SPECTRAL COEFFICIENTS OF SCATTERING AND ABSORPTION IN STRATUS

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Asymptotic formulas of radiative transfer theory for diffuse fluxes emanating from the optically dense homogeneous scattering layer are studied. Analytic expression for the parameter $s = \sqrt{\frac{1-\Lambda}{3-x_1}}$ *has been derived in terms of the radiative fluxes, which along with the formula obtained in Ref. 5 solves the inverse problem for the cloud layer. The accuracy of expressions and their range of applicability are investigated. Based on airborne measurements of spectral diffuse fluxes of solar radiation, the spectral dependences of volume scattering coefficients and the real light absorption coefficients in the cloud layer are calculated.*

Optical properties of a disperse medium (in this case the cloud layer) are connected with absorption and scattering of radiation transmitted through it and depend on effective size, chemical composition, and form of particles which compose a medium. Spectral dependences of scattering, and absorption coefficients provide an Insight into the microphysical properties of cloudiness. *In situ* measurements of microphysical and optical parameters of the atmosphere and atmospheric aerosols are very difficult and not always quite reliable. Airborne optical measurements at different altitudes in the atmosphere permit determining the values of scattering and absorption coefficients[1] and thus form a basis for the numerical calculation of atmospheric physical characteristics (in this case of a cloudy atmosphere).

At the same time scattering and absorption of solar radiation in the cloud layer are the most important factors that determine the interaction of the radiation with cloudiness and greatly affect the energy balance of the atmosphere. It is well known that the amount of radiative energy absorbed by a cloud makes sometimes up to 20% from the energy incident on the upper cloud boundary and can provide the ~ 2° rate of heating of the atmosphere a day. Thus possibly more exact values of the scattering and absorption coefficients are necessary for creating adequate climatic models of weather and climate forecasting which take cloudiness into account.[2] We will discuss absorption and scattering in detail later when the results of individual experiments for different cloud layers will be interpreted based on the technique suggested in this paper. Let us indicate that, apparently, atmospheric aerosol is of dramatic importance for absorption of solar radiation in the visible range outside the molecular absorption bands.

Problem formulation. Let us examine the plane homogeneous cloud layer, infinite in horizontal direction. Optical thickness of this layer is $\tau_0 = \sigma z$, where σ is the scattering coefficient, z is the geometric thickness of the layer, $\Lambda = \sigma/(\kappa + \sigma)$ is the albedo of single scattering, and κ is the coefficient of real absorption.

The parallel flux of solar radiation is incident on the upper boundary of the layer at an angle arcos ζ with respect to the normal to the layer, $F^{\uparrow}$ is the monochromatic flux of scattered radiation reflected from the layer in units of $4\pi S\zeta$, $F^{\downarrow}$ is the transmitted flux, and A is the albedo of the underlying surface. Let us introduce the value $s = \sqrt{\frac{1-\Lambda}{3-x_1}}$, which comprises not only absorption properties of a medium, but characterizes the scattering phase function in terms of the first coefficient of expansion of the scattering phase function in a system of the Legendre polynomials x_1.

Airborne measurements[1] of spectral fluxes of solar radiation at different heights in the atmosphere are carried out in absolute units.

After having divided the measured values into the value of the flux incident on the upper boundary of the cloud layer, we shall obtain the relative values of fluxes that are used in the subsequent theoretical transformations. After having divided the upwelling flux at the hight of the underlying surface we may assume the values $F^{\uparrow}$, $F^{\downarrow}$, and A to be well–known for the spectral range 0.4–0.9 μm with step 0.02 μm.

