BACKSCATTER AMPLIFICATION EFFECTS IN LASER DETECTION AND RANGING THROUGH A TURBULENT ATMOSPHERE

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The conditions of existence and quantitative manifestations of the effects caused by correlation between oncoming waves in laser detection and ranging through a turbulent atmosphere are studied in the paper as functions of turbulent strength, reflectivity and size of objects, and angular divergence of illuminating laser beam. Possible ways of their consideration and practical application to the development of as laser radar and lidar systems as new methods of sensing of the atmospheric turbulence are examined.

1. INTRODUCTION

The use of lasers for detection and ranging of objects in the atmosphere or through it generates a need for study of the stochastic nature of backscattered coherent optical waves. Whereas the fluctuations of radar return signals in the radio wave range occur primarily due to the random nature of scattering by locatable objects, in the optical range the main factor engendering such fluctuations becomes stochastic wave scattering by the inhomogeneities of a medium located between a laser radar and a locatable object. Due to proximity and intersection of direct and returned rays during their propagation through a randomly inhomogeneous medium whose parameters remain unchanged in many instances in the course of propagation of waves, correlated random wave fields may combine within a certain solid angle about the axis connecting an object and a laser radar. Analogous situation also arises in the image plane of a locatable object when one and the same telescope is used as transmitter and receiver. As a result, the spatial localization (focusing) of the mean energy flux of reflected wave caused by the random inhomogeneities of a medium located between a laser radar and a locatable object was found to occur.

The effect of amplification of the mean intensity of the backscattered radiation was first found when solving the problem of diffraction of a spherical wave by a specular disc of finite size[1] and by a point scatterer.[2] Later on the other fundamentally new phenomena associated with the combination of correlated random fields were discovered. An enhancement of the mean intensity of the reflected wave field turned out to be accompanied by stronger intensity fluctuations,[3] and the residual spatial correlation of these fluctuations took place.[4]

Investigations performed at the Institute of Atmospheric Optics (see Refs. 1 and 4) are of fundamental importance for laser detection and ranging in random media. Since transceiving telescopes of laser radars generally form spatially bounded beams and locatable objects may have arbitrary size, shape, and surface reflectivity, the existence of the enhancement effects and their quantitative manifestation depend strongly on the above–indicated factors. The influence of these factors was studied in detail in series of papers published by the scientists of the Institute and generalized in Ref. 5. Here a special problem is the study of the enhancement effects in the image plane of a locatable object of a receiving telescope.[6,7]

The present paper gives a review of the enhancement effects in the problems of laser detection and ranging through a turbulent atmosphere, studied at the Institute of Atmospheric Optics. The emphasis is on the study of the conditions of existence of the enhancement effects as well as ways for their allowance for the development of laser radar systems and new methods of sounding of the atmospheric turbulence.

2. FORMULATION AND METHODS FOR SOLVING THE PROBLEMS ON LASER DETECTION AND RANGING IN TURBULENT MEDIA

Let $U(x', \rho')$ be the field of laser radiation propagating along the ox' axis. In terms of the Green's function of a propagation medium, the field $U(x', \rho')$ can be represented in the form

$$U(x', \rho') = \int \mathrm{d}\mathbf{t}\, U_0(\mathbf{t})\, G(x', x_0; \rho', \mathbf{t}) , \tag{2.1}$$

where $U_0(\mathbf{t}) = U(x', \mathbf{r}')\big|_{x' = x_0}$ is the initial field, $G(x', x_0; \rho', \mathbf{t}) = \delta(\rho' - \mathbf{t})$, $\delta(\mathbf{t})$ is the Dirac delta function, and ρ' and $\mathbf{t}$ are the two–dimensional vectors.

Let the reflection occur in the plane $x' = x$. We introduce the function $V(\rho', \mathbf{r})$, characterizing a local reflectance. Then the field on a reflecting surface is given by the formula[8]

$$U^R_0(\rho) = \int \mathrm{d}\rho' U(x, \rho')\, V(\rho', \mathbf{r}), \tag{2.2}$$

and using integral relation (2.1), for the field of a reflected wave in the plane $x' < x$ we obtain:

$$U^R(x', \rho) = \int \mathrm{d}\mathbf{r}\, U^R_0(\mathbf{r})\, G^R(x', x; \rho, \mathbf{r}) . \tag{2.3}$$

The Green's functions for the direct G and backward G^R propagation satisfy the conjugate equations of parabolic type[8] and due to this fact are related by the reciprocity relation:

$$G^R(x', x; \rho, \mathbf{r}) = G(x, x'; \mathbf{r}, \rho). \tag{2.4}$$

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This enables one to reduce the problem of laser detection and ranging to that of the direct propagation.[5,8] As a result, with the use of Eqs. (2.3), (2.4), (2.2), and (2.1), for the field of a reflected wave we have

$$U^R(x', \rho) = \int \mathrm{d}\,\mathbf{t}\,\mathrm{d}\,\rho'\,\mathrm{d}\,\mathbf{r}\,U_0(\mathbf{t})\,G(x, x_0; \rho', \mathbf{t}) \times$$

$$\times\, G(x, x'; \mathbf{r}, \rho)\,V(\mathbf{r}, \rho')\,. \qquad (2.5)$$

Equation (2.5) provides a basis for analyzing the moments of the reflected field of the order $2n$

$$U^R_{2n}(x', \rho_{\underline{2n}}) = U^R(x', \rho_1)\,U^{R*}(x', \rho_2) \times \ldots \times$$

$$\times\, U^R(x', \rho_{2n-1})\,U^{R*}(x', \rho_{\underline{2n}}) \qquad (2.5')$$

and their statistical means or mutual coherence functions of the corresponding order

$$\Gamma^R_{2n}(x', \rho_{\underline{2n}}) = \langle U^R_{2n}(x', \rho_{\underline{2n}})\rangle, \qquad (2.6)$$

where $\underline{2n}$ denotes the change in the subscript from 1 to $2n$. Averaging in Eq. (2.6) is carried out on the assumption of statistical independence of the fluctuations of the parameters of a medium, initial field, and reflector, which may be random as well. As a result, for $x' = x_0$ we have

$$\Gamma^R_{2n}(x_0, \rho_{\underline{2n}}) = \int\int\int \langle U_{2n}(\mathbf{t}_{\underline{2n}})\rangle \langle V_{2n}(\rho'_{\underline{2n}}, \mathbf{r}_{\underline{2n}})\rangle \times$$

$$\times \langle \tilde{G}_{2n}(x, x_0; \rho'_{\underline{2n}}, \mathbf{t}_{\underline{2n}}; \mathbf{r}_{\underline{2n}}, \rho_{\underline{2n}})\rangle\,\mathrm{d}\,\rho'_{\underline{2n}}\,\mathrm{d}\,\mathbf{t}_{\underline{2n}}\,\mathrm{d}\,\mathbf{r}_{\underline{2n}}\,, \qquad (2.7)$$

where

$$\tilde{G}_{2n} = \prod_{j=1}^{n} \tilde{G}(x, x_0; \rho'_{2j-1}, \mathbf{t}_{2j-1}; \mathbf{r}_{2j-1}, \rho_{2j-1}) \times$$

$$\times\, \tilde{G}^*(x, x_0; \rho'_{2j}, \mathbf{t}_{2j}; \mathbf{r}_{2j}, \rho_{2j})\,, \qquad (2.8)$$

