TURBULENCE MEASUREMENTS WITH A CW DOPPLER LIDAR IN THE ATMOSPHERIC BOUNDARY LAYER

V.A. Banakh, Ch. Werner, N.N. Kerkis, F. Köpp, and I.N. Smalikho

Institute of Atmospheric Optics,
Siberian Branch of the Russian Academy of Sciences, Tomsk
Institute of Optoelectronics, DLR, Oberpfaffenhofen, Germany
Received July 3, 1995

We present here some experimental results of the investigation into the potentialities of a cw Doppler lidar in measuring the turbulent energy dissipation rate. It is shown that at small size of the volume sounded the dissipation rate can be determined with a good accuracy from the lidar measurements of both the mean square of the Doppler spectrum width and the temporal structure function or the wind velocity range. It has been found that at large size of the volume sounded and small angles between the wind direction and the sounding beam axis the structure function is directly proportional to the time shift and the wind velocity range measured by the lidar is inversely proportional to the squared frequency.

Many papers by Russian and foreign scientists are concerned with the analysis of feasibilities of using Doppler lidars for measuring the parameters of atmospheric turbulence. In particular, in Refs. 1–4 the methods and the experimental data are presented for determining the dissipation rate of turbulent energy ε_T from the estimates of the mean square of the energy spectrum width of a received signal σ_S^2 [m^2/s^2] and the range of the radial wind velocity $S_D(\omega)$ [m^2/s] measured with a Doppler lidar. There are definite applicability limits for both methods. Thus, when estimating ε_T from the measurements of σ_S^2 with a cw Doppler lidar, the longitudinal size of a sounded volume is $\Delta z=(\lambda/2)R^2/a_0^2$, where λ is the wavelength; R is the focal length (an approximate distance from the lidar to the centre of the volume sounded); a_0 is the radius of a sounding beam in the plane of a transceiving telescope, it must be less than the outer scale of turbulence $L_V (\Delta z \ll L_V)$ (Refs. 5 and 6).

In Ref. 7 the method for determining the dissipation rate ε_T has been proposed based on the estimate of the temporal structural function $D(\tau)$ [m^2/s^2] of the radial wind velocity measured by a Doppler lidar, and the formulas have been derived that show that under certain conditions one can avoid limitations on the value Δz and extract the information about ε_T from $D(\tau)$ at arbitrary ratios of the effective longitudinal size of the sounded volume Δz and the outer scale of turbulence L_V. Reference 8 deals with a theoretical analysis of the feasibility for determining ε_T from estimates of $D(\tau)$ and $S_D(\omega)$. This paper describes some results of comparative experiments on measuring the dissipation rate by a Doppler lidar and an acoustic anemometer. The experimental data are compared with the calculations made on the basis of the results from Ref. 8.

BASIC RELATIONSHIPS

Under the condition $\Delta z \ll L_V$, the mean square of the Doppler range is described by the following expression[6,8]:

$$\sigma_S^2 = C\left(\frac{2}{\pi}\right)^{2/3}(\varepsilon_T\,\Delta z)^{2/3}, \tag{1}$$

where $C \approx 1.83$ is the Kolmogorov constant.[9] In Ref. 8 the expressions are given for $D(\tau)$ and $S_D(\omega)$ in the following form:

$$D(\tau) = C_1\,\varepsilon_T^{2/3}(\Delta z)^{2/3}\int\limits_0^{\pi} \mathrm{d}\varphi\left[1-\frac{8}{11}\sin^2(\varphi-\gamma)\right]\times$$
$$\times\left[\mathrm{Re}\left(|\sin(\varphi-\gamma)| + i\,\frac{\pi}{2}\sin\varphi\,|\langle\mathbf{V}\rangle\tau|/\Delta z\right)^{2/3} - |\sin(\varphi-\gamma)|^{2/3}\right]. \tag{2}$$