Expressions for relative fluxes of diffuse radiation have the form[3,4]

$$\bar{F}^{\uparrow} = a(\zeta) - \frac{M\bar{N}Q\ u(\zeta)\exp\left(-2k\tau_0\right)}{1 - N\bar{N}\exp\left(-2k\tau_0\right)} \quad (1)$$

$$\bar{F}^{\downarrow} = \frac{M\ Q\ u(\zeta)\exp\left(-k\tau_0\right)}{1 - N\bar{N}\exp\left(-2k\tau_0\right)}\ \frac{1}{1 - Aa^{\infty}}, \quad (2)$$

where $a(\zeta)$ is the plane albedo of the layer and $u(\zeta)$ is the Miln function, which defines the angular distribution of scattered radiation transmitted through the upper boundary of the layer. There are tables of the function $u(\zeta)$ in Ref. 6. $Q = 2\int_0^1 u(\zeta)\,\zeta\,d\zeta$, M, and N are defined by the integral ralatios,[3,4] k is the parameter usually called diffuse exponent

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or inverse diffuse length, the bar over symbol means an account of reflection of the radiation from the underlying surface

$$\bar{N} = N - \frac{AMQ^2}{1 - Aa^{\infty}}, \tag{3}$$

where $a^{\infty} = 2\int_0^1 a(\zeta)\,\zeta\, d\zeta$ is the spherical albedo of the semi–infinite atmosphere.

The problem of determining of the optical thickness of the cloud layer τ_0 has been solved in Ref. 5 starting from the measured values of reflected intensity of radiation and the relation analogous to formula (1). In addition, it is assumed in Ref. 5 that the absorption in the cloud in the visible spectral range is so small that we can neglect it. Therefore, the study is performed for the value of $\Lambda = 1$, which is specified *a priori*. The albedo of the underlying surface in Ref. 5 is equal to 0.2. When we shall discuss the results of our paper, we shall show that it is necessary to take into account absorption and real values of the surface albedo in order to obtain true values of τ_0.

Solution of the problem. Expression (28) derived in Ref. 5 from the formula for the reflected intensity of the radiation is of interest for solving our problem. In a similar way starting from formula (1) we shall derive the following relation:

$$\exp(2k\tau_0) - N\bar{N} = \frac{M\bar{N}Q\, u(\zeta)}{a(\zeta) - \bar{F}^{\uparrow}}. \tag{4}$$

We also shall substitute Eq. (4) into the expression for the flux transmitted through the layer (2) and after the trivial transformations we shall obtain

$$\bar{F}^{\downarrow 2}\bar{N}\left[1 - Aa^{\infty}\right]^2 = MQu(\zeta)\left(a(\zeta) - \bar{F}^{\uparrow}\right) + N\left(a(\zeta) - \bar{F}^{\uparrow}\right)^2. \tag{5}$$

The asymptotic expansions of values and functions included in Eq. (5) in terms of the small parameters $\sqrt{1-\Lambda}$ are well known.[4] Let us write these expansions in terms of the parameter $s = \sqrt{\dfrac{1-\Lambda}{3-x_1}}$ considering the terms of the order of s^2

$$\left.\begin{aligned} M &= 8s \\ N &= 1 - 3\delta s + \frac{9}{2}\delta^2 s^2 \\ Q &= 1 - \frac{3}{2}\delta s + Q_2 s^2 \\ u(\zeta) &= u_0(\zeta)\left[1 - \frac{3}{2}\delta s\right] + u_2(\zeta)s^2 \\ a(\zeta) &= 1 - 4u_0(\zeta)s + a_2(\zeta)s^2 \\ a^{\infty} &= 1 - 4s + 6\delta s^2 \end{aligned}\right\} \tag{6}$$

where $\sigma = 4\int_0^1 u_0(\zeta)\,\zeta^2\, d\zeta = 1.427$, $u_0(\zeta)$ is the function $u(\zeta)$ for $\Lambda = 1$, and the expressions for the coefficients Q_2 and $u_2(\zeta)$ nearby s^2 will not be expanded since in subsequent calculations they will be reduced.

