VERTICAL PROFILE OF THE SPECTRAL SCATTERING AND ABSORPTION COEFFICIENTS OF STRATUS CLOUDS. PART I: THEORY

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Formulas are derived which, by using asymptotic formulas of the theory of radiative transfer, express volume scattering and absorption coefficients through hemispherical fluxes of solar radiation in the optically thick plane layer and in the system of several layers. The accuracy and applicability of formulas are assessed. From the results obtained, it is possible to determine the vertical profile of spectral dependences of the optical parameters of stratus clouds.

INTRODUCTION

The interest in studies of solar radiation interaction with stratus clouds is motivated by their great spatial and temporal coverage. Knowledge of the vertical structure of stratus cloud optical parameters is central both in studies of cloud properties and processes of cloud formation. The volume scattering coefficient in the cloud is determined by the size and number of scattering particles, while the vertical profile of scattering coefficient bears information on the vertical behavior of particle size distribution in the cloud.

In many cases stratus clouds contain atmospheric aerosols such as those of anthropogenic origin. Their vertical distribution within cloud is nonuniform and may characterize washing out and accumulation of atmospheric aerosols in stratus clouds. The spectral dependences of cloud optical parameters are sensitive to the atmospheric aerosol composition. So the possibility arises to deduce from available spectral behavior of volume absorption coefficient the presence of pollutants in the cloud.

Thus, knowledge of the vertical profile of optical parameters of real cloud is of key importance both for studying the stratus cloudiness and solving ecological problems related to the transport and transformation of atmospheric ecotoxicants.

Stratus clouds are well modeled as a plane scattering layer, with large optical depth and infinite horizontal extension, which is convenient for theoretical considerations. In the present paper we propose an approach to interpreting radiation measurements in the visible spectral range (0.4–0.95 μm) because, first, most of solar irradiance comes in the visible spectrum (0.4 μm flux is an order of magnitude larger than the 0.9 μm flux) and, second, absorption of visible radiation by clouds is weak compared with the scattering, so that asymptotic formulas of the radiative transfer theory may surely be used.

STATEMENT OF THE PROBLEM

Vertically inhomogeneous atmospheres have been thoroughly addressed in Ref. 3 where rigorous solutions of some radiative transfer problems were obtained. These results, however, are useless when solving inverse problems and derive formulas for calculating optical parameters of the medium from the data of radiation measurements. So, we shall use an approximate treatment of the vertical inhomogeneity as proposed in Ref. 4.

When considering inhomogeneous vertical structure of cloud layers one should distinguish between two cases: (1) two or three cloudy layers with noncloudy one(s) in between, and (2) single optically thick cloudy layer whose properties change with the altitude. The first case may be reduced to that of interpreting data measured at the top and bottom of a single-layer cloudiness. Earlier, similar analytical expressions for radiation fluxes emerging from cloud top and bottom were used to derive rigorous formulas for optical properties of the layer.[1,2] With the use of the above method, were obtained corresponding spectral dependences of the volume scattering and absorption coefficients averaged over the entire cloudy layer.

In the second case, it is important to remember that the radiation field inside the cloud is different from those emerging from the cloud top and bottom, thus necessitating the use of different asymptotic formulas.[3–5] For instance, in Ref. 6 radiance measurements inside the cloud layer together with the asymptotic formulas for pure scattering were used to infer the optical depth of the cloud layer. However, the above method is seriously deficient in that it neglects the true absorption in the cloud layer. Here we account for the inherent cloud layer absorption by using formulas for hemispheric radiative fluxes inside the layer, and use the procedures described in Refs. 1 and 2 to obtain formulas expressing cloud layer optical parameters in terms of the radiative fluxes measured experimentally.

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We begin with the case 1. In order to efficiently account for with the cloud inhomogeneity a simple trick was used in Ref. 4 to calculate downward and upward fluxes of scattered solar radiation in vertically inhomogeneous, optically thick medium that can be represented as a set of optically thick layers, each having different optical properties. In that paper, it was assumed that the influence of the upper-layer clouds is allowed for by the flux of scattered radiation passed through these layers.