where $\mathbf{V} = \{V_z, V_x, V_y\}$ is the vector of wind velocity; V_z is the projection of the vector of wind velocity on the beam axis (radial wind velocity); $\langle\mathbf{V}\rangle = \{\langle V_z\rangle, \langle V_x\rangle, 0\}$; angular brackets denote the ensemble averaging; $\gamma = \arcsin(\langle V_x\rangle/|\mathbf{V}|)$ is the angle between the beam axis and the wind direction; $C_1 = (2/\pi)^{2/3}\,55\,\Gamma(1/3)C/[54\sqrt{\pi}\,\Gamma(11/6)] \approx 1.2\,C$, $\Gamma(x)$ is the gamma-function, and

$$S_D(\omega) = S_r(\omega)H(\omega), \tag{3}$$

where

$$S_r(\omega) = \frac{C}{3\Gamma(1/3)}\left(1+\frac{1}{3}\sin^2\gamma\right)\varepsilon_T^{2/3}|\langle\mathbf{V}\rangle|^{2/3}\,\omega^{-5/3} \tag{4}$$

0235-6880/95/12 955-05 $02.00

is the temporal spectrum of radial wind velocity measured at a fixed point;

$$H(\omega) = \frac{55}{27}\frac{1}{4\sqrt{\pi}}\frac{\Gamma(1/3)}{\Gamma(11/6)}\left(1 + \frac{1}{3}\sin^2\gamma\right)^{-1} \times$$

$$\times \int\limits_{-\infty}^{+\infty} d\xi(1 + \xi^2)^{-4/3}\left[1 - \frac{8}{11}\frac{(\cos\gamma + \xi\,\sin\gamma)^2}{1 + \xi^2}\right] \times$$

$$\times \exp\left\{-\frac{2}{\pi}\frac{\Delta z\omega}{|\langle \mathbf{V}\rangle|}\,|\cos\gamma + \xi\,\sin\gamma|\right\} \qquad (5)$$

is the optical transfer function of the time low-frequency filter. The formulas (2) and (3) are inapplicable if the following two conditions are met simultaneously:

$$\Delta z/|\langle \mathbf{V}\rangle| \gg \tau_\eta \qquad \text{and} \qquad |\langle \mathbf{V}_x\rangle| < \sigma_x^2, \qquad (6)$$

where $\tau_\eta = (\gamma_k/\varepsilon_T)^{1/2} \sim 0.1$ C is the characteristic time microscale of the Lagrangian wind velocity[9]; γ_k is the kinematic air viscosity; σ_x^2 is the variance of the transverse to the beam axis component of the wind velocity.

Under appropriate conditions the above Eqs. (1)–(3) allow the determination of the values of ε_T based on measured quantities $D(\tau)$, $S_D(\omega)$, and σ_S^2.

EXPERIMENT

The experiments have been carried out in Germany not far from Oberpfaffenhoben in falls of 1993 and 1994. In 1994 the measurements, with a cw ground-based Doppler CO_2 lidar of the Institute of Optoelectronics,[10] were accompanied by the measurements performed with an acoustic anemometer.[11] Figure 1 presents the scheme of the experiment. The measurements have been carried out with different size of the volume sounded, Δz, by varying the focal length of a laser beam R. In this case the angle of a laser beam inclination to a horizontal plane was selected so that the beam caustic was at the same height above the ground that the acoustic anemometer. The experimental conditions roughly corresponded to the neutral atmospheric stratification. When making approximate estimates of the longitudinal scale of the wind velocity correlation L_V (the outer scale of turbulence), this scale was 30 m. The mean wind velocity varied from 3 to 4 m/s.

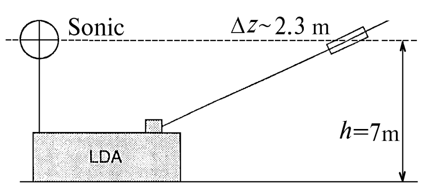

FIG. 1. Scheme of the experiment.

