FLUCTUATIONS OFSPECULARLY REFLECTED OPTICAL WAVES IN A SCATTERING ATMOSPHERE

I.P. Lukin

Institute of Atmospheric Optics
Siberian Branch of USSR Academy of Sciences
Tomsk, 634055
Received September 5, 1988

A theoretical study of the statistical properties of frequency-diversity waves propagating through discrete scatterers is reported. Variances and correlation functions of Gaussian beam log-amplitude and phase fluctuations are calculated by the smooth perturbation method. Within the framework of the continuous medium approximation, it is shown that the specular reflection of a beam whose initial size is smaller than the characteristic scale of scatterers increases the phase and log-amplitude fluctuations and improves their frequency correlation. The domain where these effects are found to occur is determined by the scatterer radius.

Multi-pass propagation through the same random medium inhomogeneities has received much attention due to recent advances in optical detection and ranging. A number of relevant references can be found in the review paper[1] and monograph [2]. In spite of the fact that optical wave fluctuations caused by discrete scatterers have been thoroughly considered (see Ref. 3-5), the statistical properties of fluctuations of waves reflected from remote objects have so far not been investigated. It is the aim of this paper to examine their behavior in a scattering atmosphere. In particular the log-amplitude and phase fluctuation correlation functions for frequency-diversity optical waves reflected from a specular plane are calculated. The variances and cross-correlation of the fluctuations are determined by the smooth perturbation method [6,7]. This study is performed within the framework of the continuous medium model[8,9], i.e., the medium is characterized by the effective statistics of dielectric permittivity fluctuations. Such atmospheric states as rain, hail, snow and mist can be adequately described by this model.

Consider two optical beams with carrier frequencies ω_1 and ω_2 respectively, propagating along the same path. The boundary conditions for the beams take the form

$$U_j(0,\rho) = U_{0j}(\rho) = U_{0j}\exp\left\{-\frac{\rho^2}{2a_{0j}^2}\right\} \tag{1}$$

where U_{0j} is the field amplitude at the center of the output aperture and a_{0j} is the effective initial radius of the relevant beam ($j = 1, 2$).

The beams are assumed to originate at the plane $x = 0$, to travelalong the x-axis through a randomly inhomogeneous medium with a thickness L, and to be reflected from an infinite specular plane. In the process, the beam return path runs through the same layer and terminates at a receiver located in the plane $x = 0$. The medium is supposed to consist of a large number of discrete scatterers whose characteristic radius a is larger than the optical wavelength, and whose mean density is m_0. It has been shown [8,9], .that in that case, the first two moments of the effective permittivity fluctuations are

$$\varepsilon(r;k_j) = \langle\varepsilon(r;k_j)\rangle + \varepsilon_{f1}(r;k_j);$$

$$\langle\varepsilon(r;k_j)\rangle = 1+2\pi i k_j^{-1} m_0 a^2,$$

$$\langle\varepsilon_{f1}(r_1;k_j)\varepsilon_{f1}^*(r_2;k_{j'})\rangle = -\langle\varepsilon_{f1}(r_1;k_j)\varepsilon_{f1}(r_2;k_{j'})\rangle =$$

$$= \iiint_{-\infty}^{\infty} d\kappa\ \Phi_\varepsilon(\kappa;k_j;k_{j'}) \exp[-i\kappa(r_1-r_2)], \tag{2}$$

where

$$\Phi_\varepsilon(\kappa;k_j;k_{j'}) = \frac{1}{2\pi}\frac{m_0 a^4}{k_j k_{j'}}\left[\frac{2J_1(\kappa a)}{\kappa a}\right]^2 \quad k_j = \omega_j/c = 2\pi/\lambda_j$$

is the optical wave number, λ_j is the wavelength in vacuum and J_1 is the first-order Bessel function.

