METHOD FOR OBJECT MOVEMENT DETERMINATION BASED ON ANALYSIS OF OPTICAL FIELD WAVEFRONTS USING REFERENCE HOLOGRAMS

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A new method is proposed for determination of magnitude and direction of point object movement based on analysis of wavefront of light reflected from objects and passed through the turbulent atmosphere. Therewith low–frequency spatial components are separated and analyzed using Fourier holograms.

An increase in accuracy and resolution of optical radars may be achieved by the use of elements of adaptive optics. It is obvious that known methods of compensation for turbulence effects allow such a problem to be solved. But at present researchers consider alternative approaches[1] together with conventional ones to solve similar problems.

Capabilities of adaptive laser measurement systems are limited by inability of compensating for amplitude fluctuations in compensating for the turbulent distortions.[2,3]

This paper is devoted to the development of the interference–holographic method for obtaining information about coordinates of a point object to be detected by separation and analysis of the low–frequency spatial components of the phase front of an optical wave reflected from the object. In this case the information coded in the amplitude distributions and phase front is separated. This separation allows the errors connected with the amplitude distortions to be eliminated and the errors introduced by elements of optical channels to be compensated for. To solve these problems, a reference hologram recorded with the light flux of the known amplitude–phase distribution in its plane can be used as a phase-sensitive element.

Consider a process of measurement of the point object movements relative to an aperture of the receiving antenna. The reference hologram is used as an antenna. Let us derive basic mathematical relations connecting the parameters of the interferogram being formed by the low–frequency spatial components in the main image plane with the movement vector of an object to be optically detected. Figure 1 illustrates the movement process. A point source of optical radiation is considered as an object to be detected. As to the positions of the points P_1^* and P_2^*, it should be noted that in the image method employed the virtual source is introduced. Position of this source is determined by a reflector. For simplicity of understanding of the field formation in the hologram plane ($z = 0$) this source is moved to certain points P_1^* and P_2^* of the upper half-space. These points are connected with the coordinates of the points P_1 and P_2 and positions of the reflector and the hologram.

The hologram depicted in Fig. 1 is recorded with a light flux of known amplitude-phase distribution in its plane.

After a movement of a real source from the point P_1 to the point P_2, the adequate movement of the virtual point source being formed by the reflector occurs from the point P_1^* to the point P_2^*. There is the relation between the coordinates of the virtual and real point sources:

$$|R_1^*| = |R_1| = R_1; \qquad \alpha_1^* = \alpha_1 + 2\alpha, \tag{1}$$

where $\mathbf{R}_1^*$ and $\mathbf{R}_1$ are the vectors of positions of P_1^* and P_1 relative to the origin of coordinates.

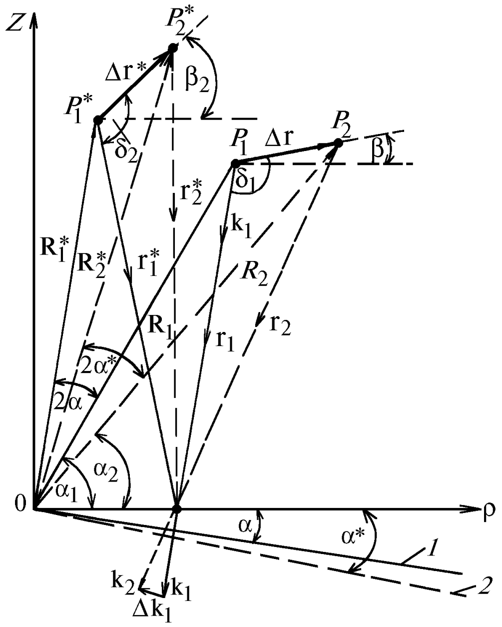

FIG. 1. Illustration of the movement of the point object to be detected: ρ is the plane of the reference hologram; 1 marks the reflector plane in recording; 2 marks the reflector plane during measurements; P_1 and P_2 are the point source coordinates before and after the movement; P_1^* and P_2^* are the virtual source coordinates before and after the movement; $\mathbf{R}_i$, $\mathbf{R}^*$, $\mathbf{r}_i^*$, $\Delta\mathbf{r}$, $\Delta\mathbf{r}^*$ are the vectors; α, α, α_1, α_2, β, β_2, δ_1, δ_2 are the angles characterizing the point object positions in space.*

