EFFECTS OF LARGE-SCALE FLUCTUATIONS OF THE REFRACTIVE INDEX ON LIGHT BEAMS PROPAGATED THROUGH THE TURBULENT ATMOSPHERE

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Numerical analysis of light beam propagation through the turbulent atmosphere is carried out by means of Monte Carlo method. The optimization of phase screens is proposed to raise the reliability of numerical prediction of such effects as random wandering and broadening of a beam. The influence of outer scale and the turbulence strength on the statistics of power characteristics of collimated and focused beams is studied.

As known the atmospheric turbulence is characterized by an extremely wide spatial spectrum of the refractive index fluctuations n, which are hard for a comprehensive presentation on a calculation grid accessible for modern computers. At the same time the fluctuation spectrum n rapidly decreases with increasing spatial frequency, therefore the optical aberrations of low orders prevail in distortions of the light wave phase. Clear confirmation of this fact is given in Ref. 1 where the experimentally measured wave fronts of a laser beam propagated through the atmospheric ground layer are presented.

This circumstance makes a basis for the conception of separate simulation of large- and small-scale fluctuations of the refractive index.[2,3] At present this conception is widely used and allows one to analyze the influence of different parts of the spectrum on statistics of beam light field. The conception is based on the assumption that random wandering and beam broadening as a whole leading to blurring of the mean intensity profile during long recording of light field are caused by the low frequency portion of the turbulence spectrum on the whole.

This present paper is devoted to numerical investigation of these effects on the base of modal representation of the atmospheric inhomogeneities. The parameters of numerical model are optimized by comparing the results obtained by the Monte Carlo method with the predictions by analytical theories. Different spectra of the atmospheric turbulence are considered and influence of the outer and inner scales on the power characteristics of light beam are determined under the conditions of weak, moderate, and strong fluctuations.

1. OPTIMIZATION OF THE PHASE SCREEN MODEL

To investigate light beam propagation through the turbulent atmosphere we use the quasi-optical approximation of diffraction theory. The equation of propagation is integrated by the method of splitting physical factors[4] on the basis of phase screen model. As known, for the modal representation of the phase distortions of light wave in the atmosphere[5–8] the diameter D of the circle of phase expansion into a series over the orthogonal polynomials (the Zernike modes, usually) is a free parameter. At the same time, the diameter D determines the variances of random coefficients of the Zernike modes, therefore its value influences the variance σ_φ^2 of the phase fluctuations on a screen. It is natural to expect that in the limit when the number of Zernike modes increases the dependence of σ_φ^2 on D unboundedly weakens. Apparently, an ideal phase screen, which is a superposition of infinite number of Zernike modes, ought to reproduce correctly the wave phase fluctuations at any D. But in the numerical experiment the number of Zernike modes is finite and, generally speaking, it is not large enough. Therefore, the estimations of the diameter of the phase expansion circle D and Zernike polynomial number J, for which such effects as wandering and broadening of a beam are reproduced satisfactorily are of interest.

Results of the analytically theory[9] developed for effective beam parameters in the atmosphere are used as a basis for making these estimations. In particular, when using the Kolmogorov model of turbulence a formula is obtained in Ref. 9 for the beam effective radius a_{ef} determining the dimension of the area which is covered by the wandering beam in the observation plane:

$$a_{\text{ef}} = a_{\text{d}}\{1 + 1.624\,(\beta_0^2)^{6/5}\, z/(k_0\, a_{\text{d}}^2)\}^{1/2}, \qquad (1)$$

where

$$a_{\text{d}}(z) = a_0 \left\{\left(\frac{z}{k_0\, a_0^2}\right)^2 + \left(1 - \frac{z}{R_{\text{f}}}\right)\right\}^{1/2} \qquad (2)$$

is the radius of diffraction-limited Gaussian beam in vacuum, a_0 is the initial beam radius, z is the path length, k_0 is the optical wave number, R_{f} is the focal length,

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$$\beta_0^2 = 1.23\ C_n^2\ k_0^{7/6}\ z^{11/6} \quad (3)$$

is the Rytov variance, C_n^2 is the structure constant of the refractive index fluctuations.

