ACCOUNT FOR THE EFFECTS OF MULTIPLE SCATTERING IN RECONSTRUCTION OF OPTICAL PARAMETERS OF CLOUDS FROM DATA OF A SPACEBORNE LIDAR

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The estimates of multiple scattering (MS) contribution into spaceborne lidar returns from clouds are obtained that allow us to describe the process of the first orders of scattering formation. A method is proposed to introduce corrections for MS into the algorithm for reconstructing the scattering coefficient profile and optical thickness τ *of a cloud under an a priori uncertainty that is applicable at* $\tau_{max} < 3.5$*. We present some results of determination of the optical parameters of the stratus clouds top from signals computed by the Monte Carlo method.*

Interpretation of the data of the spaceborne laser sounding of cloud fields has certain peculiarities connected, on the one hand, with the technical capabilities of lidars (this problem was considered earlier[1]) and, on the other hand, with large distances of sounding, hence, with the energy losses and high background due to multiple scattering (MS).

In the general case, to determine the optical and microphysical parameters of clouds it is necessary to solve a nonstationary transfer equation using the Monte Carlo method.[2] However, sometimes it is possible to eliminate the MS contribution into the backscatter power and to process it by the techniques developed in the single scattering approximation (see, for example, Ref. 3). Such an approach was used for the interpretation of the return signals from cirrus clouds.[4]

This problem is difficult, first because the MS contribution depends on the optical properties and size spectrum parameters, which are unknown *a priori*, and, in the general, should be assessed from signals (it is difficult to make additional calibration measurements from onboard a satellite). Second, no analytical estimations of contributions from scattering orders higher than 2 at the stage of lidar return formation have yet been developed. At the same time, as was shown in Ref. 3, the error of corrections for the MS background when inverting the data, results in the instability of solution, that is caused by the fact that the scattered light intensity relates to the scattering coefficient $\beta(z)$ via an integral relationship, to regularize the solution it is necessary to introduce an additional *a priori* information on multiple scattering and solution itself.

In this paper we present an algorithm for taking into account MS when solving the lidar equation. The algorithm is based on the double scattering theory[5] and is useful for interpretation of return signals from cirrus clouds obtained with a spaceborne lidar as well as from the upper boundary of stratus clouds (up to the optical thickness of $\tau_{max} < 3.5$).

RELATIONSHIPS FOR ESTIMATING THE MS CONTRIBUTION INTO THE LIDAR RETURNS

The numerical experiment carried out in Ref. 6 allowed us to isolate some peculiarities in formation of lidar return signals when sounding from space. It was noted that, irregardless of the scattering coefficient profile, the leading edge of the signal up to $\tau = 0.8–1$ is determined by the 1 and 2 orders of scattering. The intensity of scattering orders up to 5 order increases with τ increasing from 1 to 1.5, while the trailing edge of the pulse ($\tau > 2$) is formed due to higher orders of multiple scattering ($n > 5$). The signal caused by the ith order of scattering (let us designate it $P^{(i)}(z)$) has a pronounced maximum, whose amplitude is lower and displaced to the greater τ in comparison with those of $P^{(i-1)}(z)$. The values $P^{(i)}(z) < P^{(i-1)}(z)$ at small τ. The situation changes as τ increases and at some τ_* it happens that $P^{(i)}(z) > P^{(i-1)}(z)$ for $\tau > \tau_*$. One can take into account this *a priori* information.

According to the theory of double scattering,[5] when sounding clouds from space

$$P^{(2)}(z) = \delta(z)\, P^{(1)}(z), \qquad (1)$$

where

$$P^{(1)}(z) = A\gamma_\pi\, \beta(z)\, \exp\{-\,2\,\tau(z_0, z)\}/z^2 \qquad (2)$$

is the lidar equation in the single scattering approximation, A is the instrumentation constant, z_0 is the cloud top boundary, and γ_π is the lidar ratio;