The measurements have been carried out with the following size of the volume sounded: $\Delta z = 2.3$ m $\ll L_V$, $\Delta z = 62$ m $\sim 2L_V$ and $\Delta z = 260$ m $\gg L_V$. Before each measurement the azimuth angle of the beam axis direction was selected and fixed in such a way that the beam axis coincided approximately with the wind direction (toward the air flow). The angle, in this case, did not exceed 10°–15°. For one day on November 5, 1993 and for two days of November 2, 1994 and November 9, 1994 the data were obtained of 22 lidar measurements at different values of Δz. One such measurement has been done during 2 minutes. Because the time for obtaining one Doppler spectrum (the integral time is t_0) was always 5 ms, the data bulk from one measurement contained 2400 Doppler spectra. Based on these spectra we have estimated the radial wind velocity $V_D(t_j)$ (the first moment of the Doppler velocity) and the square of the power spectrum width of a received signal $V_S^2(t_j)$ (the second central moment of the Doppler velocity) where $t_j = jt_0$, $j = \overline{1,2400}$. Then using all 2400 estimates of $V_D(t_j)$ and $V_S\,(t_j)$ we have calculated the mean value of the radial component of wind velocity V_z, the structural function $D(\tau)$, and the spectrum $S_D(\omega)$ from the data on $V_D(t_j)$, the mean value of the square of the power spectrum width σ_S^2 from the data on $V_S^2(t_j)$.

EXPERIMENTAL RESULTS

The dissipation rate of turbulent energy ε_T has been estimated from the lidar data obtained at $\Delta z = 2.3$ m based on the mean square of the Doppler spectrum width by Eq. (1). Based on the measurement data on the longitudinal component of wind velocity the structure function is calculated using the acoustic anemometer. The value of ε_T is estimated within the limits of the inertial interval (2/3 power range) based on the above structure function. The results of the experiment are given in Table I. The table shows a good agreement between the estimates of ε_T obtained from the lidar data and those obtained using the acoustic anemometer.

TABLE I.

ε_T, m²/s³	time, h: min		
	14:51	15:25	15:59
from LDA data	$6.4\cdot10^{-3}$	$7\cdot10^{-3}$	$8.6\cdot10^{-3}$
from sonic data	$4.7\cdot10^{-3}$	$5.8\cdot10^{-3}$	$7.4\cdot10^{-3}$

The velocity value, estimated from experimental data, can be represented as a sum of $V_D(jt_0)$ and $\Delta V_D(jt_0)$ where ΔV_D is the error, connected with noise, in estimating the value of V_D with the zero mean value $\langle\Delta V_D\rangle = 0$. It is evident that $\langle V_D \Delta V_D\rangle = 0$ and $\langle\Delta V_D(jt_0)\Delta V_D(it_0)\rangle = \sigma_N^2\delta_{ji}$, where $\sigma_{NN}^{22} = \langle(\Delta V_D)^2\rangle$ is the variance of the noise component of the velocity estimate, $\delta_{ji} = 1$, $\delta_{ji} \neq 0$. Therefore the structure function $D_m(\tau)$ measured in the experiment at

$\tau \geq t_0$ is the sum of the sought function $D(\tau)$ and the doubled variance of the noise:

$$D_m(\tau) = D(\tau) + 2\sigma_N^2, \quad (7)$$

where $\tau = jt_0$, $j = 1, 2, \ldots, l \ll 2400$. The value of ε_T can be more accurate estimated from the difference $D_m(\tau) - D_m(t_0)$ thus eliminating the error due to noise. For a comparison of the experimental results with the theory (Eqs. (2) and (3)) we need to have information about the dissipation rate ε_T and the mean wind $\langle \mathbf{V} \rangle$ ($\langle V_z, \gamma \rangle$). In the case of a small sized volume sounded ($\Delta z = 2.3$ m), the value ε_T is determined from the measurements of the mean square of the Doppler spectrum width (using Eq. (1)), and the mean value of the radial wind velocity $\langle V_z \rangle$ is determined by averaging the estimates of the Doppler velocity V_D. In this case the angle was approximately equal to 8°–10°.