In the numerical experiment the following beam characteristics were calculated by the Monte Carlo method:

the rms displacement of the centroid

$$\rho_c = <\tilde{r}_c>\ ,\quad \tilde{r}_c = \sqrt{\tilde{x}_c^2 + \tilde{y}_c^2}\ , \quad (4)$$

$$x_c = \int\int x\ I\ dx\ dy / \int\int I\ dx\ dy\ ,$$

$$y_c = \int\int y\ I\ dx\ dy / \int\int I\ dx\ dy\ ,$$

the effective radius determined from the mean intensity is

$$a_{ef} = \left\{ \int\int (x^2 + y^2) <I>\ dx\ dy / \int\int I\ dx\ dy \right\}^{1/2}. \quad (5)$$

The averaging was performed over 100 observations for the beam with the following parameters: the initial radius a_0 = 5 cm, light wavelength λ = 1.06 μm (diffraction beam length z_d = 14.82 km). The grid of 128×128 points and the cell size A_0 = 100 cm containing was used in calculations. The number of the phase screens varied from S = 10 to S = 20, the path length of z = 7.41 km was chosen (i.e. 0.5 z_d). In these calculations we used Kolmogorov model of turbulence.

1.1. Collimated beam

When simulating the phase screens consider first the five Zernike polynomials describing aberrations of the first and second order (the piston mode is excluded from the consideration here and below). Figure 1 presents calculated values of a_{ef} as a function of z obtained for different ratios D/a_0 for the case of moderate fluctuations ($C_n^2 = 5 \cdot 10^{-17}$ cm$^{-2/3}$, $\beta_0^2 = 1.31$). One can see that the best agreement between calculated dependences and the analytical one obtained by formula (1) is observed for D/a_0 = 2.5. When D rises the beam effective radius decreases what is explained by a decrease in the variances of the optical mode describing the phase fluctuations of light wave. Figure 2 shows similar dependences obtained for D/a_0 = 2.5 for the cases of the weak (β_0^2 = 0.26), moderate (β_0^2 = 1.31), and strong (β_0^2 = 13.1) fluctuations. A good coincidence of theoretical and calculated curves in the first two cases and a considerable difference between them under the condition of strong fluctuations is observed.

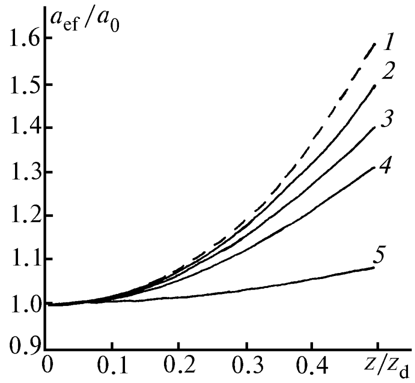

FIG. 1. Parameter a_{ef} as a function of z for a collimated beam and Kolmogorov model of the turbulence. Curves: 1— theory, 2–4 — Monte Carlo method for various values of the parameter D/a_0: 2 — D/a_0 = 2.5, 3 — D/a_0 = 4, 4 — D/a_0 = 10, 5 — propagation in vacuum. Turbulence is simulated by J = 5 Zernike modes, β_0^2 = 1.31.

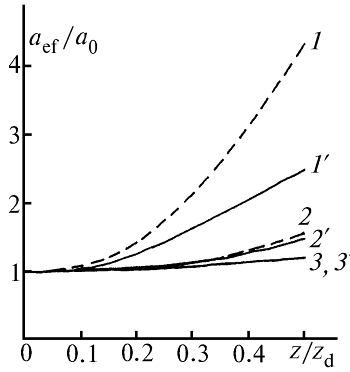

FIG. 2. Parameter a_{ef} as a function of z for a collimated beam and Kolmogorov model of the turbulence. Curves: 1, 2, 3 — theory, 1′, 2′, 3′ — Monte Carlo method (β_0^2 = 13.1, 1.31, 0.26, correspondingly). Parameters of phase screens are: J = 5, D/a_0 = 2.5.