$\tilde{G} = G(x, x_0; \rho', t)\,G(x, x_0; \mathbf{r}, \rho)$ is the laser radar Green's function (LRGF), which was first introduced in Ref. 9,

$$U_{2n}(\mathbf{t}_{\underline{2n}}) = \prod_{j=1}^{n} U_0(\mathbf{t}_{2j-1})\,U_0^*(\mathbf{t}_{2j}), \qquad (2.9)$$

$$V_{2n}(\rho'_{\underline{2n}}, \mathbf{r}_{\underline{2n}}) = \prod_{j=1}^{n} V(\rho'_{2j-1}, \mathbf{r}_{2j-1})\,V^*(\rho'_{2j}, \mathbf{r}_{2j})\,. \qquad (2.10)$$

In the case of a coherent laser source the initial distribution of the field $U_0(t)$ across the radiating aperture is usually assumed to follow the form of the Gaussian beam with an effective radius of the output aperture a, a curvature of the phase front F, and an amplitude U_0 in its center. Statistical moments of the initial field of partially coherent laser beams are assigned in a more complicated form. In particular, for the function of mutual spatial coherence of the second order $\langle U_2\rangle$ we have[10]:

$$\langle U_2(\mathbf{t}_{\underline{2}})\rangle = U_0^2 \exp\left\{ -\frac{t_1^2 + t_2^2}{2a^2} - i\,\frac{k}{2F}(t_1^2 - t_2^2) - \frac{(\mathbf{t}_1 - \mathbf{t}_2)^2}{4a_c^2} \right\}, \qquad (2.11)$$

where a_c is the effective radius of spatial coherence of radiation and $k = 2\pi/\lambda$ is the wave number. On the assumption of strong phase fluctuations of the source field the fourth moment $\langle U_4\rangle$ is given by the formula

$$\langle U_4(\mathbf{t}_{\underline{4}})\rangle = \langle U_2(\mathbf{t}_{\underline{2}})\rangle \langle U_2(\mathbf{t}_3, \mathbf{t}_4)\rangle\,, \qquad (2.12)$$

if the coherence time of the source τ_s is less than the receiver averaging time τ_r, or by the formula

$$\langle U_4(\mathbf{t}_{\underline{4}})\rangle = \langle U_2(\mathbf{t}_{\underline{2}})\rangle \langle U_2(\mathbf{t}_3, \mathbf{t}_4)\rangle + \langle U_2(\mathbf{t}_1, \mathbf{t}_4)\,U_2^*(\mathbf{t}_2, \mathbf{t}_3)\rangle\,, \qquad (2.13)$$

if the inverse condition $\tau_s > \tau_r$ is fulfilled.[11,12]

The local reflectance in the case of specular targets is assigned as[5]

$$V(\rho', \mathbf{r}) = A(\mathbf{r})\,\delta(\rho' \pm \mathbf{r})\,, \qquad (2.14)$$

where $A(\mathbf{r}) = \exp\{-r^2/2a_r^2\}$, a_r is the effective radius of a reflector, V_0 is the amplitude factor, the minus sign corresponds to a flat mirror, and the plus sign – to a corner–cube reflector.

In the case of a diffuse surface with random reflectance we have for $\langle V_2\rangle$[12,13]:

$$\langle V_2(\rho'_{\underline{2}}, \mathbf{r}_{\underline{2}})\rangle = \frac{4p}{k^2} A(\mathbf{r}_1)\,A^*(\mathbf{r}_2)\,\delta(r_1 - r_2)\,\delta(\rho'_1 - \mathbf{r}_1)\,\delta(\rho'_2 - \mathbf{r}_2)\,, \qquad (2.15)$$

and the fourth moment $\langle V_4\rangle$, depending on the relation between the time of the receiver averaging τ_r and the correlation time of surface roughness fluctuations τ_c, is represented as[12,14]

$$\langle V_4(\rho'_{\underline{4}}, \mathbf{r}_{\underline{4}})\rangle = \langle V_2(\rho'_{\underline{2}}, \mathbf{r}_2)\rangle \langle V_2(\rho'_{3,4}, \mathbf{r}_{3,4})\rangle, \quad \tau_c < \tau_r\,, \qquad (2.16)$$

$$\langle V_4(\rho'_{\underline{4}}, \mathbf{r}_{\underline{4}})\rangle = \langle V_2(\rho'_{\underline{2}}, \mathbf{r}_{\underline{2}})\rangle \langle V_2(\rho'_{3,4}, \mathbf{r}_{3,4})\rangle +$$
$$+ \langle V_2(\rho'_{1,4}, \mathbf{r}_{1,4})\rangle \langle V_2(\rho'_{2,3}, \mathbf{r}_{2,3})\rangle\,, \quad \tau_c \gg \tau_r\,. \qquad (2.17)$$

Thus to calculate the reflected wave mutual coherence functions $\Gamma^R_{2n}(x_0, \rho_{2n})$, it is necessary to know the statistical moments of the laser radar Green's function of the corresponding order. Equations for them[5,15,16] follow from the parabolic equation for the laser radar Green's function[9]

$$2ik\frac{\partial \tilde{G}}{\partial x'} + (\Delta\rho' + \Delta\rho)\,\tilde{G} + \kappa^2(\varepsilon_1(x, \rho) + \varepsilon_1(x', \rho'))\,\tilde{G} = 0 \quad (2.18)$$

with the boundary condition $\tilde{G}(x_0; x_0; \rho', \mathbf{t}; \mathbf{r}, \rho) = \delta(\rho' - \mathbf{t}) \times \delta(\mathbf{r} - \rho)$, obtained on the basis of reciprocity relation (2.4) (see Ref. 8). Therefore, the methods for solving the problems on laser detection and ranging using the functions of mutual coherence (2.7) are based mainly on one or other approximations when solving the equations for statistical moments of LRGF. Among them are the perturbation method[17] or the asymptotic technique[15,16] described in detail in Refs. 5 and 12. The approximation methods[18–20] also can be used for solving these problems with due care since application of the methods described, for example, in Refs. 19 and 20 to the solution of the problems on laser detection and ranging may yield incorrect results owing to violation of the reciprocity relation for the fields of direct and backward waves derived with the use of these approximations.

3. CONDITIONS OF EXISTENCE AND REGIONS OF LOCALIZATION OF THE ENHANCEMENT EFFECTS IN THE PROBLEMS ON LASER DETECTION AND RANGING

Let us consider the conditions of existence and regions of localization of the enhancement effects depending on the parameters of spatially bounded laser beam emitted by a transmitting telescope and size and reflectance of a locatable object for arbitrary intensity of atmospheric temperature micropulsations and path length.

Before proceeding to an analysis of concrete results, we write down the general relationship[12,16] between the even moments of the reflected $U^R_{2\,n}$ and incident $U_{2\,n}$ fields, using Eqs. (2.1), (2.5′), and (2.8)–(2.10) for the case of backscattering of a spherical wave ($U(\mathbf{t}) = 2\pi\delta(k\mathbf{t})$) by a point ($A(\mathbf{r}) = 2\pi\, V_0\delta(k\mathbf{r})$) reflector

$$U^R_{2\,n}(\,x_0,\,0) = |U_0|^{-\,2\,n}\,|V_0|^{2\,n}\,[U_{2\,n}(\,x,\,0)]^2. \qquad (3.1)$$

Equation (3.1) yields the relationship between the statistical moments of incident and reflected intensities for the case of spherical waves and a point scatterer

$$<(I^{\,R}(\,x_0,\,0))^n> = |U_0|^{-\,2\,n}\,|V_0|^{2\,n} < I^{\,2\,n}(\,x,\,0)> . \qquad (3.2)$$