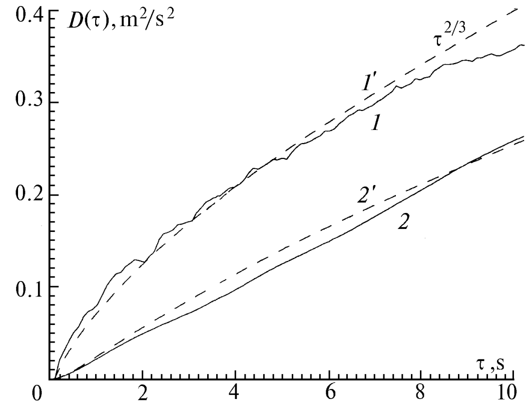

FIG. 2. Structure functions of the wind velocity measured with an acoustic anemometer (1, 1′) and a Doppler lidar (2, 2′); experiment (1, 2), theory (1′, 2′).

Figure 2 shows the structure functions ($D(\tau) = D(t_0)$) calculated from the data of simultaneous measurements of wind velocity with a Doppler lidar and an acoustic anemometer (solid curves). The values $\varepsilon_T = 3 \cdot 10^{-3}$ m^2/s^3, $\langle V_z \rangle = 3$ m/s, and $\gamma = 8°$ correspond to the above data. The dashed curve *1* shows the theoretical dependence of the structure function of wind velocity at a fixed point ($D(\tau) \sim \tau^{2/3}$). It is seen from Fig. 2 that the dependence measured with the acoustic anemometer (curve *1*) is in a good agreement with this curve. Curve *2′* is the result of calculations by formula (2) for the condition $\Delta z = 2.3$ m ($R = 50$ m) realized in the experiment. We can see from this figure that the theory (curve *2′*) and the experiment (curve *2*) are in a good agreement. Hence, the averaging of the lidar return fluctuations over space due to random scatter of velocities of the scattering particles in the lidar volume[12] results in a considerable difference in the behavior of structure functions of wind velocity, measured using the lidar (curve *2*), as compared with that measured at a point (curve *1*). In this case an approach, developed in Ref. 12 for describing the Doppler signal fluctuations in the turbulent atmosphere when $\Delta z \ll L_V$, provides quite an adequate description[8] of the time structure functions of wind velocity measured using a lidar.

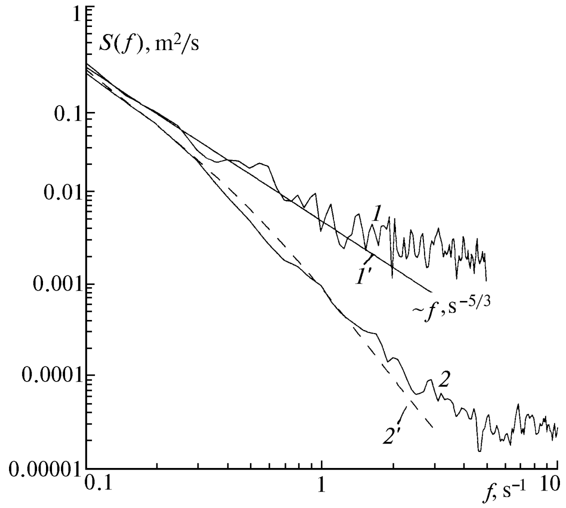

FIG. 3. Wind velocity spectra measured with an acoustic anemometer (1, 1′) and a Doppler lidar (2, 2′); experiment (1, 2); theory (1′, 2′).

The wind velocity spectra shown in Fig. 3 by solid curves were calculated based on the above experimental data. Curves *1* and *2* present the spectra calculated from the acoustic anemometer data and the lidar data, respectively. Straight line corresponds to the Kolmogorov-Obukhov spectrum, dashed line corresponds to the calculation of the $2\pi S_D(2\pi f)$ spectrum using Eq. (3). From Fig. 3 we notice that the wind velocity spectrum, calculated from the acoustic anemometer data, satisfies the Kolmogorov-Obukhov law, and the spectrum calculated from the lidar data deviates significantly from this law at high frequencies so that the lidar volume operates as a low-frequency filter. Here we can see is a good agreement between the theory and the experiment up to the highest frequencies where the noise effect becomes essential as in the case with the structure function.

The structure functions and spectra have been obtained based on the series of other two-minute lidar measurements at small sounded volumes $\Delta z = 2.3$ m, for which a comparison of the theory and the experiment shows good agreement.