The transmission coefficient T of the reference hologram used as a phase-sensitive element can be presented in the following form:

$$T = T_0 + T_1 \cos(\mathbf{k}_1 \mathbf{r}_1 - \mathbf{k}_1^* \mathbf{r}_1^*), \tag{2}$$

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where T_0 is the attenuation coefficient for the direct wave; T_1 is the attenuation coefficient for the diffracted wave; $\mathbf{k}_1$ and $\mathbf{k}_1^*$ are the wave vectors.

Movement of the luminous point P_1 is characterized by the displacement vector $\Delta\mathbf{r}$. Direction and modulus of this vector determine the spatial frequency ω of the interferogram being formed.

Field of the point source P_2 in the hologram plane can be presented in the form:

$$E_2 = E_{02}^{j\,\mathbf{k}_2\,\mathbf{r}_2} + E_{02d}\exp[\,j(\mathbf{k}_1\,\mathbf{r}_1 - \mathbf{k}_1^*\,\mathbf{r}_1^* + \mathbf{k}_2\,\mathbf{r}_2)] +$$

$$+ E_{02d}\exp[-j(\mathbf{k}_1\,\mathbf{r}_1 - \mathbf{k}_1^*\,\mathbf{r}_1^* - \mathbf{k}_2\,\mathbf{r}_2)]\,, \quad (3)$$

where E_{02} is the amplitude of the direct wave; E_{02d} is the amplitude of the diffracted wave. Field from the point source P_2^* in this plane can be written in the form:

$$E_2^* = E_{02}^*\exp(j\mathbf{k}_2^*\mathbf{r}_2^*) + E_{02d}^*\exp[j(\mathbf{k}_1\mathbf{r}_1 - \mathbf{k}_1^*\mathbf{r}_1^* + \mathbf{k}_2\,\mathbf{r}_2)] +$$

$$+ E_{02d}\exp[-j(\mathbf{k}_1\,\mathbf{r}_1 - \mathbf{k}_1^*\,\mathbf{r}_1^* - \mathbf{k}_2^*\,\mathbf{r}_2^*)]\,, \quad (4)$$

where E_{02}^* is the amplitude of the direct wave; E_{02d}^* is the amplitude of the diffracted wave.

The main image is described by the terms $E_{02}\exp[j(\mathbf{k}_2\,\mathbf{r}_2)]$ and $E_{02d}^*\exp[j(\mathbf{k}_1\,\mathbf{r}_1 - \mathbf{k}_1^*\,\mathbf{r}_1^* + \mathbf{k}_2^*\,\mathbf{r}_2^*)]$.

Intensity of the interference pattern in the main image plane can be presented as

$$I = I_0 + I_1\cos\varphi\,, \quad (5)$$

where I_0 is the term characterizing the constant light background, I_1 is the maximum amplitude of the interference fringe intensity, the phase distribution of light flux in the hologram plane is written in the form :

$$\varphi = \mathbf{k}_1\,\mathbf{r}_1 - \mathbf{k}_1^*\,\mathbf{r}_1^* - \mathbf{k}_2\,\mathbf{r}_2 + \mathbf{k}_2^*\,\mathbf{r}_2^*\,, \quad (6)$$

where

$$\mathbf{k}_2 = \mathbf{k}_1 + \Delta\mathbf{k}_1\,;\ \mathbf{k}_2^* = \mathbf{k}_1^* + \Delta\mathbf{k}_1^*\,;\ \mathbf{r}_2 = \mathbf{r}_1 - \Delta\mathbf{r}\,;\ \mathbf{r}_2^* = \mathbf{r}_1^* - \Delta\mathbf{r}^*. \quad (7)$$

In Eq. (6) the phase components caused by the atmospheric turbulence are omitted for the following reasons. The first two terms in Eq. (6) characterize the amplitude-phase distribution in the hologram plane when exposed. These terms present the phase difference of light fields from real and virtual sources and have components conditioned by the phase atmospheric distortions. Therefore, these components can compensate each other except for a controlled phase disturbance caused by an angle and distance between the hologram and the reflector. The same is valid relative to the third and fourth terms in Eq. (6). These terms characterize the phase difference of light fields from real and virtual images of the point object during the measurements.