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$$\delta(z) = \frac{2\pi\, z^2}{\gamma_\pi} \left\{ \frac{1}{\beta(z)} \int\limits_0^{\alpha_1} \Gamma(\varphi) \int\limits_{z_0}^{z} \frac{\beta(x)\, \beta(x')}{R(z, x, \varphi)}\, dx\, d\varphi + \right.$$

$$+ \frac{1}{\beta(z)} \int\limits_{\alpha_1}^{\alpha_2} \Gamma(\varphi) \int\limits_{x_1}^{z} \frac{\beta(x)\, \beta(x')}{R(z, x, \varphi)}\, dx\, d\varphi +$$

$$+ \int\limits_{\alpha_2}^{\pi} \Gamma(\varphi) \int\limits_{z_0}^{z} \frac{\beta(x)}{x^2}\, dx\, d\varphi +$$

$$\left. + \int\limits_0^{\alpha_2} \Gamma(\varphi) \int\limits_0^{\rho_1} \frac{\beta(z + \rho \cos(\varphi))}{(z - \rho)^2}\, d\rho\, d\varphi \right\}, \qquad (3)$$

where

$$\Gamma(\varphi) = \gamma(\varphi)\, \gamma(\pi - \varphi) \sin\varphi; \quad R(z, x, \varphi) =$$

$$= z^2 - (2z - x)\, x \sin^2\varphi/2;$$

$$x' = x + \rho \cos(\varphi) = x + \frac{z(z - x)}{z - x \sin^2\varphi/2};$$

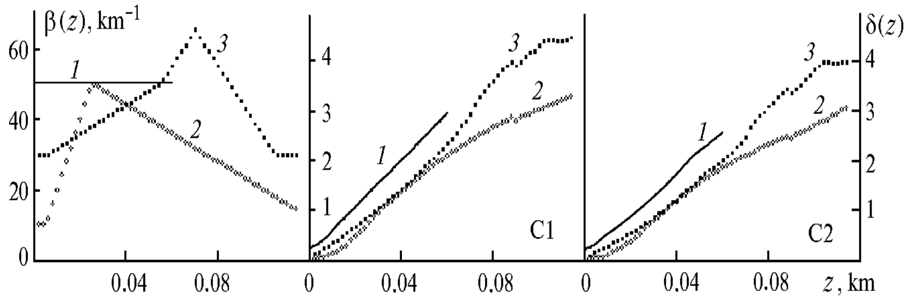
$$\alpha_1 = \arctan \left[\frac{z\, \mathrm{tg}(\varphi_0)}{z - z_0} \right];$$

$$\alpha_2 = \pi - \arctan \frac{z - z_0}{(z - z_0) + 2\,\Delta z};$$

$$x_1 = z \left(1 - \frac{(z - z_0)}{z_0}\, \mathrm{ctg}(\varphi/2) \right); \quad \rho_1 = \frac{\Delta z}{\cos^2(\varphi/2)},$$

$\gamma(\varphi)$ is the normalized scattering phase function, and φ_0 is the field of view angle of the transceiver. The relationship (3) is valid for $z \in [z_0, R_0]$, where $R_0 = z_0/(1 - \tan(\varphi_0/2))$, which is practically always valid in the cases we consider here (the generalization for $z > R_0$ may be found in Ref. 5).

Figure 1*b*,*c* presents some results on $\delta(z)$ calculated for clouds with different values of the scattering coefficients, the profiles of which are shown in Fig. 1*a*. The calculations were made for the monostatic lidar operating at $\lambda = 0.53$ μm on board a satellite 400 km above the Earth. It was assumed that the source emits isotropically in the cone $2\pi(1 - \cos\psi_0)$, where $\psi_0 = 0.2$ mrad. The return signal is collected with a receiver in the cone $2\pi(1 - \cos\varphi_0)$, where $\varphi_0 = 0.44$ mrad. The radar parameters coincided with the characteristics of the spaceborne lidar BALKAN. The cloud layer at the altitude of 1.5 to 2 km had the scattering properties corresponding to C1 and C2 clouds (Figs. 1*b* and 1*c*, respectively) of the classification[7] (numbers at the curves are the same).



a *b* *c*

FIG. 1. The ratio of the signal of the second order of scattering to the first one as a function of the sounding depth: a) model profiles of the scattering coefficient; b) the functions δ(z) for the cloud C1; c) the same for the cloud C2.