Thus, an approximate account was taken of the angular distribution of the field of scattered radiation entering from above and below into a sublayer located deep inside the system of layers. The method validation have shown its good accuracy (better than 1%) which is provided by the adopted approach[4] in the case of optically thick cloud ($\tau > 7$). In the present paper, we derive reverse formulas for determining optical parameters of cloud layers using data on measured spherical fluxes of up- and downwelling solar radiation, $F^{\uparrow\downarrow}(\tau)$, by using the above approach.

In the case 2, when deriving formulas we assume homogeneous cloud layer; so, when applying the obtained results to the field measurement data, one should first evaluate the effect of vertical inhomogeneity of a cloud layer on the accuracy of inverse problem solution.

In both cases, the true absorption in the medium is assumed negligible, and when asymptotic expansions in powers of the smallness parameter s are used, terms with orders higher than 3 are neglected.

BASIC FORMULAS

Let $\tau = \Sigma\tau_i$ be the optical depth of the system of cloud layers, where τ_i are the thicknesses of cloud sublayers. A parallel radiative flux πS is incident upon the top of the layer at an angle arccos(ζ) to the normal. The single scattering albedo (the probability of photon survival upon a single scattering event) in the layer will be denoted as ω_{0i}, so that $1 - \omega_{0i} \ll 1$. The scattering phase function used will be the Henyey-Greenstein phase function with the asymmetry parameter (mean cosine of the scattering angle) $g_i = \overline{\cos\gamma}$ as given in Ref. 9. Volume extinction coefficients in each layer are given as $\varepsilon_i = \tau_i/z_i$, where z_i are the geometrical thicknesses of the sublayers, and $k_i = \varepsilon_i\,(1 - \omega_{0i})$ and $\sigma_i = \varepsilon_i\,\omega_{0i}$ are the volume absorption and scattering coefficients, respectively. The lowest cloud layer overlays an underlying surface with the albedo A. Light scattering in the subcloud (noncloudy), optically thin layer will be neglected.

Fluxes of scattered radiation escaping an optically thick homogeneous layer and measured in units of flux incident on the cloud top, $\pi S\zeta$, are given by well-known asymptotic formulas representing a rigorous solution of the transfer equation in the case of an optically thick medium.[3,4] Let us introduce the parameter $s^2 = (1 - \omega_0)/[3(1 - g)]$.

When a multilayer system is considered, the incoming flux at the top layer from the underlying layers is thought of as coming from the surface with the albedo equal to the sum of albedos of the underlying layers. The albedo $A_1 = F^{\uparrow}(\tau_1)/F^{\downarrow}(\tau_1)$ is obtained from experiment (as the ratio of hemispherical fluxes at the interface between the 1st and 2nd layers). Radiative fluxes emerging from an optically thick layer ($\tau > 7$) are given by the formulas[4]

$$F^{\downarrow}(\tau,\zeta) = (\bar{Q}\,u(z)\,M\,e^{-k\tau})/(1 - \bar{N}\,N\,e^{-2k\tau}),$$
$$F^{\uparrow}(0,\zeta) = a(\zeta) = Q\,u(\zeta)\,M\,\bar{N}\,e^{-2k\tau})/(1 - \bar{N}\,N e^{-2k\tau}), \qquad (1)$$

where $a(\zeta)$ is the plane albedo of a semi-infinite medium, and $u(\zeta)$ is the "exit" function describing the angular dependence of the radiance emerging from a semi-infinite medium. The asymptotic constants M, Q, k, and N and functions $a(\zeta)$ and $u(\zeta)$, in the case of weak true absorption ($1 - \omega_0 \sim 0.005$), are expressed by asymptotic series expansions over a small parameter $1 - \omega_0$.[3,4] Here we write these expansions in terms of the parameter s

