PHASE DISLOCATIONS AND FOCAL SPOT

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Phase dislocations and the corresponding zeros of the wave function modulus are most likely due to the use of the complex wave model as Gabor's analytical signal. However, the azimuth oscillations observable around possible zero points[1] *and the concomitant energy transfer as well as the dichotomy of the interference fringes demonstrate the existence of new and more complicated properties of the wave process irrespective of the way its phase and envelope are determined. One more objective manifestation of that complication is the focal spots formed by a wave with dislocations passed through a Fourier lens which we study in this paper.*

The phase dislocations of light waves propagated through a randomly inhomogeneous medium have been studied in quasimonochromatic and parabolic approximations. The numerical model from Ref. 2 was used for solving the wave equation by the splitting method and FFT according to the Singleton algorithm. The Gaussian beam and its spatial-frequency spectrum were approximated by periodic functions and entered into a computer as two-dimensional matrices of their readouts. The order of these matrices was equal to 90 in order for the discrete representation be adequate to the continuous process.

Two phase-screens were used when modeling a randomly inhomogeneous medium with the spectral density of the refractive index corresponding to Kolmogorov turbulence. The law of energy conservation is held in the model with the computer accuracy. The spectral density F_s of the phase fluctuations of light and other model parameters are:

$$F_s(\varkappa) = 0.489\, r_0^{-5/3}\, (\varkappa^2 + \varkappa_0^2)^{-11/6},\ \varkappa_0 = 2\pi/L_0,$$

$$r_0 = \left\{0.423\, k^2 \int\limits_L C_n^2(l)\, \mathrm{d}l\right\}^{-3/5} = 0.05 \text{ m},$$

Here r_0 is the Fried's coherence radius; the outer scale of the turbulence, L_0, equals 1 m; the path length, L, equals 3 km; $\varkappa$ is the spatial frequency; $k = 2\pi/\lambda$ is the wave number; λ is the wavelength (0.6328 μm); C_n^2 is the refractive index structure constant.

The presence of dislocation was determined by calculating the phase gradient between adjacent points on a closed path drawn around each phase analyzed point. The phase dislocation was considered to be detected when the phase gradient was greater than or equal to 2π and less than or equal to -2π.

As seen from Fig. 1*a*, the phase dislocations appear at the points where the intensity reaches its minimum. These points correspond to zeros of the wave function. In the vicinity of these points the phase varies spirally. Along the whole length of the boundaries between white and black areas in Fig. 1*b* between two points of dislocation formation, the phase surface undergoes discontinuity of $\pm 2\pi$. Such a discontinuity cannot be removed with the use of translations of surface fragments.

These dislocations are of the first order. The focal spots from subaperture without dislocations, Fig. 1*d*, essentially differs from the spots formed by waves with dislocations, Figs. 1*e*–*h*. These spots have greater size and are doubled, and the line of minimums between two parts of the spot is directed along the tangent to the zero line of the wave imaginary part. Earlier, the double peak of the intensity function was recognized on the focal plane of Hartmann's wave-front sensor after the phase dislocation had appeared in the reference beam.[3]

If the wave function is raised to the second power, the order of the phase dislocations will also be raised. The wave function from Fig. 1 was squared and the results of the transformation are shown in Fig. 2. It was found that the second-order dislocations have the following properties:

– the focal spots from apertures with dislocations of the second order form the triplet, Figs. 2*e*–*h*;
– the zero lines of real and imaginary parts of the wave cross each other twice at the points of dislocations, Figs. 2*c* and 2*d*;
– the phase shift along the closed path around the dislocation point reaches $\pm 4\pi$, Fig. 2*b*.