The correlation functions of the log-amplitude and phase fluctuations can be written in terms of the complex phase correlation function as follows:

$$B_{\chi,S}(\rho,\rho;k_j,k_{j'}) = \frac{1}{2}\left[\langle\varphi_1(\rho;k_j)\varphi_1^*(\rho;k_{j'})\rangle \pm \pm \langle\varphi_1(\rho;k_j)\varphi_1(\rho;k_{j'})\rangle\right] \tag{3}$$

Here $\varphi_1(\rho, k)$ is the complex phase of the optical wave with wavelength λ and subscript 1 indicates that χ is determined to first order of the smooth perturbation method. In analogy with Ref. 7, the complex phase correlation functions for Gaussian collimated beams defined by Eq. (1), propagating along a sounding path with small optical depths τ of the scattering layer, $\tau = \pi m_0 a^2 L < 1$, take the form

$$\langle \varphi_1(\rho;k_j)\,\varphi_1^{*}(\rho;k_{j'})\rangle = \pi^2 k_j k_{j'} \int_0^L dx \int_0^\infty d\kappa\, \Phi_\varepsilon(\kappa;k_j,k_{j'})\kappa \times$$
$$\times\Big\{\exp\Big[i\,\frac{(2L-x)(\gamma_j^{*}k_j-\gamma_j k_{j'})}{2k_j k_{j'}}\kappa^2\Big] J_0(\kappa|\gamma_j-\gamma_{j'}^{*}|\rho) +$$
$$+\exp\Big[i\,\frac{x\,k_j\hat{\gamma}_{j'}^{*}-(2L-x)k_{j'}\gamma_j}{2k_j k_{j'}}\kappa^2\Big] J_0(\kappa|\gamma_j-\hat{\gamma}_{j'}^{*}|\rho) +$$
$$+\exp\Big[i\,\frac{(2L-x)k_j\gamma_{j'}^{*}-x\,k_{j'}\hat{\gamma}_j}{2k_j k_{j'}}\kappa^2\Big] J_0(\kappa|\hat{\gamma}_j-\gamma_{j'}^{*}|\rho) +$$
$$+\exp\Big[i\,\frac{x(\hat{\gamma}_{j'}^{*}k_j-\hat{\gamma}_j k_{j'})}{2k_j k_{j'}}\kappa^2\Big] J_0(\kappa|\hat{\gamma}_j-\hat{\gamma}_{j'}^{*}|\rho)\Big\}, \quad (4)$$

$$\langle \varphi_1(\rho;k_j)\varphi_1(\rho;k_{j'})\rangle = \pi^2 k_j k_{j'} \int_0^L dx \int_0^\infty d\kappa\, \Phi_\varepsilon(\kappa;k_j,k_{j'})\kappa \times$$
$$\times\Big\{\exp\Big[-i\,\frac{(2L-x)(k_{j'}\gamma_j+k_j\gamma_{j'})}{2k_j k_{j'}}\kappa^2\Big] J_0(\kappa|\gamma_j-\gamma_{j'}|\rho) +$$
$$+\exp\Big[-i\,\frac{x\,k_j\hat{\gamma}_{j'}+(2L-x)k_{j'}\gamma_j}{2k_j k_{j'}}\kappa^2\Big] J_0(\kappa|\gamma_j-\hat{\gamma}_{j'}|\rho) +$$
$$+\exp\Big[-i\,\frac{(2L-x)k_j\gamma_{j'}+x\,k_{j'}\hat{\gamma}_j}{2k_j k_{j'}}\kappa^2\Big] J_0(\kappa|\hat{\gamma}_j-\gamma_{j'}|\rho) +$$
$$+\exp\Big[-i\,\frac{x(\hat{\gamma}_{j'}k_j+\hat{\gamma}_j k_{j'})}{2k_j k_{j'}}\kappa^2\Big] J_0(\kappa|\hat{\gamma}_j-\hat{\gamma}_{j'}|\rho)\Big\},$$