$$k = 3\,(1 - g)\,s\left[1 + s\left(1.5g - \frac{1.2}{1 + g}\right)\right] + \ldots$$
$$M = 8s\left[1 + s\left(6 - 7.5g + \frac{3.6}{1 + g}\right)\right] + \ldots$$
$$N = 1 - 3\,\delta\,s + 4.5\,\delta^2\,s^2 + \ldots \qquad (2)$$
$$a^{\infty} = 1 - 4s + 6\delta s^2 + \left[24 - 30g + \frac{1.63}{1+g} + 9\delta^2\right]s^3 + \ldots$$
$$Q = 1 - 1.5\,\delta\,s + Q_2\,s^2,$$

where $\delta = 4\int_0^1 u_0(\zeta)\,\zeta^2 d\zeta \cong 1.427$

and $a^{\infty} = 2\int_0^1 a(\zeta)\zeta\,d\zeta$; $Q = 2\int_0^1 u(\zeta)\zeta\,d\zeta$.

The value a^{∞} is called the spherical albedo of a semi-infinite medium.

For the functions $a(\zeta)$ and $u(\zeta)$ we have the following expressions[3,4]:

$$u(\zeta) = u_0(\zeta)\,(1 - 1.5\,\delta s) + u_2(\zeta)\,s^2 + \ldots$$
$$a(\zeta) = 1 - 4\,u_0(\zeta)\,s + a_2(\zeta)\,s^2 + \ldots, \qquad (3)$$

where $u_0(\zeta)$ is the angular distribution of the emerging radiance from a semi-infinite medium in the case of pure scattering medium, and the function $a_2(\zeta)$ is given in Ref. 7 as

$$a_2(\zeta) = 3\,u_0(\zeta)\,[3/(1 + g)\,(u_0(1)\times \zeta - 0.9) + 2\delta)]. \qquad (4)$$

The bar above the symbol denotes the functions and quantities with the account for albedo, namely

$\bar{N} = N - AMQ/(1 - A\, a^\infty)$,

$\bar{u}(\eta) = u(\eta) + AQ\, a(\eta)/(1 - A\, a^\infty)$

and

$$\bar{Q} = Q/(1 - A\, a^\infty). \qquad (5)$$

In Refs. 1 and 2, simple transformations in formulas (1), substitution of expansions (2) and (3) and formula (4), and designation of the net flux $F(\tau) = F^\downarrow(\tau) - F^\uparrow(\tau)$ were used to derive expressions for s^2 and $\tau' = 3\tau(1 - g)$. In the approximation adopted here, the corresponding expressions for the top layer of the multilayer medium coincide with the formulas from Refs. 1 and 2

$$s_1^2 = \frac{F^2(0) - F^2(\tau_1)}{16\,[u_0^2(\zeta) - F^{\uparrow 2}(\tau_1)] - 2a_2(\zeta)\, F(0) - 12\delta\, F^\downarrow(\tau_1)\, F(\tau_1)}\,; \qquad (6)$$

$$\tau_1' = \frac{1}{2\,s_1} \ln \left[N_1^2 \left(1 + \frac{2\,u_0(\zeta)\,s_1\,(4 - 9\,s_1^2)}{a(\zeta) - F^\uparrow(0)} \right) \left(1 - \frac{8\,A_1\,s_1}{1 - A_1\,a_1^\infty} \right) \right]. \qquad (7)$$

For the lower layers (with the index $i > 1$), in formulas (1) the albedo $A_i = F^\uparrow(\tau_i)/F^\downarrow(\tau_i)$, from measurements of fluxes between the layers i and $i + 1$, and the functions $u(\zeta)$ and $a(\zeta)$ are replaced with the integrals of these functions, taken over the angle $\zeta - Q_i$ and a_i^∞, multiplied by the radiative flux $F^\downarrow(\tau_{i-1})$. The formulas for fluxes are converted into those for s and τ by using the procedure described in Refs. 1 and 2. Thus we arrive at expressions for s_i^2, characterizing the absorption in the layers with $i > 1$:

$$s_i^2 = \frac{F^2(\tau_{i-1}) - F^2(\tau_i)}{16\,[F^{\downarrow 2}(\tau_{i-1}) - F^{\uparrow 2}(\tau_i)] + 12\delta\,[F^\downarrow(\tau_{i-1})\,F^\uparrow(\tau_{i-1}) - F^\downarrow(\tau_i)\,F^\uparrow(\tau_i)]}\,; \qquad (8)$$

$$\tau_i' = \frac{1}{2\,s_i} \ln \left[N_i^2 \left(1 + \frac{2\,s_i\,(4 - 9\,s_i^2)}{a_i^\infty - A_{i-1}} \right) \left(1 - \frac{8\,A_i\,s_i}{1 - A_i\,a_i^\infty} \right) \right], \qquad (9)$$

where $F(\tau_i)$ are the values of the net radiative flux at the top of the total layer and at the layer interfaces