Analogously it can be shown that the mean intensity of a spherical wave reflected from an unbounded mirror ($A(\mathbf{r}) \equiv V_0$) is given by the expression

$$< I^R(\,x_0,\,\mathbf{R})> = \frac{|U_0|^2\,|V_0|^2}{4\,k^2\,L^2}\,[1 + B_{I,\,S}(\,x,\,\mathbf{R})], \qquad (3.3)$$

where $L = x - x_0$, $\mathbf{R}$ is the radius–vector of the observation point in the plane perpendicular to the direction of propagation, $B_{I,\,S}(\,x,\,\mathbf{R})$ is the correlation function of the intensity of a spherical wave on the forward propagation path, normalized to the square of the mean intensity. Analogous expression which agrees to within the constant is also derived for a spherical wave scattered by a point reflector.[2]

3.1. Enhancement of the mean intensity

It follows from Eq. (3.3) that in the strictly backward direction ($\mathbf{R} = 0$) the mean intensity of a reflected spherical wave increases as compared with the propagation in a homogeneous medium by the amount determined by the variance of the spherical wave intensity $\sigma^2_{I,\,S} = B_{I,\,S}(0)$, and this effect still retaines as the observation point is displaced at an angle $\theta \sim r_I / L$, where r_I is the scale of spatial correlation of the intensity fluctuations.

As has already been noted in Introduction, for the case of spherical wave scattering by a specular disc in the regime of weak optical turbulence, with the parameter $\beta_0^2 = 1.23\, C_n^2 k^{7/6}\, L^{11/6}$ being less than unity, backscatter amplification effect was studied in Ref. 1, and for the case of point reflector it was considered in Ref. 2. Here C_n^2 is the structure constant of the air refractive index fluctuations in the atmosphere.

In general, the amount of enhancement depends essentially on the reflector size and the angular divergence of an incident laser beam.[5,12,22]

For quantitative estimation of the mean intensity enhancement we introduce the factor $N(\mathbf{R}) = {<}I^{\,R}(x_0,\,\mathbf{R}){>}/{<}I^{\,R}(x_0,\,\mathbf{R}){>}_{\rm incoh}$, where ${<}I^{\,R}(\,x_0,\,\mathbf{R}){>}_{\rm incoh}$ corresponds to the mean intensity of a reflected wave neglecting the correlation between the incident and backward wave fields. For spatially unbounded plane ($\Omega = k\,a^2 / L \gg 1$) or spherical ($\Omega \ll 1$) waves ${<}\,I^{\,R}{>}_{\rm incoh}$ coincides with the intensity in a homogeneous medium, and in this case $N(\mathbf{R})$ characterizes absolute amplification of the mean intensity.

Figure 1 shows the results of calculation of the normalized parameter $\tilde{N}(\mathbf{R}) = (N(\mathbf{R}) - 1) / \beta_0^2$ performed in Refs. 12 and 22 for the case of weak optical turbulence and arbitrary values of the diffraction parameters Ω and $\Omega_r = k\,a_r^2 / L$ for a flat mirror, a corner–cube reflector, and a diffuse surface with random reflectivity. It is seen that the correlation between direct and backward waves leads to the decrease of the mean intensity of reflected radiation rather than to its increase, if a spatially bounded beam with finite diffraction size $\Omega_{\rm eff} \sim 1$ ($\Omega^{-1}_{\rm eff} = \Omega^{-1} + \Omega_r^{-1}$) is formed at the mirror. In the case of the plane wave incidence on an unbounded mirror ($\Omega_r \gg 1$) the amplification effect is absent, as is seen from the figure.

In contrast to a flat mirror, at $\Omega_{\rm eff} \simeq 1$ the mean intensity of radiation reflected from a corner cube reaches its maximum rather than decreases. When a spherical wave is scattered by an unbounded corner–cube reflector with $\Omega_r \gg 1$, the amplification effect, in analogy with the case of a mirror, is determined by the intensity correlation function of a plane wave rather than of a spherical wave, and $N(0) = 1 + \beta_0^2$ (see Ref. 23). The mean intensity of reflected radiation also increases when a plane wave is incident on an unbounded corner–cube reflector. In this case the amplification factor in the direction $\mathbf{R} = 0$ is defined by the parameter $N(0) = 1 + 1.56\,\beta_0^2$.

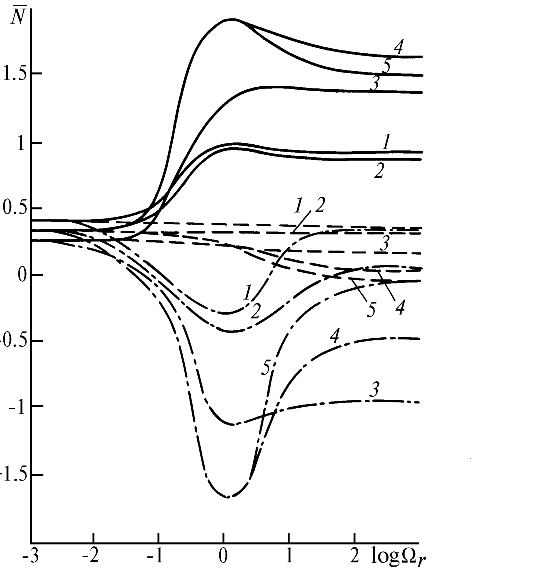


FIG. 1. Backscatter amplification factor $\tilde{N} = \tilde{N}(0)$ in the regime of weak fluctuations: $\Omega = 10^{-3}$ (1), 10^{-1} (2), 1 (3), 10 (4), and 10^3 (5); corner–cube reflector (solid curve), Lambertian reflector (dashed curve), and flat mirror (dot–dash curve).

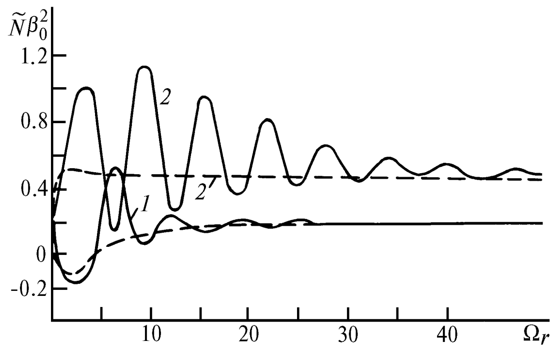


FIG. 2. Amplification factor $\tilde{N}(0)\beta_0^2$ vs. the Fresnel number of a reflector ($\beta_0^2 = 0.5$) for spherical incident wave reflected from a mirror (1) and corner–cube reflector (2); rigorous calculation (solid curves) and calculation by the Gaussian model of $A(\mathbf{r})$ (dashed curves).

Consideration of diffraction by the edges of a reflector[42,43] leads to more complicated pattern of the dependence of enhancement of backscattering on the diffraction parameters than that shown in Fig. 1 for the Gaussian model of reflectivity. The example of calculation of the amplification factor $\tilde{N}(0)\beta_0^2$ with regard for diffraction by the edges of a reflector is shown in Fig. 2. As is seen, the behavior of $\tilde{N}(\Omega_r)$ differs essentially from that shown in Fig. 1 (cf. curves in Fig. 2 with the corresponding curves for mirror and corner–cube reflector in Fig. 1). For both flat mirror and corner–cube reflector the dependence is oscillating in character. As is seen, along with the decrease of the mean intensity of a reflected wave for $1 < \Omega_r < 4$, which is also described by the Gaussian model, the allowance for the diffraction by reflector edges leads to essential amplification of the mean intensity in the case of reflection from a mirror whose size satisfies the condition $5 < \Omega_r < 10$.

