Absorption and horizontal radiative transport in 3D broken clouds: spectral variability

V.e . Zuev,[1] E.I. Kassianov,[1,2] and Y.L. Kogan[3]

[1]*Institute of Atmospheric Optics, Siberian Branch of the Russian Academy of Sciences, Tomsk*
[2]*Pacific Northwest National Laboratory, Richland, USA*
[3]*Cooperative Institute for Mesoscale Meteorological Studies, Oklahoma, USA*

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The Monte Carlo method and Large Eddy Simulation model were used to study the spectral dependence of the horizontal radiative transport in three-dimensional broken stratocumulus clouds. It is shown that the horizontal transport strongly depends on wavelength, while variations of its spectral features are mostly determined by absorption of light in water droplets and by water vapor. The neglect of this spectral dependence can be a cause of large errors in retrieval of spectral absorption of inhomogeneous clouds.

1. Introduction

The amount of solar radiation, absorbed by the cloudy atmosphere, and its spatial distribution are among the main factors controlling the dynamics of the atmospheric processes and, hence, variations of weather and climate. In this regard, the spatiotemporal variability of cloud absorption has recently become the topic of permanent concern, with most attention being currently paid to absorption of solar radiation by inhomogeneous clouds. This is motivated by the fact that the estimates of absorption by clouds, based on the model calculations and data of field measurements,[1] substantially disagree, and this could not be explained clearly and unambiguously yet (see, e.g., Refs. 2–4). An integrated experimental study, as part of the Atmospheric Radiation Measurement (ARM) Enhanced Short-wave Experiment (ARESE), which was designed to identify possible causes for the discrepancy, have demonstrated once more the high importance of the problem on absorption of light by clouds.[5–7]

The current schemes of retrieving the absorption by clouds widely use remote sensing methods and techniques (see, e.g., Refs. 8 and 9). Historically, these traditional methods are based on the spectral measurements of upward/downward radiative fluxes despite the abundant evidence that the horizontal radiative fluxes (horizontal transport) in inhomogeneous clouds can be comparable to the vertical fluxes. Hence their neglect may lead to *physically incorrect* interpretation of field measurement data, such as negative absorption values.[10–12]

One possible method of taking the horizontal transport into account was developed by Ackerman and Cox.[13] In fact it is based on simultaneous measurements of net vertical fluxes inside and outside of an absorption band, and its use involves two steps. First, the value of horizontal transport within a spectral region outside an absorption band (e.g., in the visible range) is determined, and then this value is used to estimate the absorption by a cloud in the near-infrared region. In other words, this approach assumes that the horizontal transport only weakly depends on wavelength. This method, as well as its different modifications, has been widely used to estimate the spectral and integrated absorption of inhomogeneous clouds (see, e.g., Refs. 7, 14, and 15).

In this regard, it is an important task to better understand how strongly and in what way the horizontal transport depends on the wavelength. What factors influencing the spectral dependence of the horizontal transport are most important? How accurate are the estimates of absorption by clouds obtained without taking the spectral dependence of the horizontal transport into account? All these questions are being addressed in the present work (see also Refs. 16 and 17).

Section 2 describes the model of a cloudy atmosphere and the method of solution, and indicates the spectral intervals used here to estimate the spectral variability of the horizontal transport. Section 3 discusses the main factors governing this variability and explores how sensitive are the absorption retrievals in inhomogeneous clouds to accuracy of specifying the spectral behavior of the horizontal transport. Main conclusions are summarized in section 4.

2. Model and method of solution

The three-dimensional (3D) Cooperative Institute for Mesoscale Meteorological Studies (CIMMS, Oklahoma, USA) Large Eddy Simulations (LES) model with explicit liquid phase microphysics[18,19] and the Monte Carlo method have been used to study the spectral dependence of the horizontal radiative transport. We have simulated low-level broken

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stratocumulus (*Sc*) clouds observed during the Atlantic Stratocumulus Experiment (ASTEX). The integration domain consisted of 40×40×51 grid points providing for the horizontal and vertical resolutions of 0.075 km and 0.025 km, respectively. The simulation yielded 3D spatial distributions of water vapor and water droplets. The obtained droplet size-distribution functions have then been used to calculate the optical characteristics by the Mie theory.[20]