Here L is the distance from the transmitting aperture to the retroreflector:

$$\gamma_j = \frac{1+i\alpha_j x}{1+2i\alpha_j L}; \quad \hat{\gamma}_j = \frac{1+i\alpha_j(2L-x)}{1+2i\alpha_j L};$$
$$\alpha_j = 1/(k_j a_0^2); \quad j = 1,2$$

Equations (4) were derived under the assumption that the effective dielectric permittivity fluctuations are given by Eqs. (2). The analysis of Eqs. (3)–(4) shows that the correlation functions depend on three dimensionless parameters: the transmitter aperture Fresnel number $\Omega_0 = k\,a_0^2/L$, the beam path configuration parameter ρ/a, and the parameter $d = L/(ka^2)$, which indicates the zone (near or far) in which the transmitter is located.

Consider reflection in the backward direction, i.e. $\rho = 0$; for the limiting case in which the field is a plane wave ($\Omega_0 = \infty$) and $d \gg 1$. These conditions are valid, for example, for rain ($a \propto 10^3$ m, with a typical value of $k \approx 10^7$ m^1), when $L \gg ka^2$ for paths longer than $100m$. The variance of the log-amplitude fluctuations takes the form

$$\sigma_\chi^2(\Omega_0 = \infty) \simeq \pi m_0 a^2 L. \quad (5)$$

This result is seen to be similar to the case where the plane wave travels along a one way path[10] of length L, i.e. $2\sigma^2_{\chi ow}(\Omega_0 = \infty) = \sigma^2_\chi(\Omega_0 = \infty)$. Equations (5) shows that the fluctuations of a plane wave propagating along a one-way path through a discrete scattering medium do not increase, unlike the situation for turbulent, atmospheric conditions[1,2,7]. Spherical wave propagation ($\Omega_0 = 0$) can be treated similarly, and we have

$$\sigma^2_\chi(\Omega_0 = 0) \simeq 2\pi m_0 a^2 L = 4\sigma^2_{\chi ow}(\Omega_0 = 0) \quad (6)$$

that is the log-amplitude fluctuations grow large if the wave is reflected from a plane mirror in a random inhomogeneous medium. It should be noted that for a one-way path[10] $\sigma^2_{\chi ow}(\Omega_0 = \infty) = \sigma^2_{\chi ow}(\Omega_0 = 0)$, while for the round-trip path $\sigma^2_\chi(\Omega_0 = \infty) = \sigma^2_\chi(\Omega_0 = 0)$. The phase fluctuation of the plane and spherical waves behave in the following way: in the near zone ($d \leqslant 1$),they are small compared to σ^2_χ, while in the far zone ($d \gg 1$), $\sigma^2_\chi(\Omega_0) \simeq \sigma^2_S(\Omega_0)$. This relationship can be explained by the fact that for $d \leqslant 1$, the scatterer casts a distinct geometric shadow on the receiver, while in the far zone, diffraction patterns are found to occur. It can readily be seen that in the far zone all fluctuation statistical characteristics of optical waves are mutually consistent.

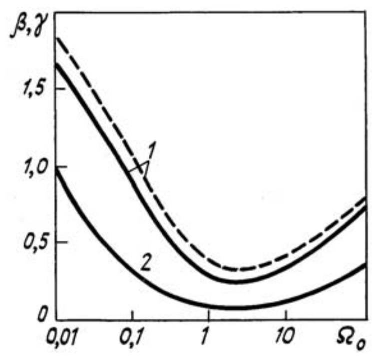

FIG. 1.

Figure 1 depicts the variances of the log-amplitude and phase fluctuations along the axis of a collimated beam as a function of the diffraction size of the transmitting aperture for the case of reflection from an infinite plane mirror in a monodisperse scattering medium.