The enhancement of backscattering for a point reflector was experimentally confirmed for scattering of a divergent laser beam under conditions of artificial convective turbulence[24,25] and on real atmospheric path.[26] The intensity amplification for a spherical wave reflected from a mirror was not observed in Ref. 24. Conceivably, this is due to the fact that after reduction of the aperture of a beam incident on a mirror in the experiment,[24] the diffraction size of a reflecting segment of the mirror gives no way of observing the amplification effect (see Figs. 1 and 2). Only recently more careful setting up of the experiment has made it possible to obtain the experimental data that confirm the existence of the amplification effect for the case of reflection from an infinite flat mirror on a real path in the atmosphere,[27] as well as nonmonotonic character of the dependence of the amplification factor on the reflecting mirror size (see Fig. 1) in the laboratory experiments.[28]

Let us consider the amplification of the mean intensity in the regime of strong optical turbulence, with the parameter β_0^2 exceeding unity. In this case the regimes of plane and spherical waves and spatially–bounded beam are characterized by the conditions $\Omega \gg \beta_0^{12/5}$, $\Omega \ll \beta_0^{-12/5}$, and $\beta_0^{-12/5} \ll \Omega \ll \beta_0^{12/5}$, respectively. The same conditions characterize an "infinite" reflector, a "point" scatterer, and a reflector of finite size: $\Omega_r \gg \beta_0^{12/5}$, $\Omega_r \ll \beta_0^{-12/5}$, and $\beta_0^{-12/5} \ll \Omega_r \ll \beta_0^{12/5}$.

Let us represent the mutual coherence function of a reflected wave in the form:

$$\Gamma_2^R(x_0, \underline{\rho_2}) = \Gamma_2^{(1)}(x_0, \underline{\rho_2}) + \Gamma_2^{(2)}(x_0, \underline{\rho_2}) , \tag{3.4}$$

where $\Gamma_2^{(1)}$ describes the coherence function of the field propagating along a direct path of doubled length without reflection, and $\Gamma_2^{(2)}$ is responsible for the correlation between direct and backward waves. As follows from the analysis of corresponding asymptotic expressions for $\Gamma_2^{(1)}$ and $\Gamma_2^{(2)}$, given in Refs. 5 and 12, the amplification of the mean intensity in the regime of strong turbulence is pronounced only in the case of a spherical wave incidence on a reflector. Both terms in Eq. (3.4) become then comparable, and the mean intensity of reflected spherical wave is more than twice as large as the intensity on a direct path of doubled length.[29] In this case the amplification factor $N(0)$ can be represented as

$$N(0) = 2 + f(\Omega_r, \beta_0)\beta_0^{-4/5}, \tag{3.5}$$

where $f(\Omega_r, \beta_0)$ is some function depending on the reflector size and the turbulence parameter β_0. For an infinite mirror and a point scatterer Eq. (3.5) assumes the form:

$$N(0) = 2 + 2.74\, \beta_0^{-4/5} \tag{3.6}$$

and can be obtained immediately from Eq. (3.3) if for $B_{I,S}$ its representation for the regime of strong turbulence is used (see Ref. 12).

In all the other cases, when $\Omega \gtrsim \beta_0^{-12/5}$, the amplification effect is insignificant and is of the order of $\beta_0^{-4/5}$ (see Refs. 5 and 12):

$$N(\mathbf{R}) = 1 + g(\mathbf{R}, \Omega_r, \beta_0)\, \beta_0^{-4/5} ,$$

where $g(\mathbf{R}, \Omega_r, \beta_0)$ is some function.

The conditions of existence and the regions of localization of the backscatter amplification effect for $\beta_0^2 >> 1$ in the cases of a corner–cube reflector and a Lambertian surface[5,12,16,23] do not differ essentially from those stated above.

3.2. Enhancement of the intensity fluctuations

Correlation between waves that propagate in forward and backward directions through the same inhomogeneities of a medium also results in the amplification of the intensity fluctuations of reflected field.[3] The amount of amplification of the intensity fluctuations varies within wide limits depending on the parameters of initial laser beam, size and type of reflector, and strength of optical turbulence β_0^2. Let us consider first the regime of weak optical turbulence.[17]

Shown in Fig. 3 are the results of calculations of relative variance of the axial intensity of a collimated beam reflected from a mirror for $\beta_0^2 < 1$. It is seen that the intensity fluctuations are minimum when the mirror size is comparable with the radius of the first Fresnel zone ($\Omega_r \simeq 1$) and maximum in the case of reflection from an "infinite" mirror.

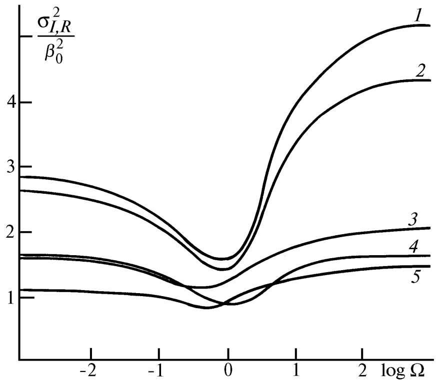


FIG. 3. Relative variance of the weak intensity fluctuations of a collimated beam reflected strictly backward, $\Omega_r = 10^3$ *(1),* 10 *(2),* 10^{-3} *(3),* 1 *(4), and* 10^{-1} *(5).*

After reflection from a corner cube, a preferred direction (optical axis) occurs, and the intensity variance of a plane wave reflected from an unbounded reflector $\sigma^2_{I,R}$ decreases from $5.23\beta_0^2$ to $3.56\beta_0^2$ as the observation point is displaced off the axis at a distance $R \gg \sqrt{\lambda L}$, while after reflection from a flat mirror $\sigma^2_{I,R}$ is independent of the position of a reception point.

In the regime of strong turbulence, the increase of the relative variance of the intensity is maximum in the case of scattering by a point reflector. As compared with the one–way propagation along a path, when the saturation level of the relative intensity fluctuations with $\beta_0^2 \to \infty$ equals unity, this level is several times higher[15,16]:

$$\sigma^2_{I,R} = \frac{5 + 46.6\,\beta_0^{-4/5}}{1 + 2.74\,\beta_0^{-4/5}} + o(\beta_0^{-8/5}),\quad \Omega \ll \beta_0^{-12/5}, \tag{3.7}$$

$$\sigma^2_{I,R} = \frac{3 + 14.6\,\beta_0^{-4/5}}{1 + 0.86\,\beta_0^{-4/5}} + o(\beta_0^{-8/5}),\quad \Omega \gg \beta_0^{12/5}\,. \tag{3.8}$$

When the condition $\Omega_r \ll \beta_0^{-12/5}$ is violated, the effect of amplification of the intensity fluctuations becomes negligible.[5,16]

Let us now turn to scattering by a diffuse surface. In this case we should take into account that the receiver can both follow the fast random variations of the reflected radiation field due to varying reflectivity of a diffuse surface, if the correlation time of the surface roughness $\tau_c > \tau_r$, and smooth them out, when the relation $\tau_c \ll \tau_r$ is fulfilled. Making use of appropriate models for the fourth moment of the function $V(\underline{\rho}'_4, \mathbf{r}_4)$ given by Eq. (2.17), for the relative variance of the weak intensity fluctuations of a spherical wave reflected from a diffuse surface for $\tau_c \ll \tau_r$ we find[5,12]

$$\sigma^2_{I,R}(\mathbf{R}) = 2\sigma^2_{I,g} + 2\,B_{I,g}(\mathbf{R})\,. \tag{3.9}$$

In Eq. (3.9) the function $B_{I,g}(\mathbf{R})$ coincides with the intensity correlation function of an incoherent source in the turbulent atmosphere[12] emitting backward from the reflector plane, $\sigma^2_{I,g} = B_{I,g}(0)$.