$F(\tau_i) = F^\downarrow(\tau_i) - F^\uparrow(\tau_i)$.

Here τ_i denotes the optical depth of the ith level into the cloud layer. The quantities b^∞ and $b(\tau)$ are called the internal albedo of a semi-infinite atmosphere and of the atmosphere of finite optical depth. The asymptotic constants and functions, expressing the dependence on the solar zenith angle in the case of weak true absorption in cloudy layers, are represented by expansions (2) and (3) over small parameter s.

DERIVATION OF ASYMPTOTIC FORMULAS FOR SOLVING INVERSE PROBLEM INSIDE AN OPTICALLY THICK LAYER (case 2)

For deriving the expressions sought, we will use formulas for radiative fluxes, both at the layer boundaries (Eq. (1)) and inside the layer. Inside the cloud layer, we will use as the main characteristics the ratio between the upward and downward hemispheric fluxes of solar radiation and the net flux $F = F^\downarrow - F^\uparrow$ given by the formulas[4,5]

$$b(\tau) = \frac{b^\infty - \bar{N}\, e^{-2k(\tau_0 - \tau)}}{1 - b^\infty\, \bar{N}\, e^{-2k(\tau_0 - \tau)}}\,;$$

$$F(\tau, \zeta) = \frac{4s\,u(\zeta)\,e^{-k\tau}}{1 - N\,\bar{N}\,e^{-2k\tau_0}}\,[1 + \bar{N}\,e^{-2k(\tau_0 - \tau)}]. \qquad (10)$$

The radiative transfer through an optically thick layer can be considered to occur in three phases[5]: (a) light penetration through the top layer with $\tau = 0$ (light pumping) lying between the levels 0 and 1; (b) diffusion through the medium (diffusion), i.e., through the internal layers between the levels $0 < i < n$; and (c) light leakage through the bottom layer with $\tau = \tau_0$ (exit). Given optically thick medium, the pumping, diffusion, and exit phases can be thought to occur independently. We will consider individual phases in the order of their occurrence. Making simple transformations in (1) and (10), using expansions (2) and (3) as well as the relations (4) and (5) for surface albedo, and retaining terms to quadratic order in s^2 we obtain equation linear, relative to s^2; its solutions are as follows.

For the top layer located between the cloud top and the first measurement level into the cloud (0, 1)

$$s_1^2 = \frac{F(0)^2 - F(\tau_1)^2}{16\, u^2(\zeta) - 4\,(F^{\downarrow} + F^{\uparrow})^2 + F(\tau_1)^2 \left(2\, Q_2 - \frac{9}{4}\,\delta^2\right) - 2\, F(0)\, a_2(\zeta)} \,. \quad (11)$$

For an internal cloud layer $(i - 1,\, i)$

$$s_i^2 = \frac{[F^2(\tau_{i-1}) - F^2(\tau_i)]\, F^2(\tau_{i-1})}{16\,[F^2(\tau_i)\, F^{\downarrow}(\tau_{i-1})\, F^{\uparrow}(\tau_{i-1}) - F^2(\tau_{i-1})\, F^{\downarrow}(\tau_i)\, F^{\uparrow}(\tau_i)]} \,; \quad (12)$$