Let us assume that the estimates $\tilde{P}^{(i)}(z)$ at $i > 2$ are the functions of $\delta(z)$ and can be presented in the form

$$\tilde{P}^{(i)}(z) = \delta_i(z)\, \tilde{P}^{(i-1)}(z),$$

$$\delta_i(z) = \frac{[\delta(z)]^{(i-1)}}{(i - 1)!}, \qquad (4)$$

where $\delta(z)$ obeys the relationship (3). Such a representation satisfies all qualitative requirements to the relationship between $\tilde{P}^{(i)}(z)$ and $\tilde{P}^{(i-1)}(z)$, formulated in Ref. 6. The introduced function $\delta_i(z)$ takes into account the information about microstructure and optical properties of clouds, that makes it possible to use it in the methods for reconstructing the cloud parameters from the MS background.[8] The estimate of the return signal power (taking into account MS up to some *n*th order of scattering) has the form

$$\tilde{P}^{(\Sigma)}(z) = \sum_{i=1}^{n} \tilde{P}^{(i)}(z) = \tilde{P}^{(1)}(z) \exp\{\delta(z)\} =$$

$$= \tilde{P}^{(1)}(z)\, \Delta\tilde{P}(z). \qquad (5)$$

The quantitative reliability and the principles of applicability of the assumptions introduced were studied in the numerical experiment. Figure 2*a* shows some results on calculated $P(z)$, $P^{(i)}(z)$ ($i = 1$ to 5) by the Monte qarlo method for C1 clouds, the profiles $\beta(z)$ of which are presented in Fig. 1*a*. The experimental conditions were analogous to the aforementioned ones (the technique for calculating is described in details in Ref. 9). Figure 2*b* presents some results of calculating $\tilde{P}^{(\Sigma)}(z)$, $\tilde{P}^{(i)}(z)$ ($i = 1$ to 5) according to Eqs. (3) to (5) obtained at the same assumptions. Figure 2*c* presents relative errors (in percent) of estimating $\tilde{P}^{(\Sigma)}(z)$ using the formulas proposed. The results calculated by the Monte Carlo method (curves 0 Fig. 2*a*) were taken as the exact value of $\tilde{P}^{(\Sigma)}(z)$. From these results one can see that the estimates (3) to (5) provide quite accurate (estimate errors ≤ 30%) description of the MS contribution up to $\tau \le 2$. At the same time these estimates may be used when correcting return signals for multiple scattering in problems on reconstructing optical parameters from space-based measurements. One can conclude from the comparison of the results presented in Figs. 2*b* and 2*c* that it is possible to use the estimates (5) for describing the MS contribution at least up to $\tau < 3.5$.