It follows from Eq. (3.9) that in the directions close to backward ($|\mathbf{R}| \gtrsim \sqrt{\lambda L}$) the amplification of the intensity fluctuations occurs. Actually, $\sigma^2_{I,R}(0) = 4\sigma^2_{I,g}$ whereas for $|\mathbf{R}| \gg \sqrt{\lambda L}$ $\sigma^2_{I,R} = 2\sigma^2_{I,g}$. In the case of a point reflector, the relative fluctuations of the reflected spherical wave intensity are maximum. The increase of the size of a diffuse surface results in their decrease. In particular, from Eq. (3.9) we have

$$\sigma^2_{I,R}(0) = 1.6\beta_0^2 + o(\beta_0^4)\,,\quad \Omega_r \ll 1\,,$$
$$\sigma^2_{I,R}(0) = 0.68\beta_0^2 + o(\beta_0^4)\,,\quad \Omega_r = 1\,,$$
$$\sigma^2_{I,R} \sim \Omega_r^{-7/6}\beta_0^2\,,\quad \Omega_r \gg 1\,, \tag{3.10}$$

whence it appears that in the limit $\Omega_r \to \infty$ the intensity of reflected radiation does not fluctuate.

The analogous results are also obtained in the case of a plane wave incident on a diffuse surface.[5,12]

In the regime of strong optical turbulence when spherical and plane waves are reflected from a diffuse surface of small transverse size $\Omega_r \ll \beta_0^{-12/5}$, the relative intensity variance is described, in analogy with the case of a mirror, by formulas (3.7) and (3.8). For a diffuse disc of finite size we have[5,12]

$$\sigma^2_{I,R}(0) = 5.48\beta_0^{-4/5} + o(\beta_0^{-8/5})\,,\quad \Omega \ll 1\,, \tag{3.11}$$

$$\sigma^2_{I,R}(0) = 2.6\beta_0^{-4/5} + o(\beta_0^{-8/5})\,,\quad \Omega \gg 1\,. \tag{3.12}$$

It follows from Eqs. (3.11) and (3.12) that with the increase of optical turbulence strength ($\beta_0 \to \infty$), the relative variance of radiation, reflected from a surface of finite size tends to zero for $\tau_c \ll \tau_r$. This means that diffuse surface illuminated by a coherent light can be considered as a source of incoherent spherical waves, whose intensities are added at a point of reception. In this case, as shown in Refs. 30 and 12, the relative variance of the strong intensity fluctuations has no nonzero saturation level.

Increase of the size of a diffuse surface ($\Omega_r \gg \beta_0^2$) results in smoothing out of the strong intensity fluctuations of reflected radiation in accordance with the law described by Eq. (3.10) as in the case of weak intensity fluctuations.

As the analysis has shown,[5,12,16] under condition $\tau_c \gtrsim \tau_r$, when a receiver have time to follow the signal fluctuations due to phase distortions engendered by a surface, the intensity variance of reflected field is defined by the formula[12]

$$\sigma^2_{I,R}(R) = 1 + 2\sigma^2_{I,RS}(R)\,, \tag{3.13}$$

where $\sigma^2_{I,RS}$ coincides with the intensity variance of reflected radiation for $\tau_c \ll \tau_r$. Thus the significant increase of the intensity fluctuation variance due to the amplification of the intensity fluctuations in accordance with formulas (3.13), (3.7), and (3.8) takes place for scattering by a "point" random reflector, whose transverse size satisfy the condition $\Omega_r \ll b_0^{-12/5}$. The saturation level of $\sigma^2_{I,R}$ with $\beta_0^2 \to \infty$ in the case of spherical wave incident on such a reflector equals eleven, while in the case of an incident plane wave it equals seven. The increase of the size of a diffuse surface results in smoothing out of the intensity fluctuations of reflected radiation, and when the condition $\Omega_r \gtrsim b_0^{-12/5}$ is satisfied, the relative variance of the intensity only slightly exceeds unity for any value of the turbulence parameter β_0^2 (see Refs. 5, 12, and 16).

In a number of particular cases the theoretical results concerning the intensity fluctuations of reflected radiation, presented in this section, are confirmed by the experimental results.[25,31,32]

4. RESIDUAL SCINTILLATIONS OF REFLECTED RADIATION RECEIVED BY A LARGE APERTURE

Reception on a large receiving aperture is one way of decreasing the fluctuations of received radiation. A telescope with an objective lens whose radius D far exceeds the intensity correlation radius r_I ($D \gg r_I$) smoothes out practically completely scintillations in the images of stars and laser sources.[33] Therefore, in measuring the intensity fluctuations of optical waves the condition $D \ll r_I$ must be satisfied.

At the same time, as was first shown in Ref. 4, the fluctuations of the reflected radiation after its double passage through the same randomly inhomogeneous medium may not smooth out by a receiver of even arbitrary large size ($D \gg r_I$).

Let a source of spherical wave and a scatterer of radius $a_R \ll r_I$ spaced at the distance L be placed in a turbulent medium. From Ref. 4 it follows that we can write the intensity correlation function of reflected wave in the source plane in the form

$$B_{I,\,R}(\rho_1, \rho_2) = \sigma_I^2 + B_{I\,r}(\rho_1) + B_{I\,r}(\rho_2) + B_r(\rho_1, \rho_2) . \quad (4.1)$$

Here σ_I^2 is the incident spherical wave intensity variance about the scatterer, $B_{I\,r}$ are the mutual intensity correlation functions of incident and returned waves at the observation points ρ_1 and ρ_2, and B_r is the intensity correlation function of the returned spherical wave.

It follows from Eq. (4.1) that the intensity fluctuations of reflected wave are inhomogeneous. The variance of the intensity fluctuations of scattered field $\sigma_{I,\,R}^2(\rho) = \sigma_I^2 + 2B_{I\,r}(\rho) + \sigma_r^2$ depends on the mutual correlation term $B_{I\,r}$ and is maximum in the strictly backward direction ($\rho = 0$).

For infinite separation of the observation points $\rho_1 - \rho_2 \to \infty$ the function $B_{I,\,R}(\rho_1, \rho_2)$ does not vanish and preserves nonzero residual correlation degree depending on the position of the observation points relative to the source ($\rho = 0$). For $|\rho_1|$, $|\rho_2| \to \infty$ the degree of residual correlation equals $\sigma_I^2/4$, whereas for $|\rho_1| = 0$ and $|\rho_2| \to \infty$ it equals $\sigma_I^2/2$.

Residual correlation is due to random nature of the source equivalent to the scatterer and is determined by the fluctuations of the incident wave field that are correlated at any point of the observation plane after reflection from the point scatterer. This effect is independent of the scattering and observation angles, and as was shown in Ref. 17, arises at any parameters of laser beam incident on point scatterer. Residual correlation also takes place when the observation plane and the source plane are separated. In this case the degree of residual correlation is determined by mutual correlation between the intensities of the incident and returned waves passed through the paths of different length $B_{I\,r}(L_1, L_2, \rho)$.