And for the bottom layer between the last but one measurement level into the cloud and cloud bottom $(n-1, n)$

$$s_n^2 = \frac{F(\tau_{n-1})^2 - F(\tau_n)^2}{4\,(F^{\downarrow}_{n-1} + F^{\uparrow}_{n-1})^2 - 16 F_n^{\uparrow 2} - F(\tau_{n-1})^2 \left(2\, Q_2 - \frac{9}{4}\,\delta^2\right) - 12\delta\, F^{\uparrow}_n\, F(\tau_n)} \,. \quad (13)$$

In Eqs. (11)–(13), the numerators represent the difference of squared net fluxes at the corresponding levels in the cloud and at its top and bottom, and also it is defined that $F^{\uparrow}_0 = F^{\uparrow}(\zeta, 0)$ and $F^{\uparrow}_i = F^{\uparrow}(\zeta, \tau_i)$. The solar zenith angle dependence enters through the functions $u_0(\zeta)$ and $a_2(\zeta)$. We note that formulas for the top and internal layers do not involve surface albedo, while those for the internal and bottom layers do not show any dependence on solar zenith angle. These findings justify the neglect of interlayer interaction in some optically thick ($\tau > 5$) layers inside the cloud.

Expressions for τ_i, the optical depths of layers between the measurement levels, follow from joint transformations of Eqs. (1) and (10) and have the forms:

$$3\,(1 - g)\,\tau_1 = \frac{1}{2s} \ln \left[\frac{(F(\tau_1) - 4\, F^{\downarrow}_1\, s\,(1 - 2s))\,(F(0)\, N + s\,(4u_0 + a_2 s))}{F(\tau_1) + 4\, F^{\uparrow}_1\, s\,(1 - 2s)\,(F(0) - s\,(4u_0 - a_2 s))} \right]; \quad (14)$$

for the top layer (0,1);

$$3\,(1 - g)\,(\tau_i - \tau_{i-1}) = \frac{1}{2s} \ln \left[\frac{(F(\tau_{i-1}) + 4\, F^{\uparrow}_{i-1}\, s\,(1 - 2s))\,(F(\tau_i) - 4\, F^{\downarrow}_i\, s\,(1 - 2s))}{F(\tau_{i-1}) - 4\, F^{\downarrow}_{i-1}\, s\,(1 - 2s)\,(F(\tau_i) + 4\, F^{\uparrow}_i\, s\,(1 - 2s))} \right]; \quad (15)$$

for the internal layer $(i - 1,\, i)$; and

$$3\,(1 - g)\,(\tau_n - \tau_{n-1}) = \frac{1}{2s} \ln \left[N\, \frac{(F(\tau_{n-1}) + 4\, F^{\uparrow}_{n-1}\, s\,(1 - 2s)) \left(F(\tau_0) - 4F^{\downarrow}_n\, s\,(1 + \frac{3}{2}\,\delta s)\right)}{(F(\tau_{n-1}) - 4\, F^{\downarrow}_{n-1}\, s\,(1 - 2s)) \left(F(\tau_0) + 4F^{\uparrow}_n\, s\,(1 - \frac{3}{2}\,\delta s)\right)} \right]. \quad (16)$$

for the bottom layer $(n - 1,\, n)$.

For the volume coefficients of true absorption and scattering, the relations $k_i = s_i^2 \tau'_i / z_i$ and $\sigma_i = \tau'_i / [3 z_i (1 - g)] - k$, where z_i is the geometrical thickness of the ith layer in kilometers. From the above formulas, by using flux values available from radiation measurements at different levels in the cloud, it is possible to determine the volume absorption coefficient throughout the cloud; and with the use of additional data on the scattering phase function, the optical depths between particular measurement levels and single scattering albedos and volume scattering coefficient throughout the cloud can be deduced.

ERRORS AND APPLICABILITY RANGE

Before using the above formulas, it is important to investigate four sources of error: (1) measurement error; (2) *a priori* setting of g; (3) use of asymptotic formulas beyond their applicability range; and (4) their application to real vertically inhomogeneous cloud layers.

Uncertainties and errors due to the use of formulas similar to formulas (6) to (9) outside their applicability range have been discussed elsewhere.[2,7] Let us consider formulas (11) to (16) in a more detail.