The function $a_2(\zeta)$ being the coefficient adjacent to s^2 has the form[3]

$$a_2(\zeta) = \frac{15(3 - x_1)}{5 - x_2}\,\upsilon(\zeta) + 6\delta u_0(\zeta). \tag{7}$$

Here x_2 is the second coefficient In the expansion of the scattering phase function in a system of the Legendre polynomials. Functions $\upsilon(\zeta)$ and $u_0(\zeta)$ are given by the integral relations in terms of azimuth–independent term in the expansion of the light reflectance $\rho_0(\eta,\zeta)$ considering only light scattering.[4]

$$\left.\begin{aligned} \upsilon(\zeta) &= \zeta^2 - 2\int_0^1 \rho_0(\zeta,\eta)\,\eta^3 d\eta \\ u_0(\zeta) &= \frac{3}{4}\left[\zeta + 2\int_0^1 \rho_0(\zeta,\eta)\,\eta^2 d\eta\right] \end{aligned}\right\}. \tag{8}$$

In the right side of Eq. (6) we take the function $u_0(\zeta)$ out of the brackets

$$a_2(\zeta) = u_0(\zeta)\left[\frac{15(3 - x_1)}{5 - x_2}\,\frac{\upsilon(\zeta)}{u_0(\zeta)} + 6\delta\right]. \tag{9}$$

Using the detailed tables for the functions $\upsilon(\zeta)$ and $u_0(\zeta)$ from Ref. 6, we calculated that $\upsilon(\zeta)/u_0(\zeta)$ is the linear function of ζ and has a form

$$\upsilon(\zeta)/u_0(\zeta) = u_0(1)\,\zeta - 0.9. \tag{10}$$

The values of $a_2(\zeta)$ were calculated for 4 types of the scattering phase functions and subsequently averaged (Table *I*). Numerical test calculations showed that in practice the mean values $\bar{a}_2(\zeta)$ provide sufficient accuracy of numerical calculations. Thus, if the scattering phase function is unknown, one can use the values $\bar{a}_2(\zeta)$ given in the table.

Taking into account expressions (8), we can write

$$2\int_0^1 \rho_0(\eta,\zeta)\,\eta^2 d\eta\left[\frac{3}{4}u_0(1)\zeta - \frac{2.7}{4}\right] + \\ + 2\int_0^1 \rho_0(\eta,\zeta)\,\eta^3 d\eta. \tag{11}$$

TABLE I. The values of function $\bar{a}_2(\zeta)$.

g \ ζ	0	0. 1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.75	1.310	2.220	3. 118	4.078	5.126	6.256	7.475	8.786	10.194	11.696	13.290
0.8	1.267	2.236	3. 151	4. 117	5. 163	6.289	7.494	8.796	10.184	11.664	13.234
0.85	1.201	2.242	3. 181	4. 148.	5. 198	6.320	7.512	8.798	10.167	11.625	13.181
0.9	1.092	2.244	3.208	4.193	5.237	6.350	7.529	8.808	10.159	11.597	13.120
$\bar{a}_2(\zeta)$	1.217	2.236	3. 165	4.209	5. 181	6.304	7.503	8.797	10.176	11.646	13.206

This relation can be useful for finding the function $\rho_0(\eta, \zeta)$ In the explicit form.

Now let us find the value s. For this we substitute expansion (6) into Eq. (5), multiply considering the terms of the order of s^2, and derive

$$s^2 = \frac{\left(1 - \bar{F}^{\uparrow}\right)^2 - \bar{F}^{\downarrow 2}(1 - A)^2}{16\left(u_0^2(\zeta) - A^2\bar{F}^{\downarrow 2} - 2\left(1 - \bar{F}^{\uparrow}\right)a_2(\zeta) - 12\delta A(1-A)\bar{F}^{\downarrow 2}\right.}. \tag{12}$$

Let us use the relation

$$k\tau_0 = \frac{1}{2}\ln\left[\frac{M\bar{N}Qu(\zeta)}{a(\zeta) - \bar{F}^{\uparrow}} + N\bar{N}\right] \tag{13}$$

analogous to that obtained in Ref. 5. By substituting expansion (6) into Eq. (13) and taking into account that $k = (3 - x_1)\cdot s$, we derive the expression for τ_0

$$\tau' = (3 - x_1)\tau_0 = \frac{1}{2s}\left[2\ln\left(1 - 3\delta s + \frac{9}{2}\delta^2 s^2\right) + \right.$$

$$+ \ln\frac{1 - A - 4As - 6A\delta s^2}{1 - A + 4As - 6A\delta s^2} +$$

$$\left. + \ln\frac{1 - \bar{F}^{\uparrow} + 4u_0(\zeta)s + a_2(\zeta)s^2 - 18u_0(\zeta)s^3}{1 - \bar{F}^{\uparrow} - 4u_0(\zeta)s + a_2(\zeta)s^2}\right], \tag{14}$$

where the notation $\tau' = (3 - x_1)\cdot\tau_0$ is introduced for convenience.