Figure 4 illustrates this agreement for the averaged (over seven spectra) Doppler velocity spectrum, which is normalized by the variance

$$\sigma_D^2 = \int_{f_1}^{f_2} f \, df \, S_D(f) \quad (f_1 = 0.1 \text{ Hz}, f_2 = 3 \text{ Hz}).$$

This figure gives analogous characteristics for the case of point measurements (the Kolmogorov-Obukhov spectrum).

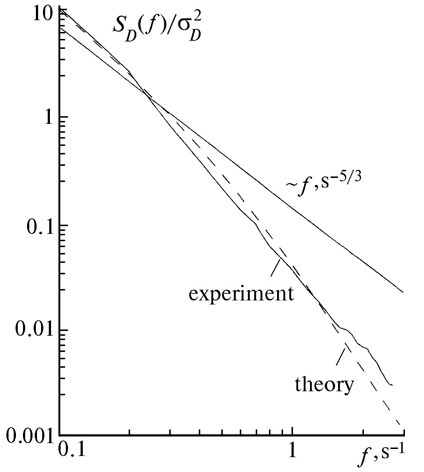

FIG. 4. The averaged normalized range of Doppler velocity.

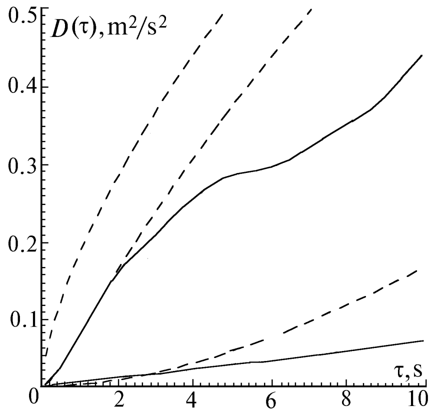

FIG. 5. Structure functions of the wind velocity measured with the Doppler lidar at Δz = 2.3 (1, 1′) and 62 m (2, 2′); experiment (1, 2); theory (1′, 2′); the structure function from point measurements of the wind velocity (1′).

Figure 5 shows the structure functions of wind velocity measured with a Doppler lidar at longitudinal dimensions of the sounded volume Δz = 2.3 m $\ll L_V$ (curve *1*) and Δz = 62 m ~ $2L_V$ (curve *2*). These two measurements have been conducted under similar turbulent conditions $\varepsilon_T = 7\cdot 10^{-3}$ m^2/s^3, $\langle V_z\rangle \approx 4$ m/s, $\gamma \approx 8°$ (Δz = 2.3 m) and 3° (Δz= 62 m). Curve *1* is the $D(\tau)$ dependence corresponding to the Kolmogorov-Obukhov "2/3" law, curves *1′* and *2′* present theoretical calculations for Δz = 2.3 m and Δz = 62 m, respectively. It is seen from this figure that while at small volume sounded there is a good agreement between the theory and experiment in the inertial interval $\tau < 3$, the theoretical and experimental curves for a large volume (Δz = 62 m) have quite different behavior in this interval. Thus, at Δz = 62 m according to the theory $D(\tau) \sim \tau^2$. At the same time, as follows from the experiment, the structure function grows linearly with increasing time shift τ. The observed discrepancy between the theory and the experiment is caused by the fact that in a given experiment both inequalities (6) are satisfied simultaneously. The above inequalities limit the application of Eqs. (2) and (3).

The data from other series of lidar measurements with a large volume sounded at the coincidence between the wind direction and the beam axis ($\gamma \ll 90°$) exhibit the analogous dependence of the structure function on τ:

$$D(\tau) = A\tau, \tag{8}$$

where A is the coefficient having the dimensionality of the dissipation rate of turbulent energy [m^2/s^3]. It should be noted that the known from the turbulence theory[9] time behavior of the structure function of the Lagrangian wind velocity $D_L(\tau)$ is also linear:

$$D_L(\tau) = C_0\, \varepsilon_T\, \tau, \tag{9}$$

where C_0 is the universal constant. At present the question on a relation of the coefficient A to the turbulence parameters and the sounded volume size is far from being settled.