$\beta = \sigma_s^2(\Omega_0)\,/\,\tau$ and $\gamma = \sigma_\chi^2(\Omega_0)\,/\,\tau$ are represented by solid and dashed curves respectively; curve 1 correspond to $d = 1$ and 2 to $d = 100$. Comparison with similar results obtained for a one-way path[10] enable us to summarize the peculiarities of the case of under consideration: i) curves for β and γ become asymmetrical with respect to the point $\Omega_0 = 1$; ii) the differences between the variances of the log-amplitude and phase fluctuations of the optical beam are smaller than for a one-way path; iii) a significant increase in beam fluctuations for small transmitting apertures ($L \le 10ka^2$) appears.

It should be added that for a plane wave reflected from an infinite plane mirror in a discrete scattering medium the fluctuations do not increase (see Eq. (5)). However, they are not amplified for spherical waves (see Eq. (6)). This can be clarified by nothing that the shadow pattern on the receiver caused by discrete scatterers is a set of individual shadows because for the plane wave the shadows from forward and backward propagation coincide. In the case of spherical waves, each scatterer casts two shadows.

Considered below are the main differences between Gaussian beam parameter fluctuations in a discrete scattering medium and those in a turbulent atmosphere[11]. These differences are due to the fact that the correlation length L_ε of permittivity fluctuations in a scattering atmosphere is less than the radius of the first Fresnel zone ($L_\varepsilon^2 \sim a \ll (L\,/\,k)^{1/2}$), while in a turbulent atmosphere it is greater than the latter [2,11], $L_\varepsilon^T \sim L_0 \gg (L\,/\,k)^{1/2}$, where L_0 is the e external turbulence scale. It follows from the foregoing discussion that for $L_\varepsilon^T \gg (L\,/\,k)^{1/2}$ the permittivity fluctuations are completely correlated in the scattering volume of importance for beam propagation, while for $L_\varepsilon^S \ll (L\,/\,k)^{1/2}$, the volume can be split into a set of separate uncorrelated domains.

For frequency-diversity plane waves and $\rho = 0$, the log-amplitude and phase fluctuation correlation functions are of the form

$$B_{\chi,S}(k,\Omega) \equiv B_{\chi,S}\left[\frac{k}{1-\Omega},\ \frac{k}{1+\Omega}\right] \equiv B_{\chi,S}(k_1,k_2) = \frac{\pi^2 k^2 L}{2(1-\Omega^2)}\int_0^1 d\xi \int_0^\infty d\kappa\ \Phi_\varepsilon\left(\kappa;\frac{k}{1-\Omega},\frac{k}{1+\Omega}\right)\kappa \times$$
$$\times\left\{\cos\left[(2-\xi)\frac{\Omega L\kappa^2}{k}\right] \pm \cos\left[(2-\xi)\frac{L\kappa^2}{k}\right] + \cos\left[(\xi-(1-\Omega))\frac{L\kappa^2}{k}\right] \pm \cos\left[(1-\Omega(1-\xi))\frac{L\kappa^2}{k}\right] + \right.$$
$$\left. + \cos\left[(\xi-(1+\Omega))\frac{L\kappa^2}{k}\right] \pm \cos\left[(1+\Omega(1-\xi))\frac{L\kappa^2}{k}\right] + \cos\left[\xi\Omega\frac{L\kappa^2}{k}\right] \pm \cos\left[\xi\frac{L\kappa^2}{k}\right]\right\}, \quad (7)$$

Here $k = 2k_1k_2/(k_1 + k_2)$ is the wave number corresponding to the average wavelength in vacuum, and $\Omega = (k_1 - k_2)/(k_1 + k_2) = (\lambda_2 - \lambda_1)/(\lambda_2 + \lambda_1)$ is the relative wave number difference. If $\Omega = 0$, i.e. $k_1 = k_2$, then $B_{\chi S}(k, \Omega) \equiv \sigma^2_{\chi,S}$. The log-amplitude and phase fluctuation correlations $b_{\chi,S}$ can be written as

$$b_{\chi,S}(k,\Omega) = B_{\chi,S}(k_1,k_2)\,/\,[B_{\chi,S}(k_1,k_1)\ B_{\chi,S}(k_2,k_2)]^{1/2}.$$

For $\Omega < d^{-1}$ their dependence on the parameter Ω is of the form

$$b_{\chi S}(k,\Omega) \simeq 1 - \alpha\, d^2\Omega^2, \quad (9)$$

and for $d^{-1} < \Omega < 1$,

$$b_{\chi S}(k,\Omega) \sim (1 \pm \Omega^2)/d^2\Omega^2.$$

Here $\alpha = 21{,}32$.