Above the simulation domain ($1.275\ \text{km} \le z \le 16\ \text{km}$), the atmosphere was represented by a set of 9 horizontally homogeneous layers with different geometrical thicknesses, specified in accordance with the existing vertical levels accepted in the atmospheric general circulation models (GCMs) (see, e.g., Ref. 21). Each layer was characterized by the constant pressure, temperature, and concentration of water vapor. To better fit the conditions of the ASTEX experiment performed during summer (June 1–28) of 1992 in the northeastern Atlantic, the model of midlatitude summer[22] was used throughout the computation. Carbon dioxide was assumed to be uniformly mixed in space. The integrated optical and radiation measurements in *Sc* clouds were conducted in a wide range (from roughly 10° to 80°) of solar zenith angles $\xi_\oplus$. The solar zenith angle of approximately 60° was frequently encountered in the observations (such as on June 2); therefore the value $\xi_\oplus$=60° was used in radiation calculations. The influence of the underlying surface was neglected, since the albedo of the ocean is close to zero at small (≤ 60°) solar zenith angles.[23] We have also neglected the influence of aerosol, whose optical depth was about two orders of magnitude smaller than the mean cloud optical depth. Subject to these assumptions, the model of cloudy atmosphere outlined above incorporated the effects of clouds, water vapor, and carbon dioxide.

As known, light absorption by water vapor and water droplets are important factors that determine spectral variations of cloud absorption and vertical solar fluxes (see, e.g., Refs. 8 and 24). How strongly does the spectral behavior of horizontal transport depend on these two factors? This question can be answered by considering these two effects separately. Such analysis can readily be made because spectral intervals exist, where only one of the factors dominates. For instance, the atmospheric absorption bands at 0.94 μm and 1.65 μm are mostly due to absorption by water vapor and water droplets, respectively.[25] These spectral intervals were chosen here to estimate spectral dependence of the horizontal transport.

In each pixel, we calculated the optical properties at three wavelengths: 0.7 μm (conservative scattering), 0.94 μm, and 1.65 μm. To denote the optical and radiative characteristics calculated for these wavelengths, we used the subscripts “0.7”, “0.94”, and “1.65”. The pixels in which the extinction coefficient $\sigma_{0.7}$ exceeded the threshold of 3 km^{-1} were designated as cloudy. The highest (lowest) height at which cloudy pixels were located was taken as the cloud top *Ht* (base *Hb*) boundary; and was set equal to 0.750 km (0.150 km). Absorption of solar radiation by water vapor and carbon dioxide was taken into consideration using transmission functions from Refs. 26–28. Note that the cloud geometrical thickness[12] and the effective droplet radius[29] are usually determined from radiation measurements in the 0.94 μm and 1.65 μm absorption bands, respectively.

Absorption and vertical and horizontal radiative fluxes within the cloudy layer, as well as the albedo (plane $z = Ht$) and transmittance (plane $z = Hb$) were calculated using Monte Carlo method and periodic boundary conditions. The developed Monte Carlo algorithm can account for not only the spatial 3D distributions of clouds and water vapor, but also those of other radiatively active gases and atmospheric aerosol, as well as reflection from the underlying surface. With use of nearly 100 million photons, the mean computation error was less than 1%.

3. Horizontal transport and absorption

At present, the dependence of the horizontal transport and absorption on the spatial structure of stratocumulus clouds has been studied quite thoroughly (see, e.g., Refs. 30 and 31), whereas the nature of the spectral *E* variations and its influence on the accuracy of cloud absorption estimates are understood much poorly. Titov[32] was the first who showed that the visible and near-IR values of *E* can differ, and that the cloud absorption retrievals must take this difference into consideration.