The formula's uncertainty due to measurement error is given as an error of indirect measurements, namely, for the function $y = f(x_1, x_2, ...)$ of measurements x_i we have

$$\Delta y \approx \left|\frac{\partial f}{\partial x_1}\right| \Delta x_1 + \left|\frac{\partial f}{\partial x_2}\right| \Delta x_2 + \dots ,$$

where Δx_i are the rms deviations of measurements. By evaluating corresponding derivatives and inserting known rms error $\Delta F^{\uparrow\downarrow} \sim 0.03$ of the flux measurements we obtain

$$\frac{\Delta s}{s} \le \frac{\Delta F}{1 - F^{\uparrow} - F^{\downarrow}} +$$

$$+ \frac{2\Delta F\ a_2(\zeta) + 16\ u_0(\zeta)\ \Delta u_0 + F(0)\ \Delta a_2}{16\ u_0(\zeta) - 2\ F(0)\ a_2(\zeta)} ,$$

$$\Delta s / s \le 0.06. \quad (17)$$

The relative error in determining $1 - \omega_0$ is calculated as

$$\frac{\Delta(1 - \omega_0)}{1 - \omega_0} = \frac{2\Delta s}{s} + \frac{\Delta g}{1 - g} . \quad (18)$$

The term $\Delta g / (1 - g)$ is the contribution from the second source of error which depends on the quality of *a priori* choice of the value of asymmetry parameter of the scattering phase function; $g = 0.85$ is frequently used to model stratiform clouds. Reference 8 presents spectral g values calculated for 8 cloud models. As model calculations show, g value in the visible vary by no more than 1%, so that $\Delta g / (1 - g)$ can be assumed to be ~ 0.02. This setting yields $\Delta(1-\omega_0)/(1-\omega_0) \le 0.08$.

The relative error of calculating optical depth from formulas (11)–(13) is given by

$$\frac{\Delta \tau}{\tau} \le \frac{2\Delta F}{\tau s\ (F^{\downarrow} - F^{\uparrow})\ (1 - g)} + \frac{\Delta s}{s} + \frac{\Delta g}{(1 - g)} . \quad (19)$$

The first and second terms, describing measurement error and error of s calculation, together contribute about 15% error which, added with 10% contribution from the third term, yields a maximum of 25% error of the optical depth retrieval.

Formulas for errors are valid within the applicability range of the asymptotic formulas. Errors in asymptotics are due to their use outside the applicability limits for the absorption and optical depth of the layers; their magnitudes depend on the values of layer absorption and optical depth and are determined by the formulas derived above to the model calculations of radiative characteristics for layers with preset optical properties. Radiative fluxes, calculated in Ref. 9 by the method of layer summation for the set $\omega_0 = 0.9999$, 0.999, 0.998, 0.995, $\tau_0 = 50$ and $\tau_i = 5$, 10, 15,...45, were introduced into the above formulas; the resulting ω_0 and τ values were then compared to the model values. The general conclusion is that reasonable retrievals of ω_0 to better than 5–10% accuracy are possible for weak absorption ($\omega_0 > 0.955$) and $\tau_i > 10$. The optical depth retrievals are more sensitive to the value of layer absorption, and are accurate to better than 10% only when $\omega_0 > 0.9995$; at the same time, the optical depth must be such that $\tau_0 - \tau_i > 5$ and $\tau_0 > 15$. We note, that this is just to check the robustness of the formulas, and not the input flux data which are considered exact.