The phenomenon of residual intensity correlation is of great practical importance. It leads to qualitatively new results at reception of reflected radiation by large–aperture telescopes. The calculations of smoothing function $G(D) = \sigma_p^2(D)/\sigma_p^2(0)$, where $\sigma_p^2(D)$ and $\sigma_p^2(0)$ are respectively the relative fluctuations of a light flux transmitted through the objective of radius D and a point aperture, performed in Ref. 4 for $D \gg r_I$, yield

$$G(D) = G_1 = 1/4 ,$$

when the transverse coordinates of the centers of transmitting and receiving apertures coincide, and for $D > r_I$ yield

$$G(D) = G_2 = 1/2 ,$$

when the receiving aperture is displaced in the transverse plane at the distance $l \gg r_I$.

Thus, for reflectors of radius $a_R \ll r_I$ the fluctuations of the scattered radiation flux do not smooth out by the receiving aperture of even arbitrary large size ($D \to \infty$). As a result, the effect of residual turbulent scintillations occurs (see Ref. 4). This effect also arises when the element of area S of a smooth surface of a body forming the reflected field at the observation point satisfies the condition $\sqrt{S} \ll r_I$. The effect of residual turbulent scintillations is confirmed experimentally.34

The phenomenon of residual scintillations can be used for lidar measurements35 of vertical profiles of the structure constant of refractive index $C_n^2(\mathrm{H})$ or of the temperature structure constant $C_n^2(\mathrm{H})$ in the atmosphere. Actually, as was first established in Refs. 35 and 36, depending on the diameter of the field diaphragm d0 placed in the image plane of the aerosol scattering volume, we can distinguish three regimes of scintillations of the light flux through the receiving aperture of diameter D:

$$\sigma_p^2 = 1 + 2\sigma_I^2 , \quad d_0 \ll \lambda F_t / D , \quad (4.2)$$

$$\sigma_p^2 = \sigma_I^2 , \quad \lambda F_t / D \ll d_0 \ll F_t \sqrt{\lambda L} / L , \quad (4.3)$$

$$\sigma_p^2 = 0 , \quad F_t \sqrt{\lambda L} / L \ll d_0 , \quad (4.4)$$

where F_t is the focal length of the receiving telescope.

In regime (4.3), the variance of the fluctuations of scattered radiation flux σ_p^2 coincides with the variance of the intensity fluctuations of a sounding beam σ_I^2, and does not depend on the diameter D. This regime is referred to as the regime of turbulent scintillations.[35] Physically by this regime we meant separation, with the use of field diaphragm, of the fluctuation component in the scattering volume image, caused by turbulent fluctuations of the refractive index on the direct path to the scattering volume. This regime allows us to increase the image brightness and signal–to–noise ratio, as compared to the conventional way of fluctuation separation ($D_1 \ll \sqrt{\lambda L}$), by a factor of $k = (D d_0 L / D_1 a_s F_t)$, where $2a_s$ is the transverse size of the sounded volume, by means of increasing the receiver aperture D on retention of the intensity fluctuation strength.

Thus in regime (4.3) lidar measurements of the intensity fluctuations of laser radiation scattered by atmospheric aerosol at different altitudes H allow us to retrieve the vertical profiles of $C_n^2(H)$ in the atmosphere in analogy with Ref. 4.

The experiment performed in Ref. 35 confirms the feasibility of measuring the intensity fluctuations caused by random inhomogeneities on the direct path to the scattering volume, by means of reception of scattered radiation by the aperture whose size far exceeds the correlation length of the intensity fluctuations of the wave incident on a reflector.

5. AMPLIFICATION EFFECTS IN THE IMAGE PLANE OF A RECEIVING TELESCOPE

When estimating the effectiveness of optical radar and lidar system operation in the atmosphere, one has to take into account both "geometric" factor of a transceiver and turbulent conditions of propagation of laser radiation on the path. From Ref. 6 it follows that the condition of existence of the effects engendered by correlation between the oncoming waves behind the focusing lens can differ significantly from those occuring in the input plane. Therefore, it is important to know how much the enhancement effect changes as the observation plane l is displaced along the axis of a receiving optical system.

Figure 4 depicts the results of calculation of the factor

$$\tilde{N}(l, \mathbf{R}) = (\langle I^R(l, \mathbf{R})\rangle - I_0^R(l, \mathbf{R})) \,/\, (\beta_0^2\, I_0^2(l, \mathbf{R}))$$

characterizing the deviation of the mean intensity of the reflected wave in a turbulent atmosphere $\langle I^R(l, \mathbf{R})\rangle$ in the regime of weak optical turbulence from that in a homogeneous medium $I_0^R(l, \mathbf{R})$, performed in Ref. 12 for different diffraction size of a source, a reflector, and a receiver at $\mathbf{R} = 0$. It is evident from Fig. 4 that in the vicinity of the plane of sharp image $q = 1 + L / l^* - L / F_t = 0$ (conjugate plane) backscatter amplification effect disappears because of turbulent broadening of the intensity distribution. In the case of a plane wave incident on a point reflector (dashed curve), the factor $\tilde{N}(l, \mathbf{R})$ for $\Omega_t = k\,a_t^2 / L \geq 1$ grows large near the focus as compared with the input plane of a telescope because of the long–range correlations,6,12 with at here being the effective radius of the telescope.

The long–range correlation effect becomes more pronounced under conditions of strong optical turbulence ($\beta_0^2 \gg 1$). Actually, when reflected plane wave ($\Omega \gg \beta_0^{12}$) is focused by a lens whose size 2at satisfies the condition $\Omega_t \gg \beta_0^{12/5}$, then the mean intensity in the focus will increase more than twice as compared with the doubled–length path without reflection.6

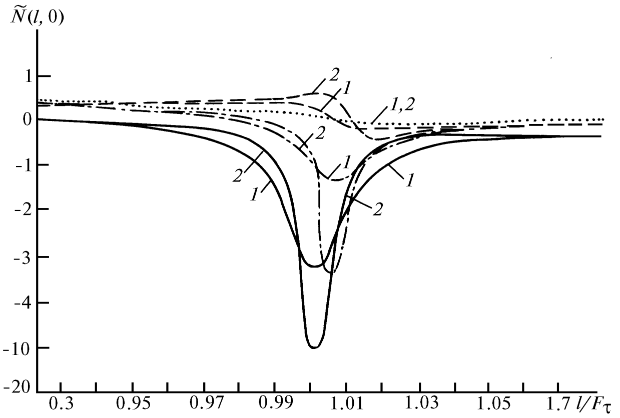


FIG. 4. Backscatter amplification effect vs. the longitudinal displacement of the image operation plane: $\Omega_t = 1$ (1) and 3 (2); plane wave and unbounded mirror (solid curve), spherical wave and unbounded mirror (dot–dash curve), plane wave and point reflector (dashed curve), and spherical wave and point reflector (dotted curve).

Thus whereas the amplification of the mean intensity of reflected plane wave is lacking in the input plane of a telescope for $\beta_0^2 \gg 1$ (see section 3.1), it occurs in the focal plane of a lens. In contrast, focusing of a reflected spherical wave by a lens for which $\Omega_t \gg \beta_0^{-12/5}$ for $\beta_0^2 > 1$ leads to vanishing of more than double amplification of the mean intensity at the input plane of a telescope, described by Eq. (3.5), in the sharp image plane of a receiving lens,[7] just as in the case of weak fluctuations (Fig. 4).