Despite their great scientific value, the results obtained by Titov[32] should only be considered as preliminary, illustrating the complexity of the problem on light absorption by clouds and demonstrating some ways for its possible solution. This is because they are based on quite simple cloud model, assuming, in particular, that the extinction coefficient of the overcast plane-parallel layer varies in just one horizontal direction and no absorption by water vapor and atmospheric gases occurs. Let us remind the main items of the problem on determining absorption in clouds, whose optical properties change at least along one direction.[32,33] According to the model of a cloudy atmosphere chosen here (see section 2), the radiative energy conservation law in inhomogeneous clouds has the form:

$$R + T + A = 1 - E. \qquad (1)$$

Here the absorptance *A*, albedo *R*, transmittance *T*, and horizontal transport *E*, all are the functions of *x* and *y* coordinates, but their arguments are omitted for convenience. From Eq. (1) it follows that the absorptance *A* can be determined only when the other three functions, *R*, *T*, and *E*, are known. In practice, however, only albedo and transmittance are usually determined, while the effect of horizontal transport is neglected (as, for instance, in the independent pixel approximation). However, the use of this approximation (the neglect of E) may result in unrealistic estimates of absorption by clouds at small (on the order of 100 m) spatial scales.[15,30,31]

The need for reliable absorption estimates has stimulated the development of new methods allowing for the horizontal radiative fluxes. An original treatment of the horizontal transport in absorption retrievals has been proposed by Ackerman and Cox.[13] The main idea behind this approach is to determine E value in the visible range (subscript "vis") as $E_{vis} = 1 - R_{vis} - T_{vis}$, and to use this value for estimating absorption in the infrared (subscript "ir") by $A_{ir} = 1 - R_{ir} - T_{ir} - E_{vis}$. This method is based on two main assumptions: (1) the horizontal transport depends only on the scattering properties of clouds (extinction coefficient σ and scattering phase function g); and (2) the spectral behaviors of the latter can be neglected in the visible and near-IR spectral ranges.

The latter assumption is physically well grounded, while the validity of the former one is not that obvious. For instance, the vertical radiative fluxes are known to depend on both the scattering and absorbing properties (single scattering albedo and water vapor absorption coefficient) of the medium. Therefore, it is expected that the horizontal transport will also depend on the absorbing properties of the cloudy atmosphere. " elow we give an example of the influence of absorption on E.

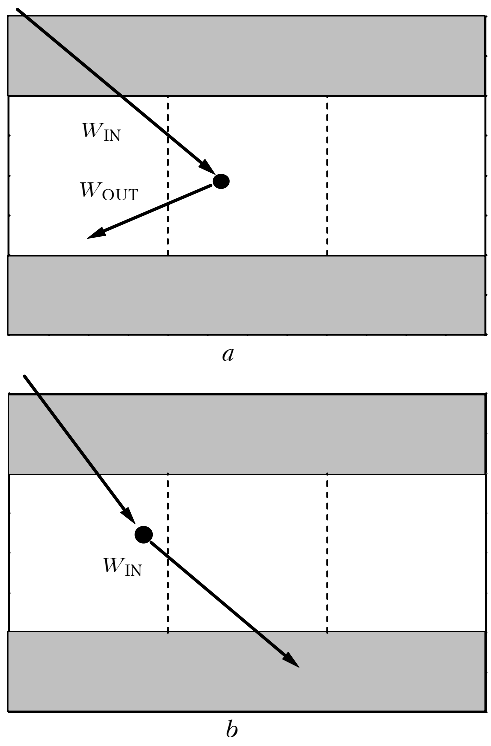

Fig. 1. Schematic illustration of the photon trajectories.

We consider an arbitrary ith pixel (Fig. 1a), and assume that a photon entered the pixel through its left-hand side, underwent scattering within it, and left the pixel through the same pixel side. In this case, the contribution of this photon trajectory segment to E will be $\Delta E = W_{IN} - W_{OUT}$, where W_{IN} and W_{OUT} are photon statistical weights at the entry (IN) and exit (OUT) points. In the absence of absorption (conservative scattering only), $W_{IN} = W_{OUT}$, and so this trajectory makes no contribution to E. Conversely, in the presence of absorption, $\Delta E > 0$. Thus, the presence of absorption will tend to increase the number of photon trajectories contributing to E. This, in turn, will lead to an *increase* of the variance of horizontal transport Var(E). Note that the physical explanation for Var(E) growth in terms of the absorption by water droplet was first proposed in Ref. 32.

