnostics, etc.) are connected with evolution of energy parameters of a light beam that passed through a turbid medium layer.[1,2] The shape of the scattering phase function, packaging density of particles, boundary conditions for a beam incident on the layer, and source–receiver geometry determine approximate methods for solution of the radiative transfer equation.

Many natural media are characterized by a strongly forward peaked scattering phase function. In the small-angle approximation,[3] the analytical equations have been derived for the angular structure of radiation near the direction of photons incident onto the layer. The domain of applicability of these equations is limited by the optical thickness $\tau' \simeq 15$. For layers with larger $\tau'$, an alternative method for investigation is the Monte Carlo method. It is convenient to take blood as a model medium with dense packaging of particles and a strongly forward peaked scattering phase function, since many problems of diagnosis and therapeutic action in the clinical medicine are connected with propagation of radiation through blood.[2] Therefore, it is interesting to follow transformation of the energy parameters of a light beam in a blood layer. Propagation of radiation from a point unidirectional source in a blood layer was studied in Ref. 4.

In this paper, we present some results of numerical simulation of the energy characteristics of a light beam in a blood layer, if the source emits radiation with the Gaussian statistics of photons.

The blood is a strongly scattering medium, and therefore the collision integral in the radiative transfer equation has high multiplicity. Computer simulation using the Monte Carlo method allows avoiding huge computational difficulties.[5] Optical properties of blood are determined by its erythrocytes, which are characterized by the following optical parameters[6]: concentration $n = 5\cdot10^6$ mm$^{-3}$, scattering $\sigma_{scat}$ and absorption $\sigma_{abs}$ cross sections at the wavelength $\lambda = 632$ nm equal to 57.2 and 0.06 μm$^2$, respectively, and the extinction cross section $\sigma_{ext} = \sigma_{scat} + \sigma_{abs}$. Let photons enter the blood layer along the normal. Let the axis $z$ coincide with the direction of the incident beam. The initial coordinates of photons are simulated by the normal law and take the values[5]:

$$x_0 = \sigma\sqrt{-2\ln\gamma_1}\,\cos2\pi\gamma_2;$$

$$y_0 = \sigma\sqrt{-2\ln\gamma_1}\,\sin2\pi\gamma_2;$$

$$z_0 = 0,$$

where $\gamma_1$ and $\gamma_2$ are generated by the random number generator; $\sigma^2 = R_0^2/2$ is the variance, $R_0$ is the beam radius.

Random wandering of a photon inside a biotissue sample is simulated from the point of its entrance to absorption or the exit point determined by the thickness of the blood layer. The free path $\tau$ between two sequential collisions with erythrocytes is distributed with the probability density

$$p(\tau) = \frac{1}{\tau_0}\exp(-\tau/\tau_0)$$

and simulated by the equation $\tau = -\tau_0\ln(1-\gamma)$, where $\tau_0 = n\sigma_{ext}$ is the mean free path; $\gamma$ is a random value with the uniform distribution on the interval [0, 1]. A photon is scattered or absorbed by an erythrocyte with the probability: $p_{scat} = \sigma_{scat}/\sigma_{ext}$ and $p_{abs} = \sigma_{abs}/\sigma_{ext}$, respectively. After a collision, the photon moves along the direction determined by the polar $\theta$ and azimuth $\varphi$ angles. The azimuth angle $\varphi$ is uniformly distributed over the interval $[0, 2\pi]$. The polar angle $\theta$ of photon scattering is simulated with the probability density determined by the Henyey–Greenstein scattering phase function[6]:

$$p(\theta) = \frac{1}{4\pi}\frac{1-g^2}{(1+g^2-2g\cos\theta)^{3/2}},$$

where $g = \langle\cos\theta\rangle$ is the mean cosine of the scattering angle. It is most convenient to simulate a random value $\xi = \cos\theta$. Using Smirnov's method,[5] we can easily derive the equation for generation of $\xi = \cos\theta$:

$$\xi = \frac{(1+g^2)\,\xi_1 - (1-g^2)^2}{2g\xi_1},$$

0235-6880/03/02 81-03 $02.00 

where $\xi_1 = (1 - g + 2g\alpha)^2$, and $\alpha$ is generated by the random number generator. It follows from this equation that the probability of photon scattering in the backward direction is ~ 0.1%, since at $\theta \geq 90°$ the random parameter $\alpha \leq 0.00104$. The new direction of photon motion after $n + 1$ scattering events is specified by a unit vector with the coordinates[7]:

