INVERSE PROBLEM IN REFRACTION FOR THE IMMERSION GEOMETRY

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We consider here the inverse problem in refraction for the case of a receiver or a source immersion into the atmosphere. The problem reduces to solution of Volterra integral equation of the 2nd kind. By numerical simulations made using radiosonde data, the dependence of the solution precision on refraction measurements errors and on the observation elevation angle has been investigated.

1. INTRODUCTION

In some papers (see Refs. 1–3) the measurements of the radiation refraction in the atmosphere were used for reconstructing the refraction index height distribution along with the related parameters such as pressure and temperature. Peculiarity of any concrete problem is determined by the relative position and motion of a source, receiver and the atmosphere along with the radiation frequency range used. Refraction angle $\varepsilon(\theta_0)$ as a function of the beam arrival elevation angle is the quantity observed. Doppler shift converted into the refraction angle can be used for refraction measurements in the radio-frequency range. Without loss of generality we can also consider the case of astronomic refraction since for every extra atmospheric source its refraction is convertible into that of equivalent source infinitely removed along its beam.

On this basis, four general kinds of the measurement geometry, namely, limb (eclipse) measurements, measurements performed at some height inside the atmosphere, those made at the Earth surface or with the variable position of a source or a receiver (immersion geometry) are recognized. For every geometry there are different equations for solving the inverse problem. The radio-frequency eclipse measurements were used with great success during investigations of the atmospheres of all planets of Solar system performed from Russian and US space vehicles (see Refs. 4–6). The eclipse measurements performed at "Salut" space station and those made inside the Earth's atmosphere in the optical range gave some interesting results (see Refs. 7–9). All inverse problems can be reduced to the Abel equations. Besides the methods for solving the inverse refraction problem in the case of ground observations where the problem is reduced to the Fredholm equation of the 1st kind have been developed and implemented in the processing of the results of astronomical observations of optical refraction of stars (see Ref. 10). Investigations of the related problem for the case of measurements of the navigation satellite (EAS) radiosignal parameters are underway (see Ref. 11).

In this paper we consider the last out of above list of the inverse refraction problem formulations. At present time this kind of the problem still remains poorly studied both mathematically and experimentally.

2. STATEMENT OF THE PROBLEM

In this case equation for the inverse problem to be solved is a Volterra equation of the 2nd kind (see Ref. (12) of the following form:

$$N(p_h) - \int\limits_{p_h}^{\infty} N(p) \frac{p\, p_h \sin q_0}{(p^2 - p_h^2 \cos q_0)^{3/2}} \mathrm{d}p = 10^6 \tan\theta_0\, \varepsilon(p_h), \quad (1)$$

where $p = nr$, $r = r_0 + h$, r_0 is the Earth's radius, n is the refractive index, $N = 10^6 (n - 1)$ is the refraction factor. The profile $N(p)$ can be converted into the height one $N(h)$ by the following expression:

$$h = \{p/[1 + 10^{-6} N(p)]\} - r\,. \quad (2)$$

Corresponding beam geometry is shown in Fig. 1.

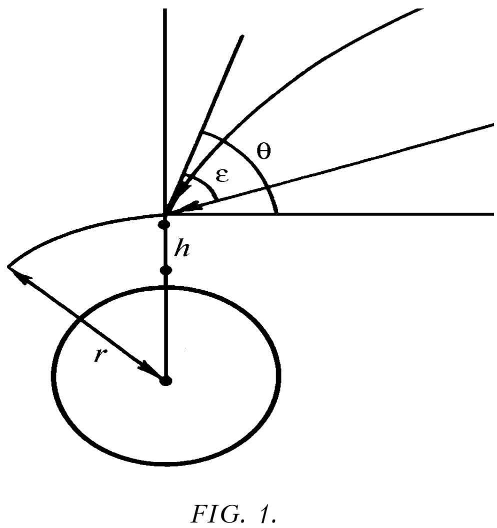

FIG. 1.

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In the general case the view of the equation kernel depends on the specific form of the immersion trajectory. For higher physical evidence let us solve the problem in the case when the object immersion occurs at a constant elevation angle.

Generally speaking Eq. (1) is nonlinear since the lower limit of integration depends on the refraction factor sought. Since $n \times r$ value only slightly differs from geocentric distance r, the equation can be solved by the iteration method using model profile of the refraction factor in the form

$$N(h) = \nu\, e^{-\beta h} \qquad (3)$$

as the first approximation.

3. RESULTS OF NUMERICAL SIMULATIONS

The $N(p)$ values calculated from the model profile were used to determine the refraction factor in the second approximation. Solution of Volterra equation of the 2nd kind is stable in computational terms. The errors in the refraction factor profile reconstructed from Eq. (2) comprise (15–20)N units, while in the second approximation these errors are 5–10 times lower. Iteration process converges quickly. Therefore two iterations are enough for the given error level.

Numerical experiment was performed on the basis of the set of data on aerological sounding obtained in the european part of Russia in summer. $N(p)$ values were calculated from the values of meteoparameters measured at heights from 0.16 to 25 km. Then, the refraction angle for a certain elevation angle was calculated from Eq. (1).

For the effect of the measurement errors on the solution accuracy to be investigated, an error simulated by a normal random, uncorrelated in angle quantity, with zero mean and standard deviation σ was "imposed" on the calculated refraction values (see Ref. 14). Climate average errors in the refractive index reconstruction δ versus height for different σ ($\theta_0 = 1°$) are shown in Fig. 2. At high heights the relative reconstruction errors were found to increase since "imposed" error became comparable to the refraction angle to be measured.

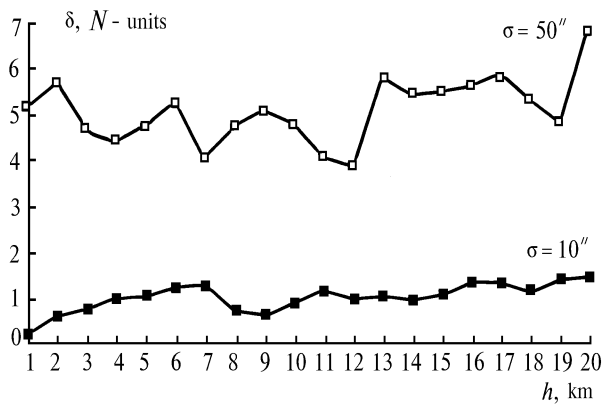

FIG. 2.

The errors in reconstruction of typical refractive index profile (pressure, temperature and relative humidity were equal to $p = 1000$ Pa, $t = 30°$C and 46%, respectively) for different elevation angles are listed in Table I. The results obtained indicate that for small measurement errors there is an angle range wherein the reconstruction errors are minimal. This is derived from the following: under Laplas-Arian theorem, as the elevation angle increases, the refraction is governed by the refractive index at the point of reception. Besides, the refraction angle absolute value decreases and at certain angles becomes comparable to the measurement errors. Thus, for every measurement error one can choose a range of elevation angles which is the most informative in measurements.

TABLE I.

	θ_0, deg						
σ, s	0.05	0.1	0.3	0.5	0.8	1	3
1	0.138	0.132	0.125	0.118	0.118	0.121	0.254
5	0.354	0.346	0.401	0.418	0.460	0.537	1.26
10	0.691	0.622	0.714	0.778	0.954	1.06	2.55
50	3.07	3.32	3.87	4.01	4.79	5.26	12.8

The prospects of the development of such a method are connected with the possibility of reconstructing the atmospheric refractive index profiles from the refraction measurements when a source or a receiver undergo immersion into the atmosphere when investigating the Solar system planets by means of descending vehicles.

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