In the derivation of Eq. (14) in the expansion of the value k we restrict ourselves to the term $\sim s$ and consider the term $\sim s^2$ under the logarithm because the numerical test calculations showed that in such a form formula (14) provides the highest accuracy.

Thus the derived formulas have quite a simple form and calculations are not difficult. At the same time these formulas makes it possible to solve rigorously the problem of finding the values $s^2 = \sqrt{\dfrac{1 - \Lambda}{3 - x_1}}$ and $\tau' = (3 - x_1)\cdot\tau_0$ for the measured fluxes of radiation emanating from the cloud layer for large optical thickness of the layer and weak absorption. The expressions (12) and (14) are derived from asymptotic formulas of transfer theory which are the unique and rigorous solution of the radiation transfer equation under conditions of $\tau_0 \gg 1$ and $1 - \Lambda \ll 1$.

It should be noted that the combination $(3 - x_1)$ which is associated with the form of the scattering phase function is included in both of the values sought. This fact follows from the nature of multiple anisotropic light scattering (here it is pertinent to mention the similarity ratio[3] which expresses the same physical meaning). For revealing the properties of the scattering phase function it would be useful to use supplemental measurements (may be of microphysical character) and techniques for their interpretation.

Let us indicate one more aspect. The coefficient of real light absorption in the layer of the thickness z may be written in the form

$$\kappa = (1-\Lambda)\tau_0/z. \tag{15}$$

From which it follows that if we know the values of τ' and s^2, we can find readily the value of κ without the use of the data on the scattering phase function

$$\kappa = \tau' s^2/z\ . \tag{16}$$

Systematic errors and range of applicability. Let us assume that we know the values f_1 of a definite function $f(x)$ and graph of $f(x)$ variations and want to find the corresponding values of argument x_1. It is clear that we can find them with higher accuracy for larger variations of the function $f(x)$. This expresses the well-known fact: if we find the function being inverse to the initial function $x(f)$, the error is inversely proportional to the derivative $f'(x)$. This fact is widely used in statistical methods of solving the inverse problems, in which the partial derivatives are calculated with respect to a number of variables sought and the matrices of the quantities are formed which are inverse to the derivatives and determine the corresponding errors.

Starting from the indicated reasons, we find the derivatives $\dfrac{\partial F^{\uparrow\downarrow}}{\partial s}$ and $\dfrac{\partial F^{\uparrow\downarrow}}{\partial \tau_0}$

$$\left.\begin{aligned} \frac{\partial \bar{F}^{\uparrow}}{\partial \tau_0} &= 2kQu(\zeta)\,\frac{M\bar{N}\exp\left(-2k\tau_0\right)}{\left(1 - \bar{N}N\exp\left(-2k\tau_0\right)\right)^2} \\ \frac{\partial \bar{F}^{\downarrow}}{\partial \tau_0} &= -\bar{F}^{\downarrow}\,\frac{k\left[1 + N\bar{N}\exp\left(-2k\tau_0\right)\right]}{1 - \bar{N}N\exp\left(-2k\tau_0\right)} \end{aligned}\right\}. \tag{17}$$