Substituting Eq. (7) into Eq. (6) we obtain

$$\varphi = (\mathbf{k}_1 + \Delta\mathbf{k}_1)\,\Delta\mathbf{r} - (\mathbf{k}_1^* + \Delta\mathbf{k}_1^*)\Delta\mathbf{r}^* - \Delta\mathbf{k}_1\,\mathbf{r}_1 + \Delta\mathbf{k}_1^*\,\mathbf{r}_1^*\,. \quad (8)$$

Assuming that $\Delta \mathrm{r} < \mathrm{r}_1$, the vectors $\Delta\mathbf{k}_1$ and $\mathbf{r}_1$ may be considered to be orthogonal. The same assumption is valid for the vectors $\Delta\mathbf{k}_1^*$ and $\mathbf{r}_1^*$. Then

$$\varphi = \mathbf{k}_1\,\Delta\mathbf{r} - \mathbf{k}_1^*\,\Delta\mathbf{r}^* + \Delta\mathbf{k}_1\,\Delta\mathbf{r} - \Delta\mathbf{k}_1^*\,\Delta\mathbf{r}^*\,. \quad (9)$$

The expression

$$(\mathbf{k}_1 + \Delta\mathbf{k}_1)\,\Delta\mathbf{r} = \frac{2\pi}{\lambda}\,\Delta\mathbf{r}\left(\cos\delta_1 - \frac{\Delta r\sin^2\delta_1}{r_1 - \Delta r\cos\delta_1}\right)$$

can be derived from the triangles formed by the vectors $\mathbf{r}_1$, $\Delta\mathbf{r}$, $\mathbf{r}_2$, and $\mathbf{k}_1$, $\Delta\mathbf{k}_1$, $\mathbf{k}_2$.

Similar expression results from consideration of the virtual sources. As a result, the expression for φ takes the form:

$$\varphi = \frac{2\pi}{\lambda}\,\Delta\mathbf{r}\left(\cos\delta_1 - \cos\delta_2 - \frac{\Delta r\sin^2\delta_1}{r_1 - \Delta r\cos\delta_1} + \right.$$

$$\left. + \frac{\Delta r^*\sin^2\delta_2}{r_1^* - \Delta r^*\cos\delta_2}\right), \quad (11)$$

where

$$\cos\delta_1 = \frac{\rho\cos\beta - R_1\cos(\beta - \alpha_1)}{R_1 - \rho\cos\alpha_1}\,; \quad (12)$$

$$\cos\delta_2 = \frac{\rho\cos\beta_2 - R_1\cos(\beta_2 - \alpha_1 - 2\alpha)}{R_1 - \rho\cos(\alpha_1 + 2\alpha)}\,; \quad (13)$$

$$\tan\beta_2 = \frac{\sin(\alpha_1 - \beta)\sin(2\alpha^* + \alpha_2) - \sin(\alpha_1 + 2\alpha)\sin(\alpha_2 - \beta)}{\sin(\alpha_1 - \beta)\cos(2\alpha^* + \alpha_2) - \cos(\alpha_1 + 2\alpha)\sin(\alpha_2 - \beta)}; \quad (14)$$

$$\tan\alpha_2 = \frac{\Delta r\sin\beta + R_1\sin\alpha_1}{\Delta r\cos\beta + R_1\cos\alpha_1}\,; \quad (15)$$

$$\Delta r^* = R_1\,\frac{\sin[(\alpha_1 + 2\alpha) - (\alpha_2 + 2\alpha^*)]}{\sin(\alpha_2 + 2\alpha^* - \beta_2)}\,. \quad (16)$$

Equations (11)–(16) allow the intensity distribution of light along the axis ρ to be determined. This distribution is a function of R_1 and α_1, i.e. known coordinates of the point P_1, angles α^*, and of the movement vector $\Delta\mathbf{r}$ sought, which is characterized by the modulus Δr and the angle β.