Typically, $d \gg 1$ for atmospheric routes. This results in a fast falloff in $b_{\chi S}$ as a function of Ω, practically to zero, since for $\Omega > d^{-1}$, $b_{\chi S}(k,\ \Omega) \approx 0$. Thus, for round trip paths in a discrete medium the frequency correlation interval of plane wave log-amplitude and phase fluctuations is found to be $\Omega_k \sim d^{-1} = ka^2/L$. This result can be also obtained from the following considerations. The angular width of the domain important for scattering is of order λ/a; hence, the path difference caused by scattering is of order $L\lambda^2/a^2$, and the relevant time delay is approximately $L\lambda^2/ca^2$. The frequency bandwidth is connected with the time delay by the uncertainty relation; thus, $\Omega_k \sim ka^2/L = d^{-1}$.

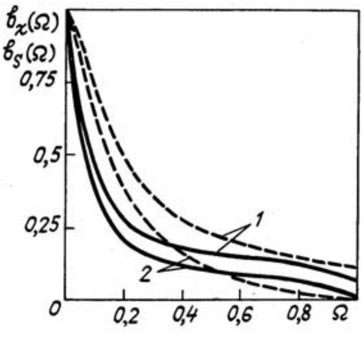

FIG. 2.

Figure 2 illustrates the calculated behavior of the normalized correlation function for frequency-diversity plane waves travelling through a monodisperse scattering atmosphere ($d = 5$). Here solid and dashed curves 1 and 2 depict the log-amplitude and phase statistics respectively. The log-amplitude and phase fluctuation correlation functions for a round-trip path and a one-way path of length $2L$ are seen to be the same.

The foregoing treatment was applied to plane waves. Similar calculations can readily be carried out for spherical waves. The dashed curves in Fig. 2 show the normalized log-amplitude (1) and phase (2) fluctuation correlation functions of the frequency-diversity spherical waves ($d = 5$). Obviously, the frequency correlation in the spherical wave is greater than that for the plane wave. The asymptotic equations for the spherical wave correlation function $b_{\chi S}(k, \Omega)$ are in good agreement with numerical calculations, and give the same results as Eqs. (9) at $\alpha = 0.53$.

It has thus been shown that the log-amplitude and phase fluctuation frequency correlations for frequency-diversity waves travelling over a round-trip path are higher than those for a one way route of length $2L$. For reflected spherical waves the degree of frequency correlation is the same as that for a one-way trip of length L. It might be said that for spherical waves the frequency correlation is improved over a round-trip path. The degree of correlation of a finite Gaussian beam lies somewhere between the limiting cases of plane and spherical waves. Numerical estimates predict that a round-trip through the same inhomogeneities in a discrete scattering medium results in better frequency correlation for beams initial size $a_0 < a$.

It is known[7] that the statistical characteristics of a reflected wave travelling in an inhomogeneous medium strongly depend on the path configuration. The same is true of spherical wave propagation in a discrete scattering medium. Here the propagation geometry is defined by the distance p from the transmitter to the receiver. The correlation is greatest for backward reflection, i.e., for $\rho = 0$. As ρ increases, the correlation decreases monotonically to a limiting value of 0,5 $B_{\chi S}(k, \Omega)$. The characteristic scale for significant variations of $B_{\chi S}(k, \Omega)$ is the scatterer radius a. It should be noted that for the plane waves no dependence of the optical radiation statistical characteristics on the path configuration was found.