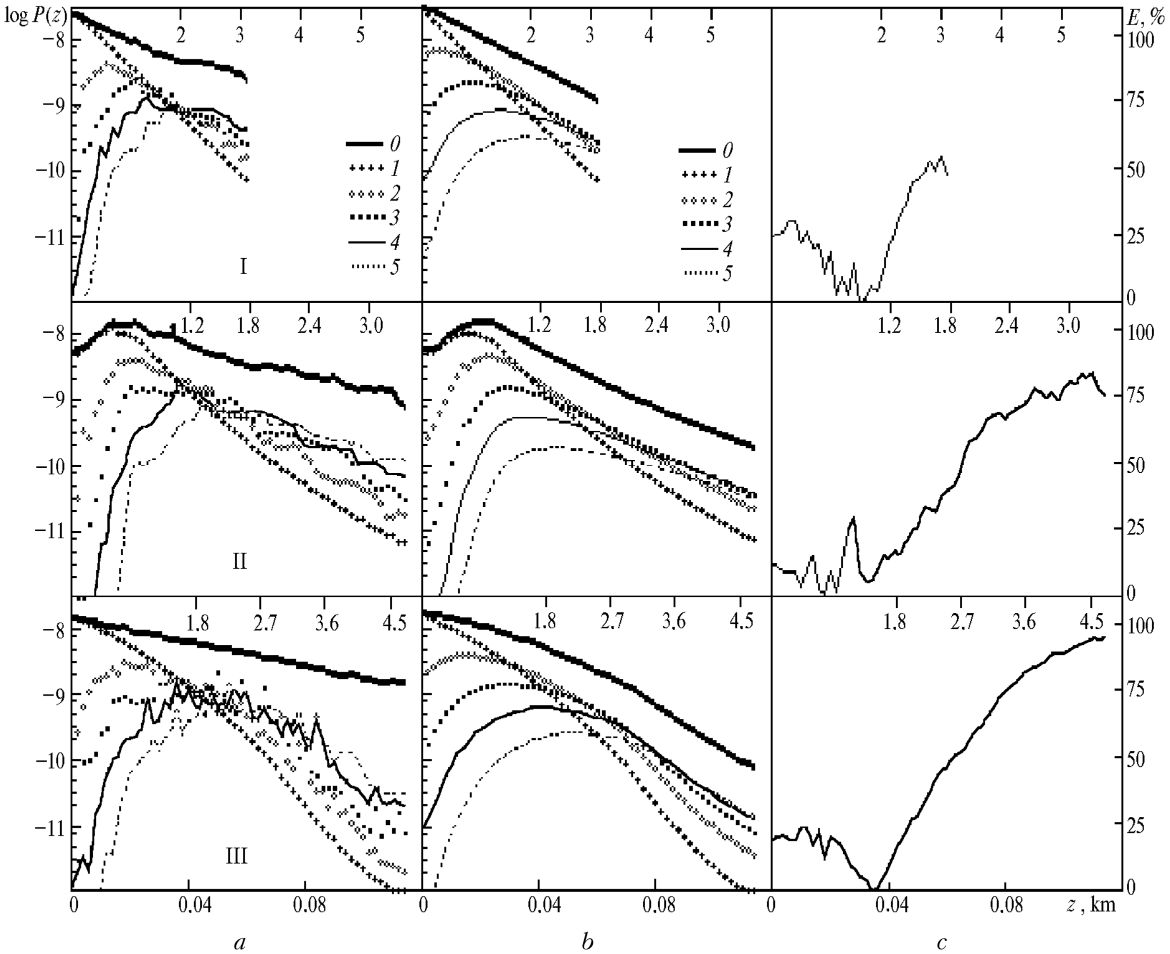


FIG. 2. Comparison of the methods for calculation of return signals obtained with a lidar from space using first orders of multiple scattering: a) calculations by Monte Carlo method; 0 – total signal; 1 to 5 – signals from 1 to 5 orders of scattering; b) estimates allowing for MS; c) relative errors in estimating total return signal (model profiles of $\beta(z)$ *for versions* I, II, *and* III *were taken from Fig. 1a (curves 1 to 3).*

ALGORITHM FOR RECONSTRUCTING PROFILES OF SCATTERING COEFFICIENT IN A CLOUD

In order to apply the algorithm for reconstructing the optical parameters of clouds one needs for certain *a priori* information about the object under study. In practice, such a comprehensive *a priori* information can hardly be obtained, so it is necessary to develop the methods under conditions of an *a priori* uncertainty. Without the loss of generality, let us consider the case of a complete absence of any additional information about the cloud (if it is available there are no problems how to use it).

Then the solution of Eq. (2) taking into account the MS contribution relative to $\beta(z)$ has the form

$$\beta(z) = \frac{\tilde{P}^{(1)}(z)\, z^2}{2\varepsilon\Psi(z_0, z_*) + 2\Psi(z, z_*)} \, ; \qquad (6)$$

$$\tilde{P}^{(1)}(z) = \tilde{P}^{(1)}(\beta(z), \gamma(\varphi), z) = P(z)/\Delta\tilde{P}(z); \qquad (7)$$

$$\Psi(z_1, z_2) = \int_{z_1}^{z_1} \tilde{P}^{(1)}(z)\, z^2\, dz;$$

$$\varepsilon = \varepsilon(\tau, z_*) = 1/\{e^{[2\tau(z_0,z_*)]} - 1\} \quad (8)$$

is the dimensionless parameter represented by the *a priori* value of the optical thickness of the sounding path portion $[z_0, z_*]$; z_* is an arbitrary point from the interval $[z_0, z_{max}]$. In order to make use of Eqs. (6)–(8) it is necessary to know:

1) profile of the scattering coefficient (use of $\beta(z)$ in Eq. (7)). Since $\beta(z)$ is also the sought parameter, let us use the iteration procedure (j is the iteration number)

$$\bar{\beta}^{(j)} = \beta^{(j-1)};\ \ P^{(1),(j)} = \tilde{P}^{(1)}(\bar{\beta}^{(j)}, \gamma);$$

$$\beta^{(j)} = \beta(\varepsilon(\tau), P^{(1),(j)}), \quad (9)$$

starting from the zeroth approximation $\beta^{(0)} = \bar{\beta}$ (model or calculated);

2) optical thickness of the path interval $[z_0, z_*]$ (regularization of the solution by means of introducing the parameter $\varepsilon = \varepsilon(\tau)$ into Eq. (8)). To estimate ε, traditionally,[10] one should to minimize the purpose function that is chosen based on the requirements to the sought profile $\beta(z)$. Let us determine the calibration point at the end of the sounding path ($z_* = z_{max}$) and set

$$\Phi(\tau) = [\beta_{max}(\tau^{(j,l)}) - \beta_{max}(\tau^{(j,l-1)})]^2, \quad (10)$$

(l is the number of iteration when estimating the optical depth). As calculations show, this function has a stable minimum on τ in the vicinity of $\beta(z)$ value sought (the function of discrepancy between profiles

$$\Phi(\tau) = \int_{z_{max}}^{z_0} [\beta_{max}(\varepsilon(\tau^{(j,l)}), z) - \beta_{max}(\varepsilon(\tau^{(j,l-1)}), z)]^2\, dz$$

that is traditionally used or the “smoothness functionB

$$\Phi(\tau) = \int_{z_0}^{z_{max}} \sum_{k=1}^{n} \left[\frac{d^k\, \beta(\varepsilon(\tau^{(j,l)}), z)}{dz^k}\right]^2 dz$$

have no extrema at $\tau > 1$). Minimization of Eq. (10) with respect to τ together with the iteration procedure (6)–(9) give the sought solution $\beta(z)$ under the conditions of *a priori* uncertainty.

As was mentioned above, the value τ_{max} is also related to the number of scattering orders which take part in the formation of return signal and can be used for correct estimation of $P^{(1)}(z)$ from Eq. (1) at $\tau_{max} < 1$ or from Eq. (5) at $\tau_{max} \geq 1$.

3) setting the cloudiness type (setting $\gamma(\varphi)$ in Eq. (7)); if no special methods are used, it seems to be most reasonable to identify clouds according to the distance, and in the case of the upper layer (altitude more than 6 km) to use $\gamma(\varphi)$ for crystal clouds and for water clouds in other cases.

NUMERICAL EXPERIMENT ON RECONSTRUCTION OF CLOUD OPTICAL PARAMETERS

In order to study efficiency of the algorithm proposed, we have carried out a closed numerical experiment. The conditions of the experiment were similar to the aforementioned ones.

Figure 3 shows the results on the scattering coefficient profiles reconstructed under conditions of *a priori* uncertainty relative to $\tau(z_0, z_{max})$ at a known cloud type (C1). The functions $\beta(z)$ sought are shown in Fig. 3*a*; Figure 3*b* illustrates the reconstruction of $\beta(z)$ according to Eqs. (6)–(9) at the exactly known τ_{max} which are, in some sense, the standard for resolution of the method. Curve *1* in this figure shows a solution to lidar equation without taking into account contribution from multiple scattering. This solution was selected as the initial approximation in the iteration algorithm. The result of its application after 10 iterations is shown by curve *2* (let us note that the method has shown the stability to the initial approximation; for providing the convergence it is enough to make ≈ 10 iterations). Figure 3*c* shows the results of reconstructing $\beta(z)$ at a known τ_{max}, the estimate of which is found by minimizing Expr. (10) (numbers at the curves are the same).