$$a_{n+1} = a_n \left( \delta_0 - \frac{a_n\delta_1 + b_n\delta_2}{1 + |c_n|} \right) + \delta_1;$$

$$b_{n+1} = b_n \left( \delta_0 - \frac{a_n\delta_1 + b_n\delta_2}{1 + |c_n|} \right) + \delta_2;$$

$$c_{n+1} = c_n\delta_0 - \delta\,(a_n\delta_1 + b_n\delta_2),$$

where $a_n$, $b_n$, $c_n$ are the coordinates of the unit vector before the collision; $\delta_0 = \xi$; $\delta_1 = \sqrt{1 - \xi^2}\cos\varphi$, $\delta_2 = \sqrt{1 - \xi^2}\sin\varphi$;

$$\delta = \begin{cases} 1, & \text{at } c_n \geq 0, \\ -1, & \text{at } c_n < 0. \end{cases}$$

The radiation is detected at different parts of the path with the depth from 100 to 800 μm. For this purpose, the plane normal to the beam axis is divided into ring-shaped zones, where detectors record photons in each zone. The number of statistical tests for estimation of the studied parameters was $10^5$. The numerical tests have demonstrated insignificant widening of the beam (~10%) due to multiple scattering. Diffraction divergence on the studied interval can be neglected. The typical dependences of the relative intensity $I/I_0$ ($I_0$ is the maximum intensity of light within the input beam) on the radial variable $\rho$ in the beam cross section are shown in Fig. 1.

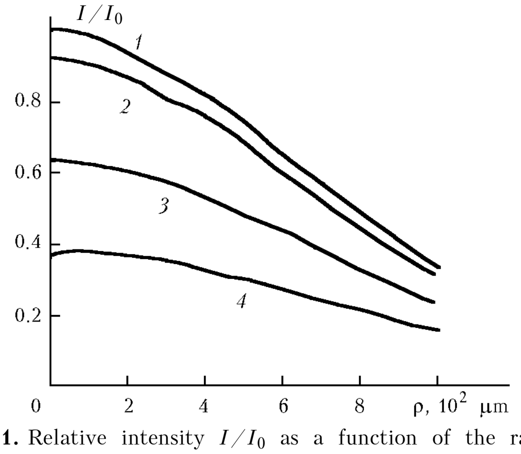


**Fig. 1.** Relative intensity $I/I_0$ as a function of the radial variable $\rho$ in the beam cross section at different depth $z$: 0 (*1*), 100 (*2*), 400 (*3*), and 700 μm (*4*); $I_0$ is the maximum intensity on the beam axis at the point with the coordinate $z = 0$.

The characteristic dimensions of the beam (at the level $e^{-1}$) begin to exceed the initial value $R_0 = 1$ mm at the depth $z$ greater than 600 μm.

Multiple scattering changes the Gaussian structure of the beam over the cross section at large optical thickness (Fig. 1, curve *4*). Figure 2 shows the log dependence of the peak intensity $I_{max}/I_0$ on the layer thickness $z$. The intensity of the non-scattered radiation at the beam axis decrease by the exponential law with the extinction coefficient $\alpha_{ext} = 1.44$ mm$^{-1}$. The dependence of the radial intensity $I(z, \rho, \theta)$ on the polar angle $\theta$ at different values of the cross coordinate $\rho$ is illustrated in Figs. 3 to 5. The shape of the brightness curves at the fixed layer thickness $z$ appears to be independent of the cross coordinate $\rho$. Radiation penetration deep into the layer is accompanied by transformation of the angular dependence of the radial intensity. The angular spectrum broadens with simultaneous straightening in the angle range of 0–80°.