Taking account of the expansion (6) within an accuracy to s we obtain the derivatives

$$\left\{\begin{aligned} \frac{\partial F^{\downarrow}}{\partial s} &= F^{\downarrow}\left[\frac{1}{s} - \frac{3\delta}{Q} - (3 - x_1)\tau_0 - \right. \\ &\left. - \frac{2N\exp\left(-2k\tau_0\right)\left(3\delta + N\tau_0(3 - x_1)\right)}{1 - N^2\exp\left(-2k\tau_0\right)}\right], \\ \frac{\partial F^{\uparrow}}{\partial s} &= 4u_0(\zeta) - Ne^{-k\tau_0}F^{\downarrow}\left[\frac{1}{s} - \frac{3\delta(Q+N)}{QN} - \right. \\ &\left. - 2(3-x_1)\tau_0 - \frac{2N\exp\left(-2k\tau_0\right)\left(3\delta + N\tau_0(3 - x_1)\right)}{1 - N^2\exp\left(-2k\tau_0\right)}\right]. \end{aligned}\right. \tag{18}$$

To estimate the errors it is sufficient to take into account the terms which determine the order of magnitudes. We then can write for small s

$$\frac{ds}{s} \sim \frac{dF^{\downarrow\uparrow}}{F^{\downarrow\uparrow}} \text{ and } \frac{d\tau}{\tau} \sim \frac{dF^{\uparrow\downarrow}}{F^{\uparrow\downarrow}} \frac{1}{k\tau_0}, \tag{19}$$

i.e., the relative errors in the values sought are proportional to the relative errors of measuring the radiative fluxes. But relations (19) are valid only in the range of applicability of asymptotic formulas (1) and (2) and expansions (6) and (18). The range of applicability has been studied in detail with the help of both theoretical considerations[4,7] and numerical calculations.[18] Here we shall determine the error calculated from the derived formulas as functions of the values τ_0 and $1 - \Lambda$ on the basis of the model calculations of the scattering radiation fluxes for $\tau_0 = 3$–50, $\Lambda = 0.7$–1.0, and $x_1 = 2.55$ that have been performed by the method of summation over the layers.[8] Figure 1 shows the corresponding dependences of relative errors $\delta\tau_0$ and δs on Λ for a fixed value of $\tau_0 = 25$ and on τ_0 for a fixed value of $\Lambda = 0.999$. It is evident that for larger values of absorption (smaller Λ) the error in determining τ_0 increases sharply. It can be readily understood since τ_0 is determined by the logarithmic dependence. Therefore, in formula (14) we considered the term of the order of s^3 to improve the accuracy of the numerical calculation of the expression under logarithm. It should be noted also that uncertainty in assigning the albedo of underlying surface A and real absorption in the layer In terms of the value s results in gross error in calculating from formula (14), and it is difficult for us to agree with the author of Ref. 5 who stated that the relation analogous to Eq. (14) for $\Lambda = 1$ for the intensity along with the measurements of reflected radiation are sufficient for determining optical thickness of the layer. In this case only quite a rough estimate of τ_0 is possible. For the exact determination of τ_0 from the data of measuring the reflected radiation, the measurements performed under the cloud layer, at least ground–based measurements, are also necessary. To investigate the cloud layers in detail, of course, the airborne measurements with heigh-range resolution performed outside and inside the cloud layer are preferable.