Figure 4 illustrates the effect of the error in setting the scattering type of the cloud on the accuracy of determining $\beta(z)$. The sought function $\beta(z)$ is shown in Fig. 4*a*; the results of using the iteration algorithm (6)–(9) at a known τ_{max} are shown in Fig. 4*b*; the results from Fig. 4*c* are obtained by the same method with estimating τ_{max}. Curves *1* correspond to the profiles reconstructed without the account of MS; curves *2* are the result of reconstruction for the known type of cloud (C1), curves *3* are obtained by inversion with the use of $\gamma(\varphi)$ for a C2 cloud, curves *4* were calculated using $\gamma(\varphi)$ of the crystal cloud according to Ref. 11; in addition the values τ_{max} obtained in each case are shown in Fig. 4*c* (numbers at curves are the same).

The analysis of results of the numerical simulation makes it possible to draw the following conclusions:

1) processing of lidar signals by the algorithm proposed provides for a detailed reconstruction of $\beta(z)$ up to $\tau_{max} < 3$; resolution becomes worse as τ_{max} increases; for using the method at $\tau > 4$ it is necessary to describe the MS contribution more strictly (Fig. 3*b*);

2) the method is effective for estimating τ_{max} under conditions of *a priori* uncertainty (the error does not exceed 10%, Fig. 3*c*);