It is interesting to make an attempt of achieving a better agreement between calculated and theoretical dependences by increasing a number of the Zernike modes J when simulating the phase screens. In this case J = 14 for the modes of the 4th order and J = 44 for the modes of the 6th order inclusive. Calculated dependences of a_{ef} on z obtained for different numbers of the Zernike modes are presented in Fig. 3 (β_0^2 = 1.31, D/a_0 = 4, Fig. 3*a*) and (β_0^2 = 13.1, D/a_0 = 8, Fig. 3*b*). One can see from the figures that increase of the number of modes when simulating the phase screens produce an appreciable effect at the moderate fluctuations and far weaker effect at the strong fluctuations. It is necessary to note that the optimal diameter of the area of phase expansion D increases with increasing number of modes from D/a_0 = 2.5 for J = 5 to D/a_0 = 8 for J = 44.

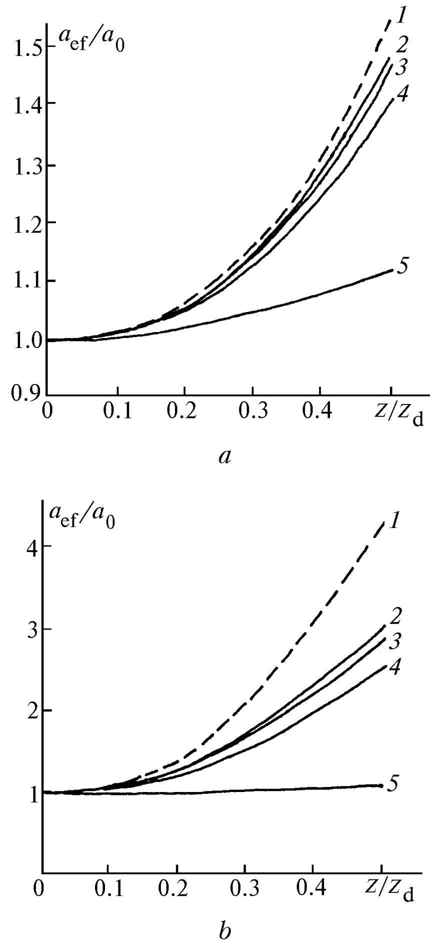

FIG. 3. Parameter a_{ef} as a function of z for a collimated beam and Kolmogorov model of the turbulence. Curves: 1 – theory, 2–4 – Monte Carlo method for various numbers of Zernike modes J on the phase screens: 2 – J = 44, 3 – J = 14, 4 – J = 5, 5 – propagation in vacuum. Figure 3a – $D/a_0 = 4$, $\beta_0^2 = 1.31$; Fig. 3b – $D/a_0 = 8$, $\beta_0^2 = 13.1$.

Considerable difference between the calculational data based on theoretical predictions under conditions of strong fluctuations (Fig. 3*b*) can not be explained by the limitedness of the basis when generating the phase screen because the increase of J from 14 to 44 leads to an increase in a_{ef} not more than 2%. From our point of view it is natural to assume that under the conditions considered the theory gives too high values of a_{ef}. Really, the results from Ref. 9 are based on the assumption that a profile of the beam mean intensity is Gaussian for long-exposure recording. Analysis of the mean intensity profiles obtained during the numerical experiments for various J (Fig. 4*a*) shows that under the condition of strong fluctuations this assumption is true only partially. This fact becomes obvious when analyzing the curves in Fig. 4*b* where the plots of the functions $\ln(\langle I\rangle_{max}/\langle I(x)\rangle)$ obtained for the profiles $\langle I(x)\rangle$ presented in Fig. 4*a* are shown (here $\langle I\rangle_{max}$ is the maximum value of intensity for the given profile). Straight line in this figure corresponds to the Gaussian profile with the effective radius obtained by formula (1). One can clearly see the difference in the slopes of the calculated dependences from the slope of the theoretical straight line which is especially essential in the beam periphery.