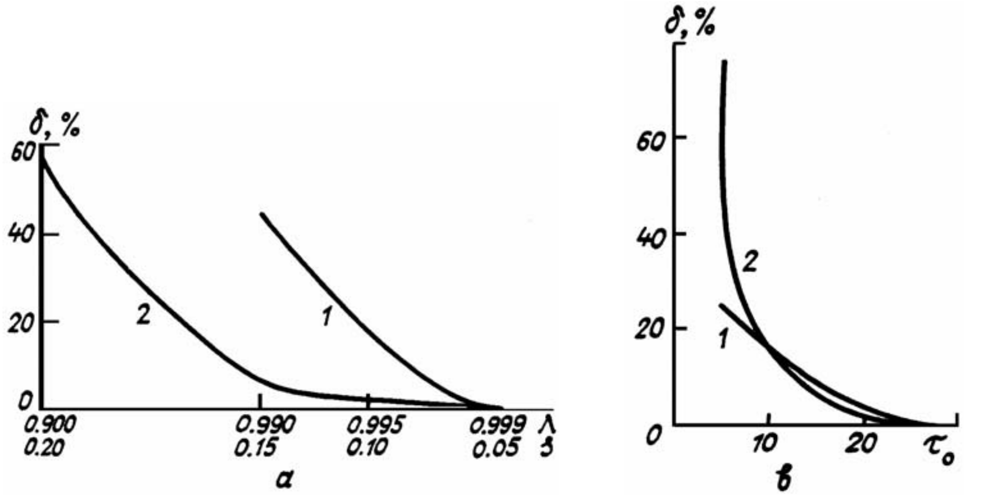

FIG. 1. Relative errors in determining τ_0 and s from the formulas (21) *and* (22) *(*$\delta\tau$ *is indicated by curve 1 and* δs *is indicated by curve 2; g = 0.85): a) as a function of* Λ *for* $\tau_o = 25$ *and b) as a function of* τ_0 *for* $\Lambda = 0.999$.

Application of the formulas to the interpretation of the experimental results. Let us apply the derived formulas to the interpretation of experimental data from Ref. 1. Let us describe briefly the conditions and the results of the experiment. The measurements were carried out on April 20, 1985 on board aircraft–laboratory "IL–18" over Lake Ladoga. The underlying surface was covered by ice and snow with rather high reflectance, $\cos\theta = \zeta = 0.647$, and the thickness of the cloud layer z was ~ 1.1 km. The values of radiative fluxes are given with step 0.02 μm for the visible spectral range 0.4–0.9 μm. The measurement errors are about 2%. Figure 2*a* shows the hemispherical fluxes of solar radiation emanating from the upper and lower boundaries of the layer which are scaled to the flux incident on the upper boundary of the cloud layer and to the albedo of the underlying surface.

The spectral dependence $s(\lambda)$ (curve *2*), which is shown in Fig. 2*c*, is determined from formula (12) applied to the data of measurements at each wavelength. Using formula (14) and the assumption that elongation of the scattering phase function is described by the parameter $x_1 = 2.55$ and is independent of the wavelength, we obtained the spectral values of optical thickness of the cloud layer $\tau_0(\lambda)$. If in addition we take into account that the thickness of the cloud layer is 1.1 km, we may pass over to the values of scattering coefficient $\sigma = \tau_0/z$ shown in Fig. 2*b*. Spectral values of the coefficient of real light absorption $\kappa(\lambda)$ calculated from formula (16) are given in Fig. 2*c* (curve *1*).

Now let us estimate the relative errors in the results that have been obtained hare. We shall discuss the relative error $\sigma\tau_0$ since it coincides with the relative error $\sigma\tau$. In the spectral range 0.4–0.6 μm, where the value $s \le 0.06$; $\delta s \sim \delta F = 2\%$ and $\delta\tau = \delta\tau_0 \sim \delta F/\kappa\tau_0 = \delta F/0.5 \sim 4\%$. The quantity s increases up to 0.08 with the increase of the wavelength and according to Fig. 1*a* the relative error $\delta\tau_0$ increases and for $s \sim 0.08$ it is approximately equal to 10–12%. The accuracy of determining s almost does not decrease and for $s = 0.08$ the error $\delta s \sim 3\%$. The spread of points in Fig. 2*b* for $\lambda > 0.6$ μm is explained by the increased error $\delta\tau_0 \sim 10\%$.

The discussion of the properties of the cloud layer based on the obtained spectral dependences $\sigma(\lambda)$ and $\kappa(\lambda)$ falls outside the scope of this paper; this problem will be the subject of a new paper. It should be noted that in finding the values s and τ' and more correctly, in deriving the corresponding formulas, some limitations were imposed (for example, that the plane layer is horizontally infinite and homogeneous). We intend to derive the analogous formulas for solving the inverse problem for the layers inclosed inside the single layer homogeneous and inhomogeneous in the vertical direction.