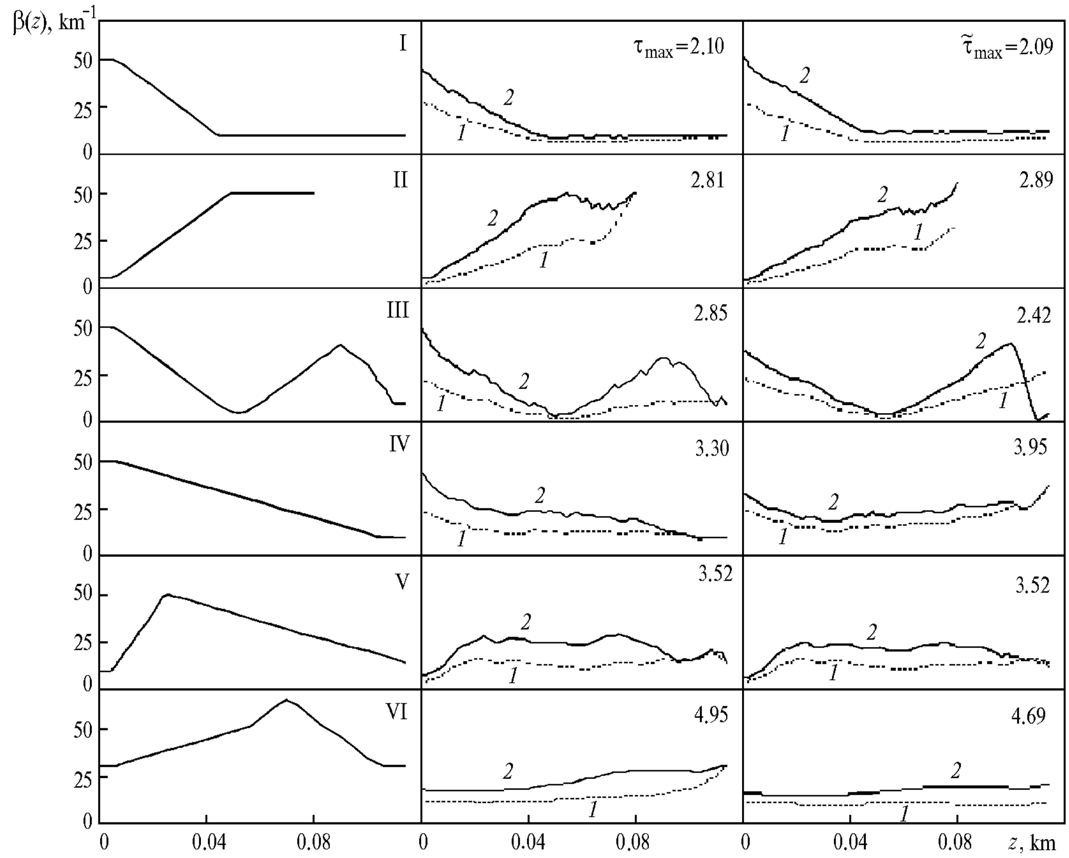


FIG. 3. Comparison of different methods of reconstruction of the scattering coefficient from signals calculated by the Monte Carlo method: a) exact value β(z), result of reconstruction without taking into account MS (1) and taking into account MS after 10 iterations (2); b) at a known τ_{max}; c) under the conditions of a priori uncertainty in τ (P(z) profiles for V *and* VI *models correspond to the total signal from Fig. 2a, curves 0, for versions* II *and* III*).*

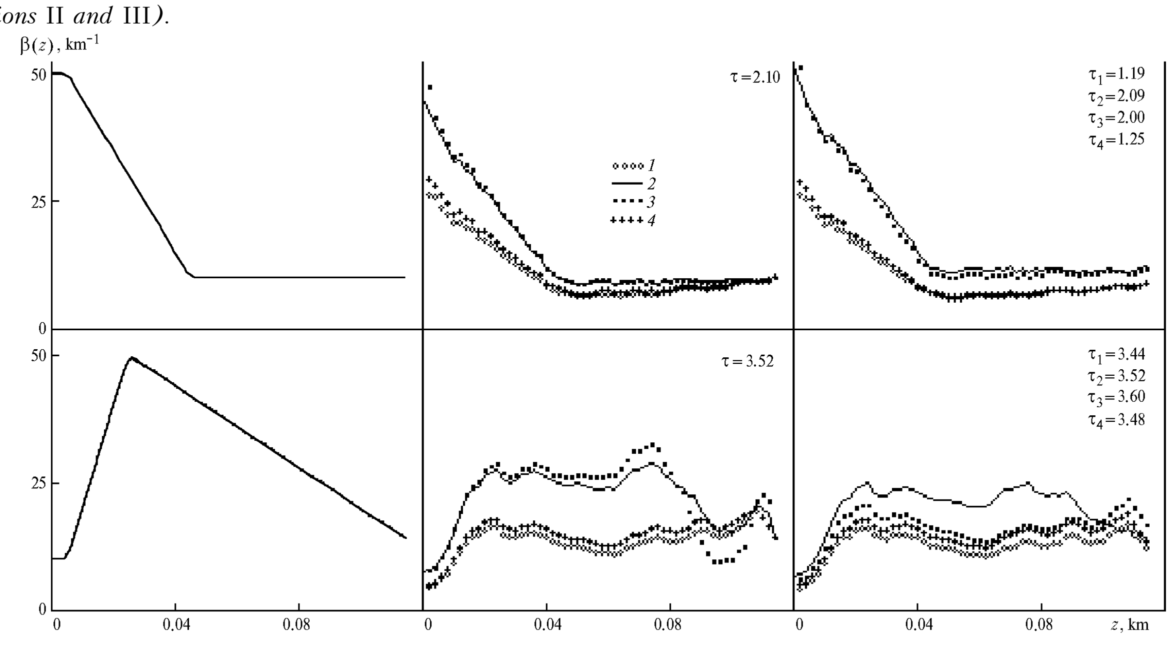


FIG. 4. Reconstruction under a priori uncertainty in the cloud type: a) exact value β(z) for a cloud C1; the result of reconstruction without taking into account MS (1), using γ(φ) for C1 (2), for C2 (3); for a crystal cloud (4); b) τ_{max} is known; c) τ_{max} is to be estimated.

3) the necessary stage in the use of the method is distinguishing of the phase composition of the cloud: the use of γ(φ) for the crystals when processing the signal from a water droplet cloud results in a “smoothedB profile β(*z*) reconstructed that is close to the values found from the lidar equation without the account for MS (in other case, when processing signals from crystal clouds with γ(φ) for C1 cloud reconstruction of β(*z*) is not stable already at $\tau_{max} = 1$, see Ref. 4); the choice of specific form of the scattering phase function of water droplet clouds does not essentially affect the quality of reconstructing β(*z*) (Fig. 4).

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