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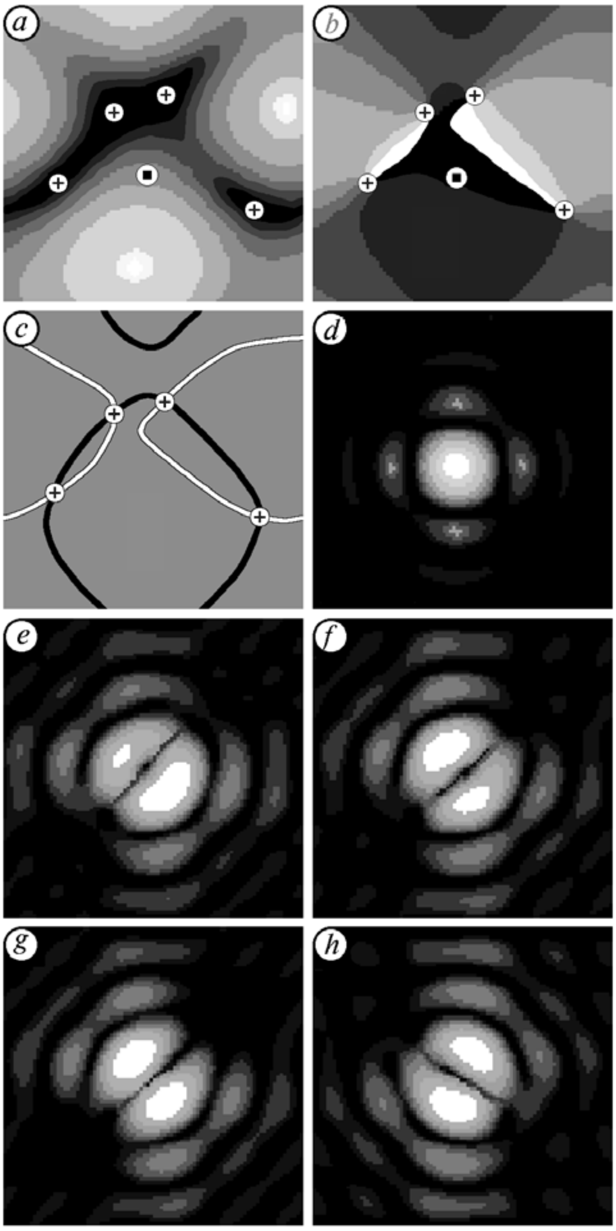

FIG. 1. First order phase dislocations and focal spots (Log scale) created by them. Crosses denote the intensity zeros and the points of phase dislocation appearance. Wave intensity (a), wave phase (b), zero-lines of the real (black) and imaginary (white) parts of the wave function (c); the focal spot for subaperture without dislocations (d); see square (■) in (a) and (b); focal spots when the dislocation points are at the following subapertures, see crosses (+): the upper left (e), the upper right (f), bottom left (g), bottom right (h).