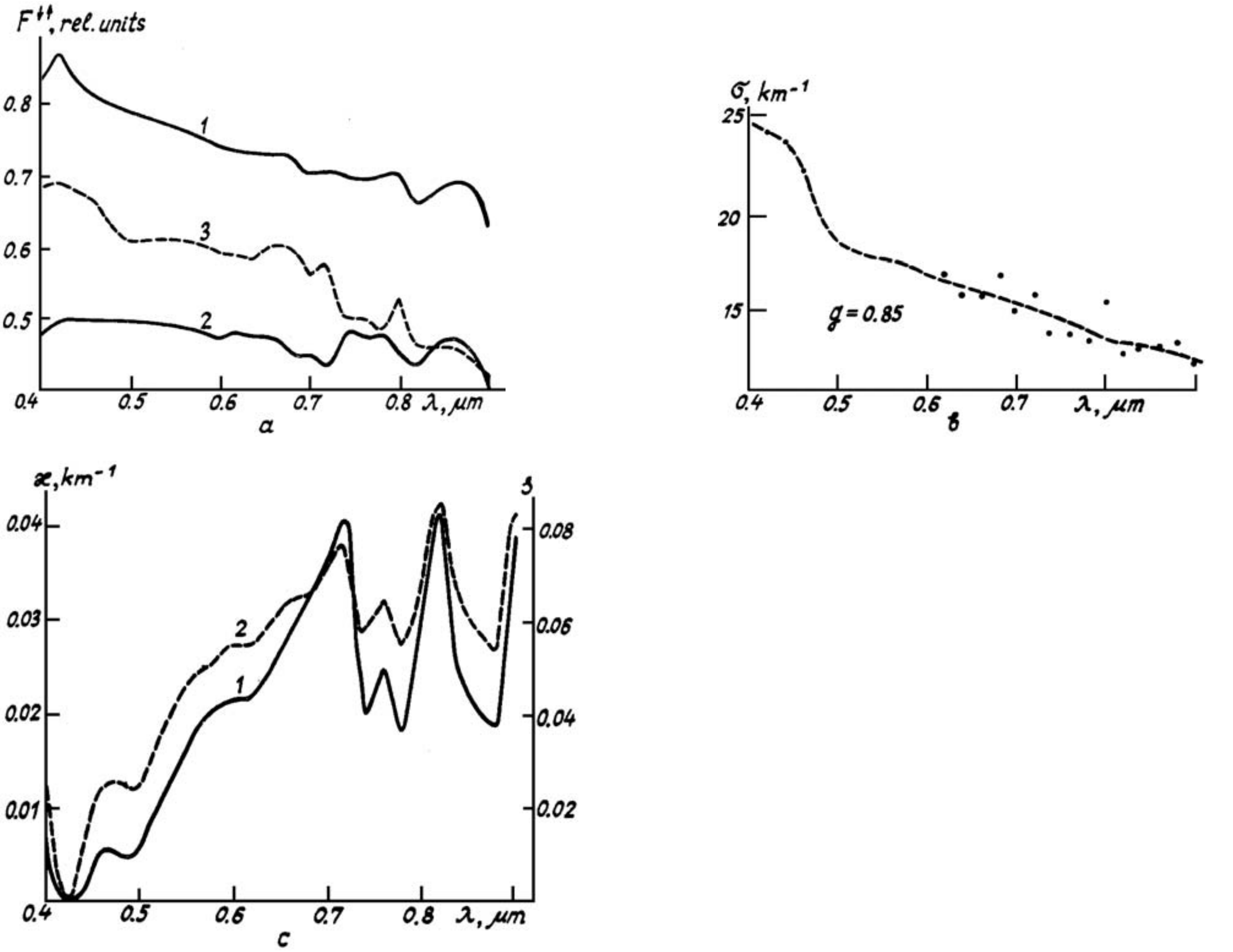

FIG. 2. Spectral dependences: a) hemispherical fluxes of diffuse solar radiation emanating from the cloud layer derived from the data of Ref. 1 (curve 1 indicates $F^{\uparrow}$, 2 — $F^{\downarrow}$, and 3 — A; b) volume light scattering coefficient; and, c) volume coefficient of real light absorption.

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