For quantitative estimation of an excess in the mean intensity of a reflected wave $\langle I^R(l, \mathbf{R})\rangle$ over that of wave passing the one–way path of doubled length $\langle I^{(1)}(l, \mathbf{R})\rangle$ (in accordance with Eq. (3.4)), we introduce the factor

$$N(l, \mathbf{R}) = \langle I^R(l, \mathbf{R})\rangle \,/\, \langle I^{(1)}(l, \mathbf{R})\rangle =$$

$$= 1 + \langle I^{(2)}(l, \mathbf{R})\rangle \,/\, \langle I^{(1)}(l, \mathbf{R})\rangle.$$

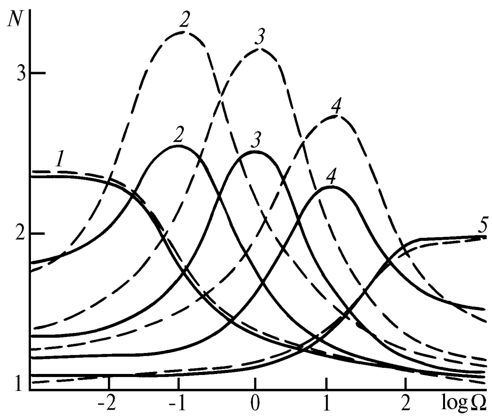


FIG. 5. Amplification factor in the focus of a telescope for a Lambertian reflector, $D_s = 50$: $\Omega_t = 10^{-3}$ (1), 10^{-1} (2), 1 (3), 10 (4), and 10^3 (5); $\Omega_r = 10^{-3}$ (solid curves) and 10^3 (dashed curves).

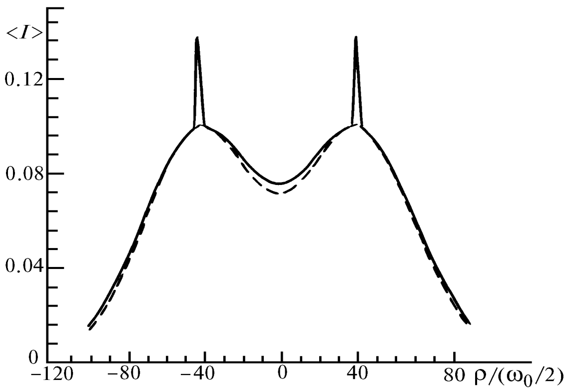
Figure 5 shows the results of calculation of the amplification factor $N = N(F_t, 0)$ of the mean intensity in the focus ($l = F_t$) of a telescope performed in Ref. 7 for different diffraction size of a source, a reflector, and an objective of a telescope. The calculation was done for a diffuse reflector taking account of the terms of the order of $\beta_0^{-4/5}$ (see Ref. 7). The figure shows that the amplification takes place only in the case of close proximity of the size of the source and receiver apertures. It is maximum when a spatially bounded light beam ($\beta_0^{-12/5} \ll \Omega \ll \beta_0^{12/5}$) is incident on the reflector and increases with the increase of the reflecting surface size.

This result becomes clear if we take into account the circumstance that the rays are coherent due to long–range correlations only in the region bounded by the size of the output aperture.[7] Therefore, the receiver whose size is less than $2a$ collects not all coherent rays, whereas the use of the receiver of larger size results in relative decrease of the contribution from the coherent component of scattered radiation $\langle I^{(2)}\rangle$ as compared with the increased contribution from the incoherent component $\langle I^{(1)}\rangle$.

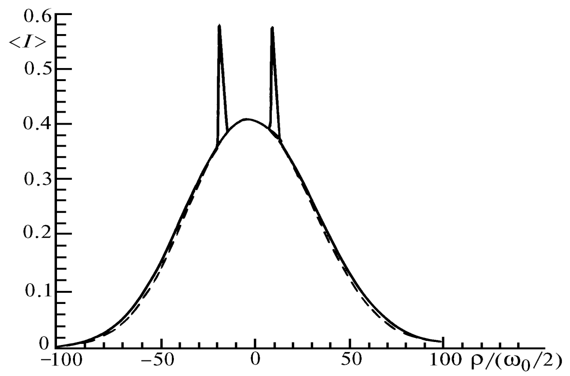
As shown in Refs. 44 and 45, correlation of the oncoming waves can lead to increasing resolution of a telescope when viewing coherently illuminated objects through the turbulent atmosphere if transmitting and receiving apertures are adjusted. We note that the increase of telescope resolution takes place in the focal plane ($l = F_t$) rather than in conjugate one ($l = l^*$).

Depicted in Fig. 6 are the results of calculation of the intensity distribution of the images of a two–point object for different distances r_s between the points. The distance r_s is scaled to the coherence length of a plane wave in a turbulent atmosphere $\rho_p = (L / 1.22\, k\, \beta_0^{12/5})^{1/2}$, the distance in the plane transverse to the telescope optical axis, scaled to $l/ka_t = \omega_0/2$, is plotted on the abscissa. It is seen from Fig. 6 that in the absence of correlation between incident and return waves the objects become unresolved both in focal and conjugate planes as the distance r_s decreases. The correlation between oncoming waves gives rise to the narrow peaks in image intensity distribution of the two–point object in the focal plane, and coherently illuminated objects can be resolved by a telescope at such separations at which they are no longer resolved in incoherent light.

Long–range correlations of a reflected field also manifest themselves through the intensity fluctuations of reflected radiation focused by a receiving lens. In particular, the saturation level of the intensity fluctuations for a plane wave scattered by a point reflector was found to grow from the value $\sigma^2_{I,\,R} = 3$ in the plane of an input pupil of a lens up to the value $\sigma^2_{I,\,R}(F_t,\, 0) = 5$ in the lens focus.[37]



a



b

FIG. 6. Intensity distribution of a two–point object image in the focal plane (solid curve) and in the conjugate plane (dashed curve), $\Omega_t = \Omega = 10$, $\beta_0 = 50$, *and* $r_s = 150$ (*a*) *and* 50 (*b*).

If the reflected spherical wave is focused, the level of the intensity variance saturation proves to be lower in the lens focus than in the input plane.[37,38]

However, these changes are of local nature. As the observation plane is displaced from the focus along the axis of an optical system ($l \neq F_t$), the relative variance of the reflected radiation intensity acquires finally the same values as those observed in the lens plane.[5,12,38] The amount of the intensity fluctuation amplification (reduction) was found to depend on the receiving lens size. The amplification of the strong intensity fluctuations as well as of the mean intensity for $\beta_0^2 \gg 1$ (Fig. 5) becomes pronounced only in the case of identical size of the source and receiver apertures.

Along with the image intensity fluctuations, jitter in the locatable object image as a whole takes place in the plane l. These random image displacements are generally characterized by the variance of random shifts of the image energy centroid ρ_t in the plane l: $\sigma_t^2 = \langle\rho_t^2\rangle - \langle\rho_t\rangle^2$.

Variance of the shifts of image of a locatable target $\sigma^2_{l,\,R}$ was considered in Ref. 39 where it was shown that under conditions of weak optical turbulence in the case of reflection of a plane wave from an unbounded mirror the variance of the jitter in the image could exceed that for a plane wave source on a one–way path by a factor of four (the effect of variance quadrupling), whereas the image of a large corner–cube reflector did not shifted (displacement compensation effect).

Investigations into the jitter in the image on ranging paths in general case of a finite reflector illuminated by a spatially bounded laser beam for arbitrary strength of the optical turbulence were performed in Refs. 11, 14, 40, and 41 and were summarized in Refs. 5 and 12.