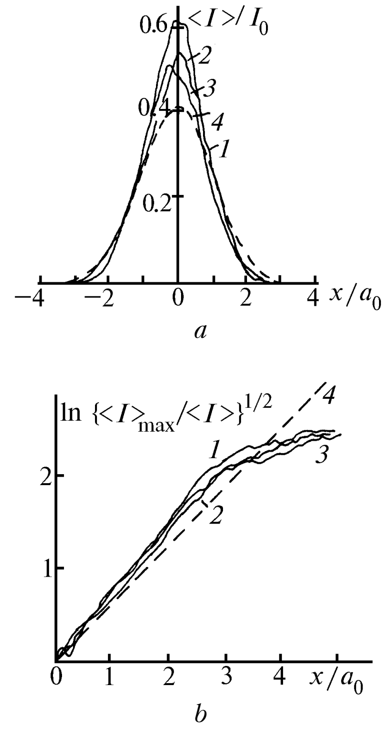

FIG. 4 Mean intensity profiles obtained by Monte Carlo method for the Kolmogorov model of turbulence. Curves: 1 – J = 5, 2 – J = 14, 3 – J = 44 ($D/a_0 = 4$), 4 – Gaussian profile with a_{ef} obtained theoretically (a). Logarithmic representation of the profiles (b) presented in Fig. 4a.

Therefore, it should be expected that with the increase of the fluctuation strength the formula (1) will overestimate the effective beam radius as compared with the Monte Carlo method.

1.2. Focused beam

When analyzing numerically propagation of a focused beam we are limited by a simple model of the phase screens containing the modes of the first and second order ($J = 5$) with an optimal for this case ratio $D/a_0 = 2.5$. The beam focused at the distance $R_f = 7.41$ km ($= 0.5z_d$) is considered. Calculated and theoretical dependences of a_{ef} on z are presented in Fig. 5 for three values of C_n^2 corresponding to $\beta_0^2 = 0.26$; 1.31; and 13.1. One can see that, as it was for the collimated beam under the condition of weak and moderate fluctuations the theoretical and numerical dependences agree quite satisfactorily, under the condition of strong fluctuations the formula (1) gives too high values for a_{ef}. At the same time the coordinates of the effective turbulent waist of the beam determined by Monte Carlo method and by the analytical relation (1) well agree over a wide range of C_n^2 values. As it was to be expected the turbulent waist was displaced in the direction toward the emitting aperture as the strength of the turbulence increases.

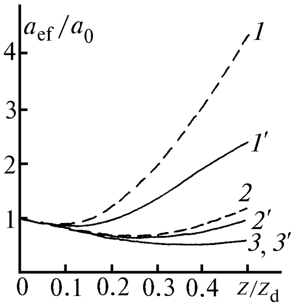

FIG. 5. Parameter a_{ef} as a function of z for a focused beam and Kolmogorov model of the turbulence. Curves: 1, 2, 3 – theory, 1′, 2′, 3′ – Monte Carlo method (β_0^2 = 13.1, 1.31, 0.26, respectively). Parameters of the phase screens are: J = 5, D/a_0 = 2.5. Focusing radius is $R_f = 0.5z_d$.

2. THE ROLE OF THE OUTER SCALE OF TURBULENCE

The estimation of the influence of the outer scale of turbulence on the power characteristics of the beam are performed here using Karman spectrum of atmospheric inhomogeneities.[10] Beam propagation conditions are chosen as in the section 1.

Figure 6 presents the calculated values of a_{ef} as a function of z obtained for a collimated beam under the condition of moderate fluctuations ($C_n^2 = 5{\cdot}10^{-17}\,cm^{-2/3}$) for various values of the outer scale of turbulence L_0. For a comparison the results obtained for Kolmogorov turbulence from Fig. 1 are also shown here, i.e., the calculated curve for $L_0 = \infty$ and the curve by formula (1). One can see that with a decrease in the outer scale of turbulence the beam effective radius decreases. Moreover, the decrease in a_{ef} is most essential in the region of small L_0. This effect is explained, on the whole, by a decrease in the variance of the beam random displacements and, in the lesser degree, by a decrease in the turbulent broadening, because the outer scale value influences the variances of the wavefront tilts most strongly.[3,11]

Results of the analysis made for a focused beam are presented in Fig. 7 for $R_f = 0.5z_d$. The dependences of a_{ef} on z obtained under the condition of weak and moderate fluctuations for both Kolmogorov and Karman models of turbulence with $L_0 = 100\ a_0$ (5 m) are shown in this figure. To compare the curves obtained by formula (1) are presented in Fig. 7. One can see that when the outer scale decreases the turbulent waist moves off from the emitting aperture under all the above considered conditions. When C_n^2 increases at a fixed L_0 the turbulent waist moves in the direction of a source as it takes place in the Kolmogorov model too.