Let us consider another possible situation (Fig. 1b), in which the photon scattered in the neighboring pixel ($i - 1$), entered the ith pixel through its left-hand side, and left the pixel through its bottom. In this case, the contribution of this photon trajectory segment to the horizontal transport will be $\Delta E = W_{IN}$. Obviously, ΔE value will be less in the absorbing than in the non-absorbing medium. Thus, the presence of absorption will tend to *decrease* the relative contribution of photon trajectories to E. This, in turn, will lead to a *decrease* of Var(E). Thus, two *competing* effects associated with absorption will determine the variance Var(E).

Numerous theoretical and experimental studies (see, e.g., Refs. 25 and 34) show that in clouds the extinction coefficient and scattering asymmetry factor can vary by roughly 2–5% per micrometer of wavelength, whereas the single scattering albedo and absorption coefficient of water vapor can vary by several orders of magnitude within quite a narrow spectral interval. It should be expected, therefore, that the spectral behavior of the horizontal transport will be mainly determined by those of the water vapor and water droplets.

To check up the validity of this hypothesis, we have performed calculations for two cases. In one case, we neglected the absorption by water vapor and water droplets, and considered the spectral variations of extinction coefficient and scattering phase function. The σ and g values for the three wavelengths, 0.7, 0.94, and 1.65 μm, were used. In the other case, in addition, the absorption by water vapor and water droplets was also taken into consideration.

The calculated results on the horizontal transport E, corresponding to the first case (conservative scattering), are presented in Fig. 2. As is seen from this figure, insignificant spectral variations of σ and g have only a minor influence on E. Thus the horizontal transport can be calculated without the account of the spectral dependence of the cloud scattering properties over quite wide (about 1-μm) spectral intervals. Note that the absolute E values may exceed 1. This means that a considerable contribution to E may come from direct (unscattered) radiation. We would like to note that pixels with $E < 0$ ($E > 0$) predominately gain (loose) energy through their sides.

Let us proceed to the second case (Fig. 3), in which the absorption by water vapor and water droplets is taken into consideration. The absorption by water droplets influences the diffuse radiation. The scattering processes are most intense in the optically *dense* top portion of the cloud. For these reasons, the effect of absorption by water droplets on Var(E) is most appreciable near the cloud top boundary (Fig. 3*b*): the difference between Var($E_{0.7}$) and Var($E_{1.65}$) peaks in the cloud top portion.

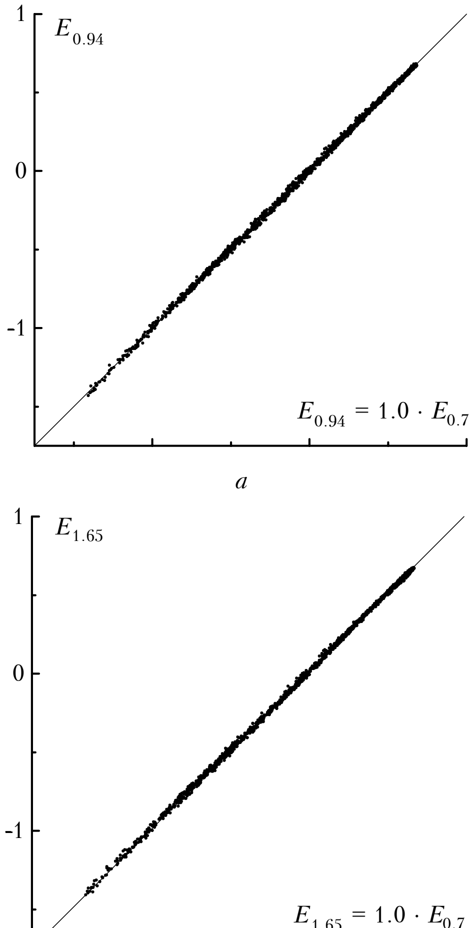

Fig. 2. Linear regressions of (*a*) $E_{0.94}$ vs $E_{0.7}$ and (*b*) $E_{1.65}$ vs $E_{0.7}$.