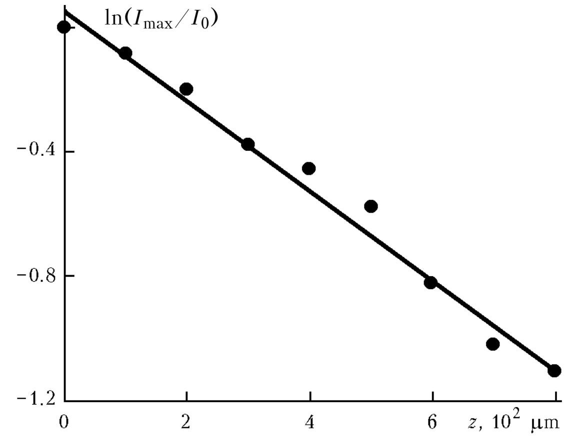


**Fig. 2.** Log dependence of the relative peak intensity $I_{max}/I_0$ on the depth $z$.

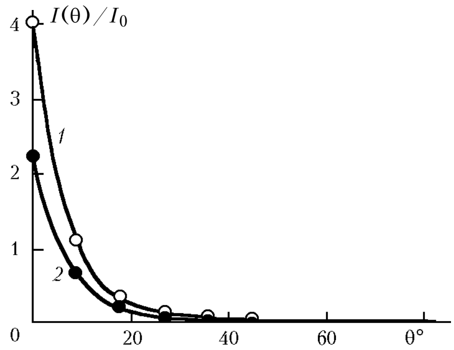


**Fig. 3.** Dependence of the normalized radial intensity $I(z, \rho, \theta)/I_0$ on the polar angle $\theta$ at the depth $z = 100$ μm for different values of the cross coordinate $\rho$: 200 (*1*) and 800 μm (*2*).

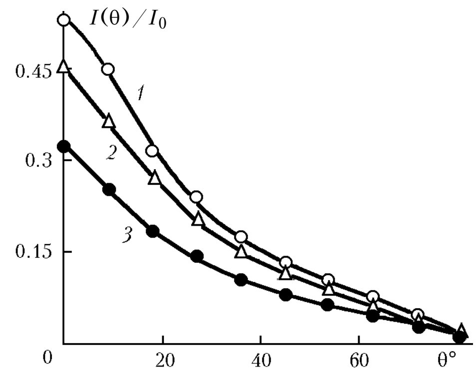


**Fig. 4.** Dependence of the normalized radial intensity $I(z, \rho, \theta)/I_0$ on the polar angle $\theta$ at the depth $z = 400$ μm for different values of the cross coordinate $\rho$: 200 (*1*), 500 (*2*), and 800 μm (*3*).

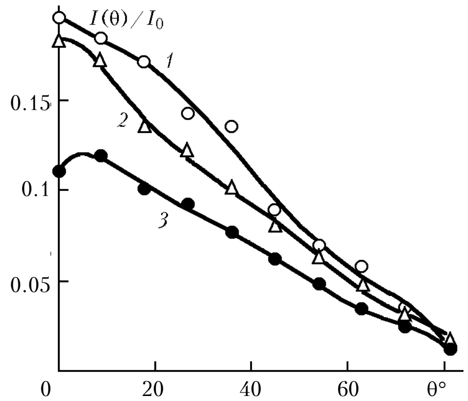


**Fig. 5.** Dependence of the normalized radial intensity $I(z, \rho, \theta)/I_0$ on the polar angle θ at the depth $z$ = 700 μm for different values of the cross coordinate ρ: 200 (*1*), 500 (*2*), and 800 μm (*3*).

Using computer simulation by the Monte Carlo method, we have studied propagation of a light beam with the Gaussian statistics of photons in the blood layer up to 800 μm thick. Statistical estimates of functionals, such as radiation brightness, intensity over the beam cross section, and the peak intensity on the beam axis have been obtained.

## References


1. V.A. Donchenko and M.V. Kabanov, *Scattering of Optical Waves by Disperse Media*. Part 2. *System of Particles* (TA SB AS USSR, Tomsk, 1983), 185 pp.
2. A.V. Priezzhev, V.V. Tuchin, and L.P. Shubochkin, *Laser Diagnostics in Biology and Medicine* (Nauka, Moscow, 1989), 73 pp.
3. L.S. Dolin and V.A. Savel'ev, Izv. Ros. Akad. Nauk, Ser. Fiz. Atmos. Okeana **36**, No. 6, 794–801 (2000).
4. V.I. Bukatyi, T.K. Kronberg, and D.V. Mikheev, Atmos. Oceanic Opt. **14**, No. 3, 210–212 (2001).
5. I.M. Sobol', *Monte Carlo Method* (Nauka, Moscow, 1985), 78 pp.
6. A. Ishimaru, *Wave Propagation and Scattering in Random Media.* Vol. 1. *Single Scattering and Transport Theory* (Academic Press, New York, 1978).
7. N.N. Chentsov and A.N. Subbotin, Vopr. Atom. Nauki i Tekhn., Ser. Metod. i Program. Chisl. Resh. Zadach Mat. Fiz., Issue 1(9), 3–9 (1982).