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REFERENCES

1. M.S. Belen'kii and V.L. Mironov, Kvant. Elektron., No. 5(11), 38–45 (1972).
2. A.G. Vinogradov, Yu.A. Kravtsov, and V.I. Tatarskii, Izv. Vyssh. Uchebn. Zaved. SSSR, ser. Radiofiz. **16**, No. 7, 1064–1070 (1973).
3. A.G. Vinogradov and Yu.A. Kravtsov, in: *Abstracts of Reports at the VIth All–Union Sumposium on Wave Diffraction and Propagation,* Moscow–Erevan (1973), Vol. 1, p. 294.
4. M.S. Belen'kii and V.L. Mironov, Kvant. Elektron. **1**, No. 10, 2253–2262 (1974).
5. V.A. Banakh and V.L. Mironov, *Lidar in a Turbulent Atmosphere* (Artech House, Boston–London, 1987), 185 pp.
6. A.B. Krupnik and A.I. Saichev, Izv. Vyssh. Uchebn. Zaved. SSSR, ser. Radiofiz. **24**, No. 10, 1234–1239 (1981).
7. V.A. Banakh, Izv. Vyssh. Uchebn. Zaved. SSSR, ser. Radiofiz. **29**, No. 12, 1507–1509 (1986).
8. V.I. Gel'fgat, Akust. Zh. **22**, No. 1, 123–124 (1976).
9. V.P. Aksenov and V.L. Mironov, J. Opt. Soc. Am. **69**, No.11, 1609–1614 (1979).
10. A.I. Kon and V.I. Tatarskii, Izv. Vyssh. Uchebn. Zaved. SSSR, ser. Radiofiz. **15**, 1547 (1972).

11. V.A. Banakh, I.N. Smalikho, and B.N. Chen, Opt. Spekstrosk. **61**, No. 3, 582–586 (1986).
12. V.E. Zuev, V.A. Banakh, and V.V. Pokasov, *Optics of the Turbulent Atmosphere* (Gidrometeoizdat, Leningrad, 1988), 270 pp.
13. V.M. Orlov, I.V. Samokhvalov, G.M. Krekov, et al., *Signals and Noise in Laser Detection and Ranging* (Radio i Svyaz', Moscow, 1985), 264 pp; Journal of Soviet Laser Research **8**, No. 4, 281–428 (1987).
14. V.P. Aksenov, V.A. Banakh, V.L. Mironov, et al., Opt. Spektrosk. **61**, No. 4, 839–844 (1986).
15. V.P. Aksenov, V.A. Banakh, V.L. Mironov, et al., in: *Abstracts of Reports at the Second All–Union Conference on Atmospheric Optics,* Tomsk (1980), Vol. 2, pp. 120–130.
16. V.P. Aksenov, V.A. Banakh, and V.L. Mironov, J. Opt. Soc. Am. **A1**, No. 3, 263–274 (1984).
17. V.A. Banakh and O.V. Tikhomirova, Opt. Spektrosk. **56**, No. 5, 857–863 (1984).
18. V.A. Banakh and V.L. Mironov, Opt. Lett. **1**, No. 5, 172–174 (1977).
19. V.P. Aksenov and V.L. Mironov, Opt. Lett. **3**, No. 5, 184–186 (1978).
20. S. Frankenthal, A.M. Whitman, and H.J. Beran, J. Opt. Soc. Am. **A1**, No. 3, 585 (1984).
21. V.P. Aksenov, V.A. Banakh, and V.L. Mironov, in: *Abstracts of Reports at the Fourth All–Union Conference on Physical Principles of Information Transfer Using Laser Radiation*, Kiev (1976), p. 149.
22. V.A. Banakh, V.M. Buldakov, and I.N. Smalikho, in: *Abstracts of Reports at the XIth All–Union Symposium on Propagation Waves and Diffraction – 85*, Tbilisi (1985), Vol. 1, pp. 410–413.
23. V.P. Aksenov and V.A. Banakh, in: *Abstracts of Reports at the Sixth All–Union Symposium on Laser Radiation Propagation in the Atmosphere*, Tomsk (1981), Vol. 3, pp. 67–70.
24. A.S. Gurvich and S.S. Kashkarov, Izv. Vyssh. Uchebn. Zaved. SSSR, ser. Radiofiz. **20**, No. 5, 794–796 (1977).
25. S.S. Kashkarov, T.N. Nesterova, and A.S. Smirnov, Izv. Vyssh. Uchebn. Zaved. SSSR, ser. Radiofiz. **27**, No. 10, 1272–1278 (1984).
26. S.S. Kashkarov, Izv. Vyssh. Uchebn. Zaved. SSSR, ser. Radiofiz. **26**, No. 1, 44–48 (1983).
27. A.S. Gurvich, A.P. Ivanov, S.S. Kashkarov, G.Ya. Patrushev, and A.P. Rostov, in: *Abstracts of Reports at the XIth All–Union Symposium on Laser Radiation Propagation in the Atmosphere and Water Media*, Tomsk (1991), p. 6.
28. B.S. Agrovskii, A.N. Bogaturov, A.S. Gurvich, S.V. Kireev, and V.A. Myakinin, J. Opt. Soc. Am. **A8**, No. 7, 1142–1147 (1991).
29. V.P. Aksenov and V.L. Mironov, Izv. Vyssh. Uchebn. Zaved. SSSR, ser. Radiofiz. **22**, No. 2, 141–149 (1979).
30. V.A. Banakh, V.M. Buldakov, and V.L Mironov, Opt. Spektrosk. **54**, No. 6, 1054–1059 (1983).
31. P.A. Pincus, M.E. Fossey, J.F. Holmes, and J.R. Kerr, J. Opt. Soc. Am. **68**, No. 6, 760–762 (1978).
32. G.Ya. Patrushev, A.I. Petrov, and V.V Pokasov, Izv. Vyssh. Uchebn. Zaved. SSSR, ser. Radiofiz. **26**, No. 7, 823–831 (1983).
33. V.I. Tatarskii, *Wave Propagation in a Turbulent Medium* (Dover, New York, 1968).
34. M.S. Belen'kii, A.A. Makarov, V.L. Mironov, and V.V. Pokasov, Izv. Vyssh. Uchebn. Zaved. SSSR, ser. Radiofiz. **21**, No. 2, 299–301 (1978).
35. M.S. Belen'kii, V.L. Mironov, P.I. Netreba, V.V. Pokasov, and A.P. Shelekhov, Izv. Vyssh. Uchebn. Zaved. SSSR, ser. Fiz., No. 12, 103–105 (1986).
36. M.S. Belen'kii and A.P. Shelekhov, in: *Abstracts of Reports at the Third All–Union Conference on Atmospheric Optics and Actinometry,* Tomsk (1983), pp. 14–16.
37. V.P. Aksenov, V.A. Banakh, V.M. Buldakov, V.L. Mironov, and O.V. Tikhomirova, Kvant. Elektron. **11**, No. 5, 1022–1026 (1984).
38. V.P. Aksenov, V.A. Banakh, V.M. Buldakov, V.L. Mironov, and O.V. Tikhomirova, Kvant. Elektron. **12**, No. 10, 2136–2140 (1985).
39. V.L. Mironov and V.V. Nosov, Izv. Vyssh. Uchebn. Zaved. SSSR, ser. Radiofiz. **20**, No. 10, 1530–1533 (1977).
40. V.P. Aksenov, V.A. Banakh, and B.N. Chen, Opt. Spektrosk. **56**, No. 5, 864–868 (1984).
41. V.P. Aksenov, V.A. Banakh, and B.N. Chen, Opt. Spektrosk. **56**, No. 4, 732–734 (1984).
42. V.A. Banakh, Proc. SPIE **1968**, 312–318 (1993).
43. V.A. Banakh, Atmos. Oceanic Opt. **6**, No. 4, 229–232 (1993).
44. V.A. Banakh and B.N. Chen, Proc. SPIE **2222** (1994).
45. V.A. Banakh and B.N. Chen, Atmos. Oceanic Opt. **7**, No. 11 (1994), (in print).