Quite the opposite situation occurs in the spectral interval at 0.94 μm (Fig. 3*b*). The absorption by water vapor is important for both diffuse and *direct* radiation. The highest water vapor concentration is found near the cloud base (Fig. 3*a*). For this reason, the influence of water vapor on Var(E) is, accordingly, strongest in this same part of the cloud (Fig. 3*b*). In contrast to absorption by droplets, the absorption by water vapor *decreases* Var(E), primarily through diminishing the relative contribution of direct radiation to the variance Var(E). As seen, the vertical profiles of Var(E), corresponding to different spectral intervals, may have not only considerable quantitative but also *qualitative* differences (see Fig. 3*b*). The variance of the horizontal transport is found to affect considerably the accuracy of cloud absorption estimates (see, e.g., Ref. 31); therefore, for correct determination of the absorption within a cloud layer, it is necessary to take into account the strong wavelength dependence of the vertical distribution of Var(E).

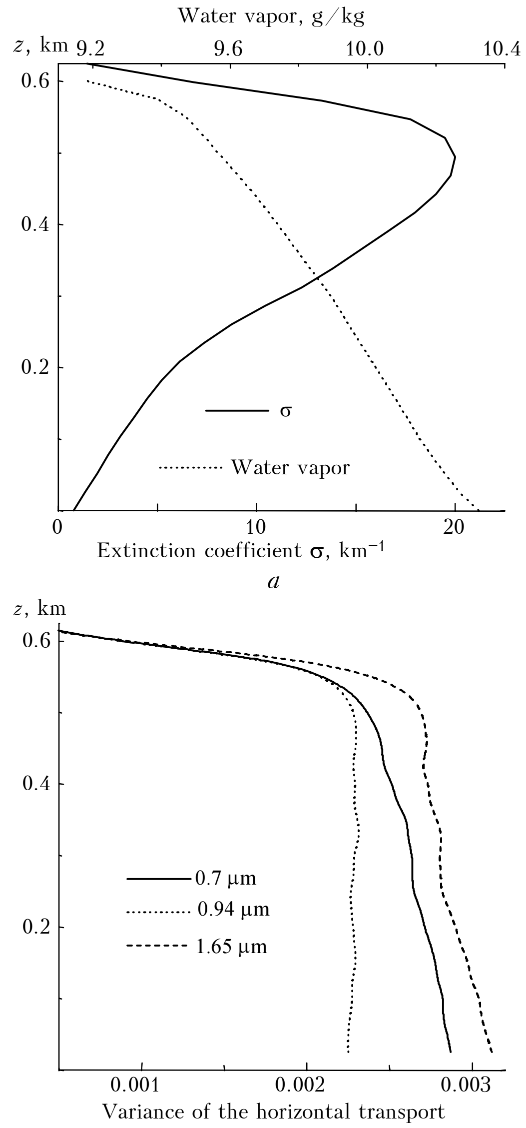

Fig. 3. Vertical profiles of the mean extinction coefficient (at 0.7 μm wavelength) and of the mean water vapor concentration (*a*); and variance of the horizontal transport for three wavelengths (*b*).

” ecause of the absorption by water droplets, Var(E) increases at all vertical levels inside the cloud layer (see Fig. 3*b*), and so does the variance of the *total* (between cloud top and bottom boundaries) horizontal transport, whose probability density becomes wider (Fig. 4). This conforms with the results obtained using a simpler model of the overcast *Sc* clouds.[31,33] However, in contrast to the overcast cloud, the probability density of E for the broken clouds is not symmetrical and, hence, cannot be approximated by a simple Gaussian distribution.[33] The variance of the total horizontal transport increases by approximately 20% (from 0.295 to 0.35) due to absorption by water droplets and, conversely, decreases by 15% (from 0.295 to 0.25) due to the absorption by water vapor. In the overcast clouds, where the contribution of direct radiation to the horizontal transport can be neglected, it is expected that the increase in the variance of E due to absorption by water vapor will be less significant.