Thus in a discrete scattering medium, specular reflections of beams whose initial size is smaller than the scatterer characteristic size lead to amplification of the log-amplitude and phase fluctuations, as well as to their enhanced frequency correlation. Both effects are observed in the immediate vicinity of the beam axis. The transverse size of this neighborhood is determined by the scatterer characteristic radius.

The above considerations provide the basis for a new method of measuring the atmospheric turbulence inner scale I_0 pertaining to precipitation conditions. It should be noted that conventional methods for determining the turbulence inner scale based on the statistical characteristics of optical beam fluctuations cannot be used for a turbid atmosphere[6,11]. The proposed approach to the estimation of I_0 relies on measurements of the log-amplitude fluctuation frequency correlation. It has been shown in an earlier paper[7] that for the frequency-diversity waves the normalized correlation function of the log-amplitude fluctuations only depends on one meteorological parameter, namely the turbulence inner scale. In the realm of weak fluctuations, atmospheric turbulence and discrete scatterers additively affect optical wave fluctuations[8]. If $\Omega > \Omega_k$, the log-amplitude fluctuation correlation of frequency-diversity waves travelling through a discrete scattering medium is negligible, according to Eq. (9). For the same initial values of the diffraction parameter at ω_1 and ω_2, the difference between the variance of the log-amplitude fluctuations of the two beams is determined by the variances of the fluctuations for waves propagating through a turbulent atmospheric layer. Thus, for the frequency-diversity Gaussian beams, the ratio of the log-amplitude fluctuation correlation function to the variance difference depends solely on the meteorological parameter I_0, both for clear and turbid atmospheric conditions. The effect of the scatterer on the optical log-amplitude fluctuations can be eliminated, and there is consequently a way to measure the atmospheric turbulence inner scale without any distortion due to discrete scatterers.

The potential of our method can be illustrated by the following fact. The measured intensity-fluctuation correlation function for frequency-diversity waves ($\Omega = 0{,}18$) were reported in Ref. 12 and 13. The experimental data for the weak turbulence regime were processed using the proposed procedure to yield an average turbulence inner scale of 0.5 mm. The meteorological data given in Refs. 12 and 13 lead to $I_0 = 0.4 \div 0.9$ mm.

Thus, the results obtained by our method for optical measurement of the turbulence inner scale are in good agreement with the available meteorological data.

REFERENCES

1. Ju.A. Kravtsov and A.I. Saichev Usp. Fiz. Nauk **137**, 501, (1982)
2. V.A. Banah, V.L. Mironov *Laser Sounding in a Turbulent Atmosphere* (Nauka, Novosibirsk, 1986)
3. A.G. Borovoj, V.A. Krutikov Opt. Spektrosk., **40**, 728, (1976)
4. V.A. Krutikov Izvestiya Vyssh. Ucheb. Zaved., Radiofizika, **22**, No. 1, 84, (1979)
5. A.G. Borovoj Izvestiya Vyssh. Ucheb. Zaved., Radiofizika, **25**, No. 4, 391 (1982)
6. S.M. Rytov, Yu.A. Kravtsov, V.I. Tatarskii *Introduction to Statistical Radiophysics* (Nauka, Moscow, 1978)
7. I.P. Lukin Akust. Zh., **24**, 899, (1978)
8. I.P. Lukin Kvant. Elektron., **6**, 1756 (1979)
9. I.P. Lukin Opt. Spektrosk., **51**, 1083 (1981)
10. I.P. Lukin Manuscript registed at VINITI, No. 894-80 DEP.
11. A.S. Gurvich, A.I. Kon, V.I. Mironov and S.S. Khmelevtsov *Laser Radiation in a Turbulent Atmosphere* (Nauka, Moscow, 1976)
12. A.S. Gurvich, V. Kan Izvestiya Vyssh. Ucheb. Zaved., Radiofizika, **22**, No. 7, 843 (1979)
13. A.S. Gurvich, V. Kan and Vl.V. Pokasov Optika Akta, **26**, 555 (1979)