From the measurements of the intensity minima the system of the transcendental equations can be obtained for determination of the parameters Δr and β, which can be solved by a numerical technique. To eliminate the ambiguities, the results of measurements at various α^* are convinient.

The case when the third and fourth terms in Eq. (11) can be ignored due to the smallness of the ratio $\Delta r/R_1$ should be explained in detail. To do that, we expand Eqs. (12) and (13) into the power series[4] over the parameter ρ/R_1 retaining the squared terms. Then we calculate the derivative $d\varphi/d\rho$ and obtain the relation for the intensity spatial frequency along the axis ρ in the form:

$$\omega = \frac{d\varphi}{d\rho} = \frac{2\pi}{\lambda}\frac{\Delta r}{R_1}[\cos\beta - \cos\alpha_1 - \cos^2\alpha_1\cos(\beta - \alpha_1) - \cos\beta_2 +$$

$$+ \cos(\alpha_1 + 2\alpha)\cos(\beta_2 - \alpha_1 - 2\alpha)\,] + \frac{4\pi}{\lambda}\frac{\Delta r\,\rho}{R_1^2}[\,\cos\beta\cos\alpha_1 -$$

$$- \cos^2\alpha_1\cos(\beta - \alpha_1) - \cos\beta_2\cos(\alpha_1 + 2\alpha) +$$

$$+ \cos^2(\alpha_1 + 2\alpha)\cos(\beta_2 - \alpha_1 - 2\alpha)\,]\,. \quad (17)$$

Near the coordinate origin, i.e. for small ρ/R_1, the frequency is determined by the first term in Eq. (17) and is a complicated function of the parameters Δr and β to be determined.

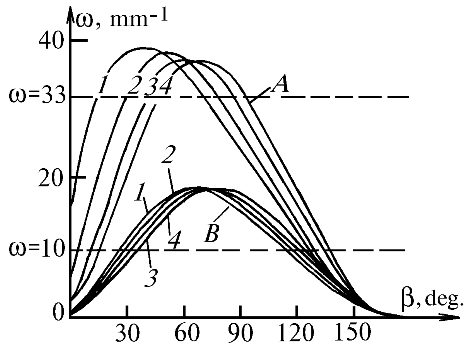

FIG. 2. Spatial frequency of the interferogram as a function of direction and magnitude of source movement: Δr = 4 (*1*), 5 (*2*), 7 (*3*), *and* 10 m (*4*).

To study the capabilities of the method developed, the mathematical simulation of the measurement process has been carried out on a computer. The reference hologram was obtained for R_1 = 1000 m, α_1 = 83°, and α = 3°. Simulation results are presented in Fig. 2. The family of curves *A* corresponds to α^* = 3.1°, and the family of curves *B* is obtained when the measurement are carried out at α^* = 5.05°. Let us assume that the measurement at α^* value gave the result ω_1 = 33°. Such a frequency corresponds to several movement vectors (for example, Δr = 4 m, β = 15°; Δr = 5 m, β = 30°; and others) as it is clear from Fig. 2. To eliminate the obtained ambiguity, we use the measurement for α^* at which we obtain, for example, the value ω_2 = 10. It is clear from the family of curves *2* that in this case the movement can be different too (Δr = 4 m, β = 26.4°; Δr = 5 m, β = 30°, and others). But from the whole set of possible values of Δr and β the values Δr = 5m and β = 30° only are obtained in the first and second cases at the same time, therefore, these values characterize the real movement of the point object.

Thus, the algorithm of determination of the point source movement vector reduces to the measurement of spatial frequencies near the origin of coordinates ($\rho \approx 0$) at different α^* and equating of the first term in Eq. (17) to these values, finding the roots of the obtained equations, and their comparison.

It should be noted that the measurements at various α^* should be carried out simultaneously, what is technically easy, by the way. Moreover, the use of measurement is possible for a greater number of α^* values that can increase a reliability of the results and reduce the time of the movement determination.

Based on the method developed an optical arrangement of a system for point object movement measurements has been proposed.[5]

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