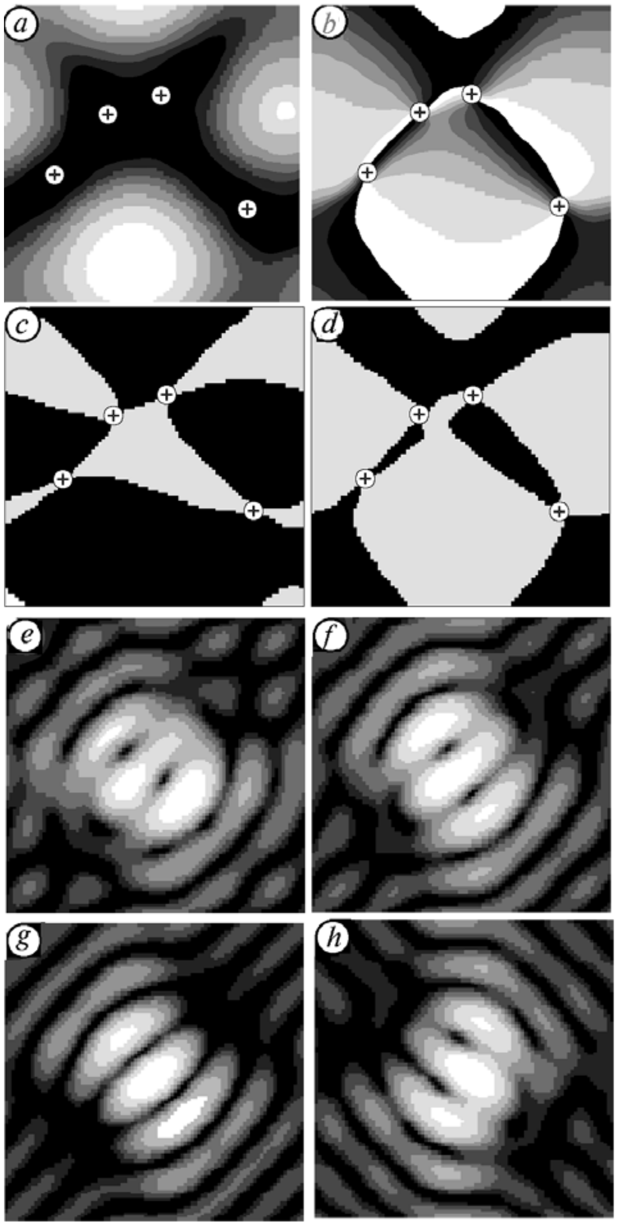

FIG. 2. Second-order phase dislocations and focal spots (Log scale) created by them. Crosses (+) denote the intensity zeros and the points of phase dislocation appearance. Wave intensity (a), wave phase (b), imaginary part of the wave function (c), real part of the wave function (d); focal spots when the dislocation points are at the following subapertures, see crosses (+): upper left (e), upper right (f), bottom left (g), bottom right (h).

The experiment has been conducted to confirm the existence of the second-order phase dislocations in a wave propagated through a linear randomly inhomogeneous medium. The results of this experiment are shown in Fig. 3. The conditions of propagation and the model of the medium were the same as in previous experiment. These dislocations were recognized using the above-mentioned features. The second-order phase dislocations appear when the spatial-frequency spectrum of the wave is sufficiently wide. In that case zero-lines are closer to each other.

Also, it is interesting that the dislocation of the second order formed without a co-dislocation of the same order but surrounded with dislocations of the first order, see Fig. 3. Therefore a second-order phase dislocation may be considered as a combined pair of the first-order dislocations.

The above wave functions with dislocations can be simulated using two or three quasiplane waves because these wave functions have two or three peaks in the spatial-frequency plane.

The focal spot width was estimated as a function of the turbulence strength for the cases when the phase dislocations were observed in the subaperture, when there were no dislocations, and for the alternating case. The estimations were calculated as average values. The number of experiments varied from 20 to 300 depending on the presence of the dislocation points within the subaperture during the numerical experiments.

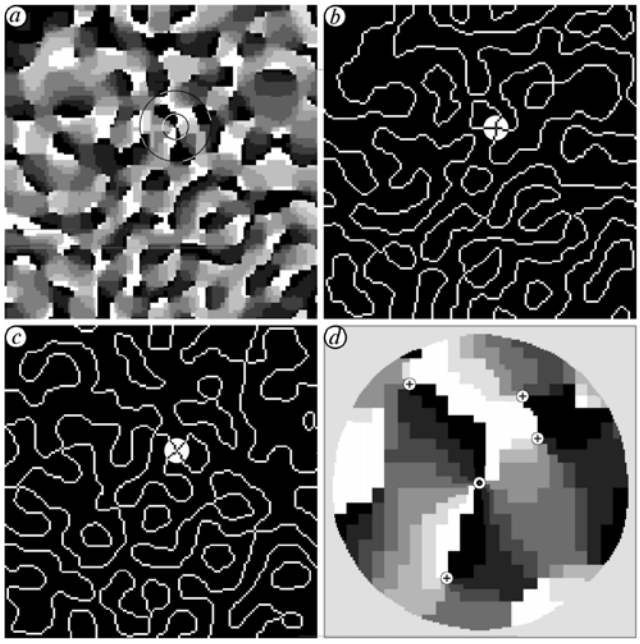

FIG. 3. Second-order phase dislocations in an inhomogeneous medium: random phase of the wave function (a), zero-lines of the real part (b) and imaginary part (c) of the wave function. A magnified image of the large circular area from Fig. 3a is shown in Fig. 3d. Point of the second-order phase dislocation is in the small white circles (b, c) and at the circle center (d). Crosses (+) denote the points of the first-order phase dislocations.