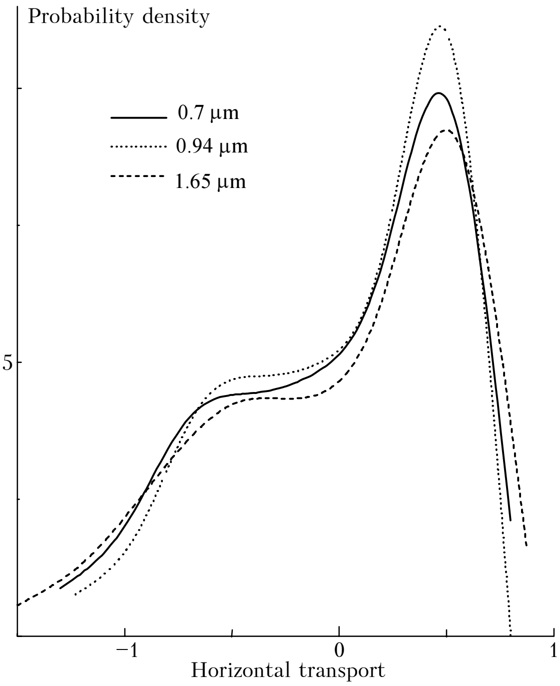

Fig. 4. Probability density function of the total horizontal transport for three wavelengths.

How strongly does the spectral dependence of E influence the accuracy of absorption estimates? To answer this question, we compared the true absorption A with the reconstructed one, $A' = 1 - R - T - E_{0.7}$. Note that the absorption in the IR spectral range (0.94 and 1.65 μm) was reconstructed using the values of horizontal transport calculated at a visible wavelength (0.7 μm). As seen, at small (~100 m) spatial scales, the reconstructed absorption, A', may differ from the true one, A, by a factor of two, once $E_{0.7}$ value is used instead of $E_{0.94}$ (Fig. 5*a*). This means that the E value obtained outside absorption bands (e.g., in the visible range) cannot provide reliable estimates of the *spectral* absorption in the near-IR spectral range. The utility of this method for retrieval of *integrated* absorption will be addressed in future studies.

Since the absorption by water vapor and water droplets are two *competing* factors influencing the spectral dependence of horizontal transport, their net effect is expected to be less than individual contributions. The magnitude and sign of the net effect will depend on the optical and geometrical parameters of a cloud (cloud top and base heights, concentration and size spectrum of cloud droplets, etc.), the concentrations and spatial distributions of water vapor, and other radiatively active gases, as well as on meteorological conditions.

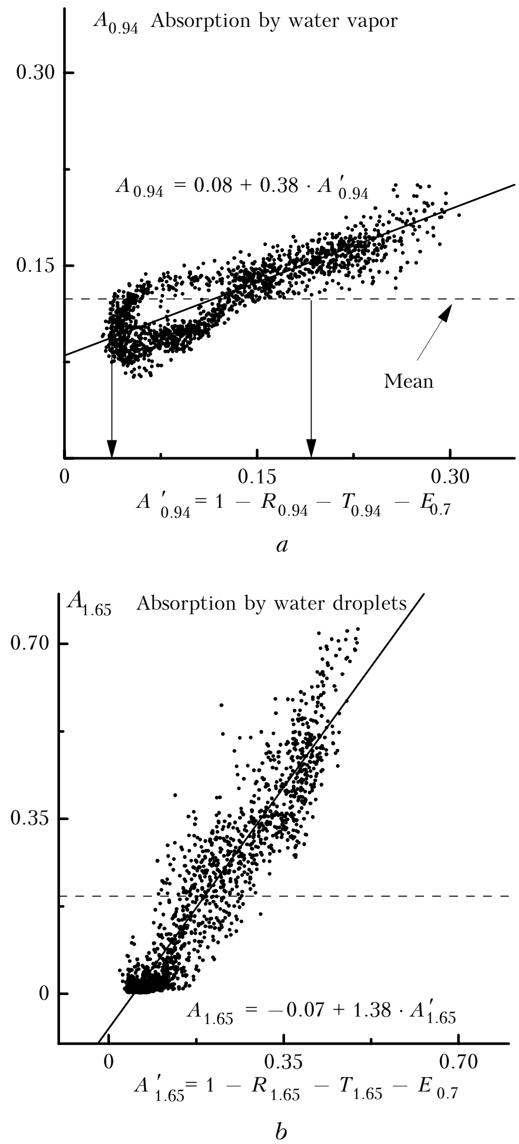

Fig. 5. True absorptance A versus the reconstructed one A' for two wavelengths: (*a*) 0.94 μm and (*b*) 1.65 μm.