The spectra calculated based on lidar data, measured for a large sounded volume Δz and small angles γ, are inversely proportional to the squared frequency, $S_D(f) \sim f^{-2}$. This result is illustrated in Fig. 6 where solid curve shows the result of the experiment, and dashed line presents the dependence $S_D(f) \sim f^{-2}$. In this case Δz = 260 m.

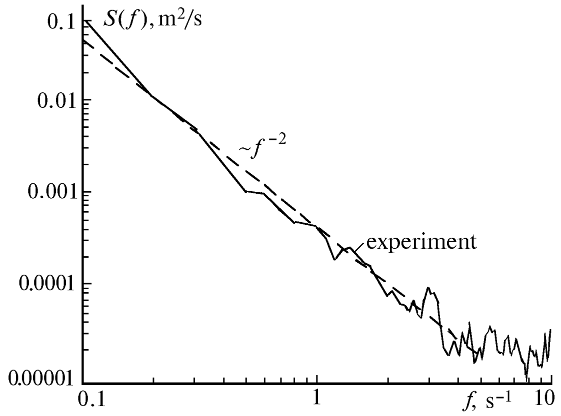

FIG. 6. The wind velocity spectrum measured using a Doppler lidar for a large sounding volume (Δz = 260 m) with the wind direction along the beam axis.

Thus, it is shown in this paper that at small size of the volume sounded by a lidar the information about the dissipation rate of turbulent energy can be obtained quite accurately both from the measurements of the mean square of the Doppler spectrum width and from the measurements of the structure function or the velocity spectrum. It has been found experimentally that for large volumes and for wind direction along the sounding beam axis the structure function of wind velocity linearly depends on the time shift, and the wind velocity spectrum is inversely proportional to the square frequency.

Further research in this field needs for additional theoretical and experimental studies of the characteristics considered for large size of a sounded volume, in particular, for the determination of the parameter A, appearing in the empirical formula (8), and the experimental test of the dependences $D(\tau) \sim \tau^{5/3}$ and $S_D(\omega) \sim \omega^{-8/3}$ resulting from Eqs. (2) and (3) for large volumes and strong side wind.

ACKNOWLEDGEMENT

The work has been performed under financial support from the International Scientific Foundation and the Russian Government (Grant No. J29100).

REFERENCES

1. T.Gal-Chen, Mei Xu, and W. Eberhard. J. Geophys. Research **97**, No. D17, 18409–18423 (1992).
2. V.M. Gordienko et al., Optical Engineering **33**, No. 10, 3206–3213 (1994).
3. G.M. Ancellet, R.I. Menzies, and W.B. Grant, J. Atmos. Oceanic Technol. **6**, No. 1, 50–58 (1989).
4. R.J. Keeler et al., J. Atmos. Oceanic Technol. **4**, 113–128 (1987).
5. R.J. Doviak and D. Zrnic, *Doppler Radar and Weather Observations* (Academic Press), 354 pp.
6. V.A. Banakh et al., Proc. SPIE **1968**, 483–493 (1993).
7. V.A. Banakh, N.N. Kerkis, I.N. Smalikho, F. Köpp, and Ch. Werner, in: *Topical Meeting* "*Coherent Laser Radar*" (1995).
8. I.N. Smalikho, Atmos. Oceanic Opt. **8**, No. 10, 788–793 (1995).
9. A.S. Monin and A.M. Yaglom, *Statistical Hydromechanics* (Nauka, Moscow, 1967), 720 pp.
10. F. Köpp, R.S. Schwiesow, and Ch. Werner, J. Climate Appl. Meteorol. **3**, No. 1, 148–151 (1984).
11. G.Ya. Patrushev, A.P. Rostov, and A.P. Ivanov, Atmos. Oceanic Opt. **7**, Nos. 11–12, 890–891 (1994).
12. V.A. Banakh et al., Appl. Optics **34**, No. 12, 2055–2067 (1995).

0235-6880/95/12 955-05 $02.00