Formulas for τ_i and s_i calculation were derived under the assumption of vertically homogeneous scattering layer, which is seldom in nature. To estimate the error due to the presence of vertical inhomogeneity, the model was applied to the vertically inhomogeneous layer simulated as a set of 5 homogeneous layers with different optical properties (Table I). Data on diffuse fluxes at the layer interfaces were taken from Ref. 9. Table I presents the initial values of the optical parameters (columns 2 through 5), calculated values obtained using the above method outlined (columns 6 and 7), and the errors of s and τ' retrievals (columns 8 and 9); also shown are the results for two cases of homogeneous layer whose optical depth is equal to the total thickness of inhomogeneous layer. For the homogeneous layer, we used the same single scattering albedo and asymmetry parameter as for the inhomogeneous layer, and considered one of its internal sublayers. As seen, the layer inhomogeneity adds little to the total error because its contribution is within the error due to the use of asymptotics outside their applicability range. The relative errors for inhomogeneous media are close in value to those for a homogeneous medium. These are physically clear results, for it is due to multiple scattering that photon "forgets" about distant points in the cloud, just bearing to the sensor only the information on the last collision. Thus diffuse flux measurements provide information on parts of the cloud on the order of photon mean free path apart from the measurement point, which for stratus cloud is about 20–50 m. Therefore, it is quite justifiable to study properties of 100 m thick cloud layer inside the cloud by means of the measurements described in the paper.

TABLE I. Influence of inhomogeneity on the accuracy of optical depth retrievals inside a scattering medium.

i	g	ω_0	τ'_{model}	s^2_{model}	τ'	s^2	δs^2,%	$\delta\tau'$,%
Inhomogeneous layer								
1	0.85	0.999	2.25	0.00222	2.29	0.00235	4.0	4.8
2	0.85	0.999	2.25	0.00222	2.27	0.02287	2.7	3.6
3	0.85	0.970	2.25	0.06667	2.17	0.07042	5.8	5.2
4	0.85	0.950	2.25	0.10870	2.54	0.11620	7.3	9.4
5	0.85	0.930	2.25	0.15556	2.95	0.15732	8.8	15
Homogeneous layer								
1	0.85	0.999	4.59	0.00222	4.72	0.00228	3.0	3.1
3	0.85	0.970	4.59	0.06667	4.80	0.06349	5.4	5.0

PURE SCATTERING CASE

This case is assumed to occur in certain cloud layers at some wavelengths by setting the single scattering albedo $\omega_0 = 1$. Formulas for the upward going and incident fluxes have simple forms, while the expression for the product $3(1 - g)\tau = \tau'$ is readily obtained by using appropriate formulas from Ref. 3.

Optical depth of the cloud top layer is expressed in terms of the radiative fluxes as

$$\tau' = \frac{4\,u_0(\zeta) - 2\,(F_1^{\downarrow} + F_1^{\uparrow})}{F(\tau_1)} - 1.5\,\delta. \tag{20}$$

Corresponding formula for the bottom layer is

$$\tau'_n - \tau'_{n-1} = \frac{2\,(F_{n-1}^{\downarrow} + F_{n-1}^{\uparrow})}{F(\tau_{n-1})} - 1.5\,\delta - \frac{4A}{1 - A}\,. \tag{21}$$

When this formula is used in the case of two levels i and $i - 1$, the optical depth between the levels is given as

$$\tau'_i - \tau'_{i-1} = 2\,\frac{(F_{i-1}^{\downarrow} - F_i^{\downarrow}) + (F_{i-1}^{\uparrow} - F_i^{\uparrow})}{F(\tau_i)}\,. \tag{22}$$

Validity tests showed that these formulas are applicable at the optical depth calculation in the case of quite small optical depth ($\tau > 3$) and, more important, weak absorption $\omega_0 > 0.998$ (with the accuracy of 5%). This finding is important and means that formulas (20)–(22) can be used to calculate optical depth in the case of optically thin media where formulas (14)–(16), which account for absorption in the layer, lead to large errors. As the absorption increases, formulas (20)–(22) rapidly become invalid. Expression similar to (22) was derived in Ref. 6 to calculate radiance from aircraft measurements inside the cloud layer.

CONCLUSION

The set of formulas is derived which express cloud optical parameters through experimentally measured solar radiative fluxes and make it possible to determine the vertical profiles of fundamental properties of stratus clouds from airborne radiation measurements in the cloudy atmosphere. Their validation will be performed in the second part of the paper. The method can be applied to cloud layers sufficiently extended in a horizontal plane (> 10 km), due to the use of aircraft platform. Note also that, in view of the monochromatic nature of the method, its results, especially for the optical depth, should change quite slowly with the wavelength; this represents an extra check on the method robustness and increases its accuracy.

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