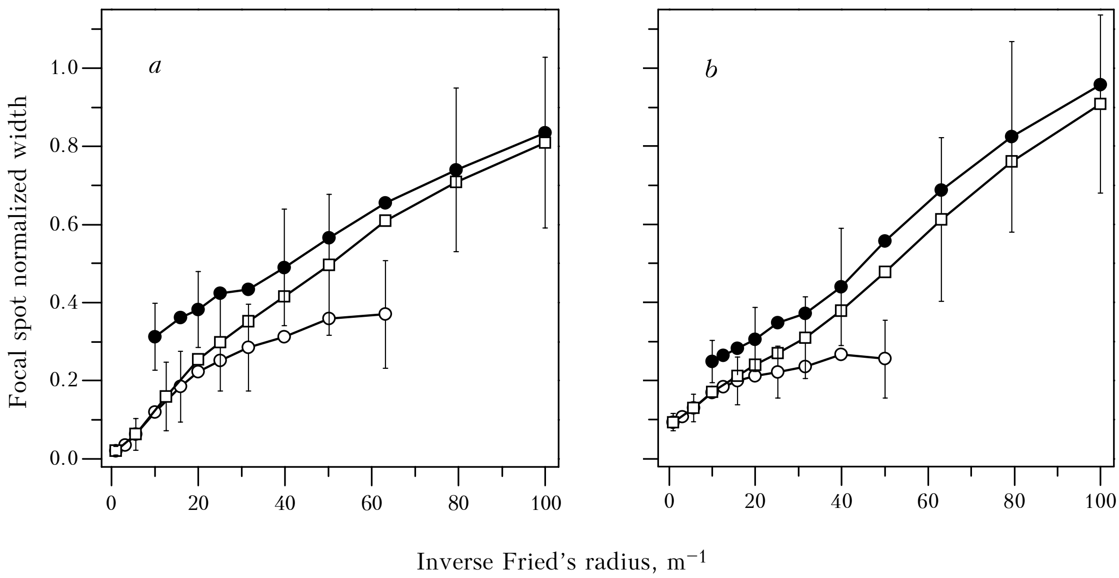

FIG. 4. Estimates of the normalized width of the focal spot versus turbulence intensity. Normalization is done by the subaperture diameter. The ratios of the Hartmann subapertures size to the matrix order are: 0.05 (*a*) *and* 0.1 (*b*). *Estimated are subapertures with dislocations (—●—), all subapertures (—□—), subapertures without dislocations (—○—). Bars in the figures show the standard deviations. The number of the experiments varied from* 20 *to* 300.

Results of these experiments are shown in Figs. 4*a* and *b*. The width of the focal spots or the width of spatial frequency spectrum of the wave with dislocations increases monotonically in the entire region of the variance of Fried's coherence radius (from weak to strong fluctuations). This fact demonstrates that the number of readouts used is quite sufficient for the wave representation.

The tendency to saturation of the width of the focal spots, created by subapertures without dislocations, demonstrates the fact that the probability of dislocation occurrence becomes high when the width of the spatial spectrum reaches a certain value.

REFERENCES

1. N.N. Mayer and V.A. Tartakovski, Atmos. Oceanic Opt. **8**, No. 3, 231–234 (1995).
2. P.A. Konyaev and V.P. Lukin, Izv. Vyssh. Uchebn. Zaved., Fizika, No. 2, 79–89 (1983).
3. V.P. Lukin and B.V. Fortes, Atmos. Oceanic Opt. **8**, No. 3, 223–230 (1995).