More reliable estimates of the spectral absorption can be obtained using the method proposed in Ref. 31. Its main idea is as follows. First, the horizontal transport and other radiative properties entering in Eq. (1) are averaged over space, and a linear regression between values of the horizontal transport in the visible and near-IR is plotted. Then, thus obtained regression equation is used to estimate the absorption in the IR spectral range. The results shown in Fig. 6 well illustrate how this method works. Averaging over the spatial cell of 0.6×0.6 km^2 size substantially reduces the variances Var($E_{0.7}$) and Var($E_{1.65}$), and the linear regression is a good approximation for the relationship between $E_{1.65}$ and $E_{0.7}$ (Fig. 6*a*). Use of the linear regression $E_{1.65} = 1.09 \cdot E_{0.7}$ improves the accuracy of retrieval of the spectral absorption $A_{1.65}$ (Fig. 6*b*).

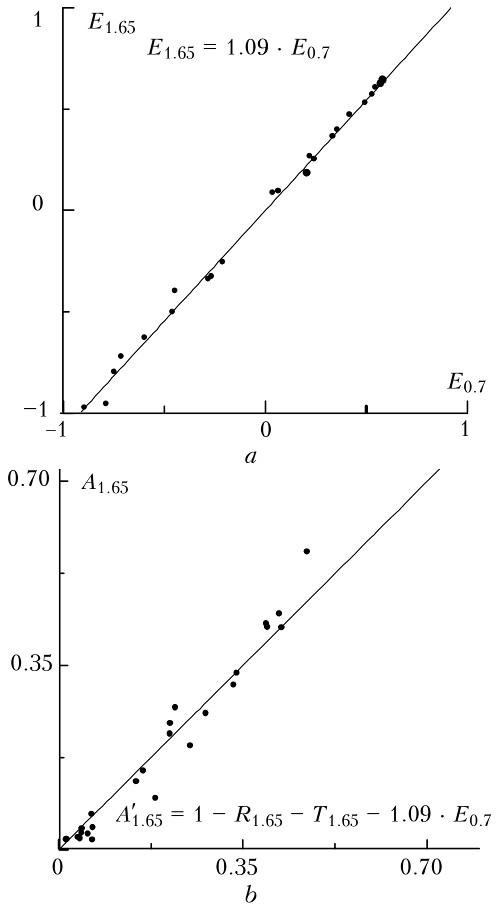

Fig. 6. Linear regression of $E_{1.65}$ versus $E_{0.7}$ (*a*), as well as between the true absorptance $A_{1.65}$ and an improved estimate of the reconstructed absorptance $A'_{1.65}$ (*b*) for the spatial resolution of 0.6×0.6 km^2.

Note that the scale of spatial averaging depends quite strongly on the structure of a cloud field. For instance, in an overcast cloud layer, this averaging scale is approximately 0.05 km,[32] i.e., about an order of magnitude smaller than for broken clouds. Obviously, different spatial averaging scales should be used depending on whether the spectral or broad-band absorption is reconstructed. The issue of how large this possible difference may be needs for further theoretical and experimental study.

4. Conclusion

The CIMMS LES (Oklahoma, USA) model with explicit liquid phase microphysics and Monte Carlo method were used to study the spectral dependence of the radiative horizontal transport. The influence of this dependence on the accuracy of spectral (near-IR) absorption retrievals for broken stratocumulus clouds has been estimated.

Recently, the assumption of weak spectral dependence of horizontal transport has been widely used to study the small-scale (on the order of 100 m) variations of cloud absorption (see, e.g., Refs. 13 and 14). This work has demonstrated that the horizontal transport depends significantly on wavelength. In particular, as wavelength increases by roughly 1 μm, the variance of the horizontal transport may change by 15 to 20%.

The sign and magnitude of these spectral variations are primarily determined by the absorption of light by cloud droplets and water vapor. The influence scattering properties of a cloud on the spectral behavior of the horizontal transport can be neglected over quite wide (nearly 1-μm) spectral intervals. The absorption by cloud droplets (water vapor) generally increases (decreases) the variance of the horizontal transport.

When absorption within the water vapor (0.94 μm) and cloud droplet (1.65 μm) absorption bands is retrieved without the account of the spectral dependence of horizontal transport, considerable errors may appear in the final results. For instance, if this spectral dependence is treated incorrectly, at small (on the order of 100 m) spatial scales the retrieved and true absorption (at 0.94 μm) may differ by a factor of two.

Acknowledgments

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