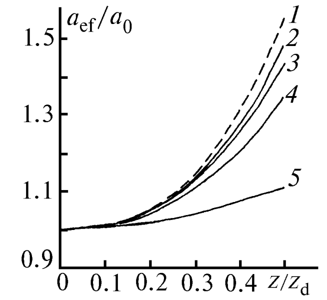

FIG. 6. Parameter a_{ef} as a function of z for a collimated beam when using Karman model of turbulence (β_0^2 = 1.31). Curves: 1, 2 – theory and Monte Carlo method for $L_0 = \infty$; 3, 4 – theory and Monte Carlo method for L_0 = 5 m and 2 m, respectively; 5 – propagation in vacuum.

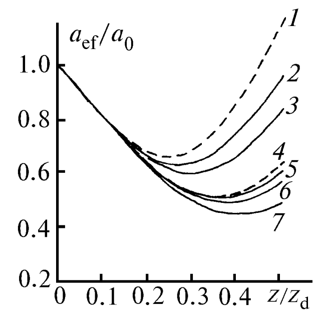

FIG. 7. Parameter a_{ef} as a function of z for a focused beam for the Karman model of turbulence. Curves: 1, 2 – theory and Monte Carlo method for $L_0 = \infty$, $\beta_0^2 = 1.31$; 3 – Monte Carlo method for L_0 = 5 m, β_0^2 = 1.31, 4, 5 – theory and Monte Carlo method for $L_0 = \infty$, β_0^2 = 0.26; 6 – Monte Carlo method for L_0 = 5 m, β_0^2 = 0.26, 7 – propagation in vacuum. Focusing radius is $R_f = 0.5z_d$.

3. RANDOM DISPLACEMENTS OF THE BEAM CENTROID

Important advantage of the Monte Carlo method is that this method allows the statistics of different characteristics of light beam to be investigated separately. In particular, this method can be used to confirm the analytical results obtained in Ref. 12 for the variance of beam random displacement. In approximation of a given field of collimated Gaussian beam the following formula is obtained in Ref. 12

$$\rho_c^2 = \frac{4p^2 z^3}{3} \int\limits_0^\infty \kappa^3\, \Phi_n(\kappa)\, \exp(-\kappa^2 a_0^2/2)\, d\kappa\ , \qquad (6)$$

where Φ_n is the spectrum of the refractive index fluctuations. For the Kolmogorov model of turbulence the integral (6) is calculated analytically and leads to the expression

$$\rho_c^2 = \frac{0.132\ p^2\ G(1/6)}{3{\cdot}2^{5/6}}\ C_n^2\ z^3\ a_0^{-1/3}. \qquad (7)$$

For other spectra of the atmospheric turbulence the calculation of the integral (6) is performed easily by any quadrature formula.

Table I presents the results of numerical analysis of random displacements of the centroid of a collimated beam compared with the data obtained by the formulas (6) and (7). One can see a good coincidence of the theory with the Monte Carlo method over a wide range of variation of the outer scale and turbulence strength.

TABLE I. The rms displacement of power center of collimated beam ρ_c/a_0 *as a function of the outer scale of turbulence* L_0 (a_0 = 5 cm, λ = 1.06 μm, $z = 0.5z_d$).

C_n^2, $cm^{-2/3}$	β_0^2	$L_0/a_0 = \infty$		$L_0/a_0 = 100$		$L_0/a_0 = 40$	
		Theory	Monte Carlo method	Theory	Monte Carlo method	Theory	Monte Carlo method
$1{\cdot}10^{-17}$	0.26	0.358	0.356	0.309	0.314	0.240	0.244
$5{\cdot}10^{-17}$	1.31	0.800	0.806	0.710	0.708	0.53	0.55
$5{\cdot}10^{-16}$	13.1	2.58	2.55	2.11	2.18	1.77	1.75

CONCLUSION

The main result of this paper is a development of the phase screen method for numerical analysis of light beam propagation through the turbulent atmosphere. Procedure to select the diameter *D* of a circle of phase fluctuation expansions over Zernike modes which is a free parameter for the modal approach to the phase screen generation is proposed. It is established that the obtained value of *D* ensures a reliable prediction of the broadening and wandering of collimated beam as well as focused one for the case of the weak and moderate fluctuations.

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