FORMATION OF SHEAR INTERFEROGRAMS BY DIFFUSELY SCATTERED LIGHT FIELDS AT A DOUBLE–EXPOSURE RECORDING OF THE FOURIER HOLOGRAM WITH A MICROSCOPE

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Received March 21, 1994

A lateral shear interferometer has been analyzed based on a double–exposure recording of the Fourier hologram with the use of mat–screen microscope. It is shown both theoretically and experimentally, spatial filtering, being performed in the hologram plane, allows one to separate out wave aberrations introduced by the microscope over its field of view.

As was shown in Ref. 1, a double–exposure recording of a mat screen image using collimating Kepler optical system results in the formation of lateral shear interferograms in the far diffraction zone, in the bands of infinite width, which characterize wave aberrations over field of a two–component optical system. As this takes place, prior to the second exposure, the compensation for phase shift in light waves was performed by tilting waves, used for illumination of the mat screen, and reference wave front. Similar results can be obtained by matching the subjective speckle fields of two exposures in the mat screen image plane.[2]

For a two–component optical system like a Galilean telescope, a double–exposure recording of a hologram of a virtual image of a mat screen results in the formation of a lateral shear interferograms in the near diffraction zone. These interferograms characterize wave aberrations over the optical system field and phase distortions of a quasiplanar wave front of radiation used for illumination of the mat screen.[3] Superposition of subject speckle fields of the two exposures at displacement of the telescope and a photographic plate prior to its reexposure allows the recording of interference patterns characterizing only the telescope wave aberrations over its field to be done.[4]

In the present paper the conditions of lateral shear interferogram formation in the bands of infinite width are analyzed for the case of double–exposure recording of the Fourier hologram of a mat screen using a microscope with spatial filtration of the diffraction field at the stage of reconstruction.

As shown in Fig. 1, a mat screen *1* placed in (x_1, y_1) plane is illuminated with an aberrationless diverging spherical wave with the curvature radius R, which is formed with lens L_0 and point aperture in an opaque screen p_0 located in its focus. Its image is constructed in the front focal plane of lens L_2 (microscope ocular) with the use of lens L_1 (microscope objective). The recording of mat screen Fourier hologram is performed on a photographic plate *3* placed in (x_4, y_4) plane during the first exposure using off–axis quasiplanar reference wave *2*. Prior to recording the second exposure, the mat screen is shifted in its plane, for example, along the x axis at a distance a, and the reference wave front angle in (x, z) plane is changed from θ_1 to θ_2.

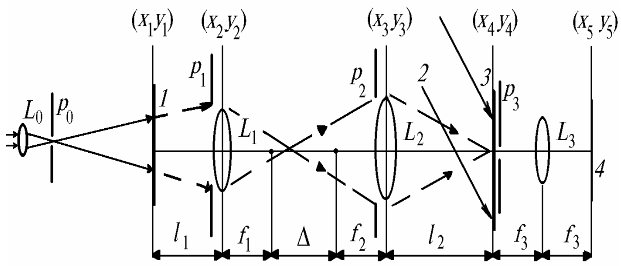


FIG 1. The optical scheme used for recording and reconstruction of double – exposure Fourier hologram: mat screen (1), reference beam (2), photographic plate – hologram (3), interferogram plane (4), lenses L_0, L_1, L_2, L_3, filtering diaphragms p_0 and p_3, and aperture diaphragms p_1 and p_2.

Thus recorded double–exposure hologram is reconstructed using initial reference wave, and spatial filtering of the diffraction field in the hologram plane using an opaque screen p_3 with a hole makes it possible the lateral–shear interferogram to be recorded in Fourier plane *4* in bands of infinite width. The interferogram characterizes the microscope wave aberrations.

In the Fresnel approximation (constant amplitude and phase factors are omitted) the complex amplitude of the object field, corresponding to the first exposure, in (x_4, y_4) plane of a photographic plate can be written in the form:

$$u_1(x_4, y_4) \sim \int\int\int_{-\infty}^{\infty}\int\int\int t(x_1, y_1) \exp\left[\frac{i\,k}{2\,R}(x_1^2+y_1^2)\right] \times$$

$$\times \exp\left\{\frac{i\,k}{2\,l_1}\left[(x_1-x_2)^2+(y_1-y_2)^2\right]\right\} p_1(x_2, y_2) \times$$

$$\times \exp\left\{-i\left[\frac{i\,k}{2\,f_1}(x_2^2+y_2^2) - \varphi_1(x_2, y_2)\right]\right\} \times$$

$$\times \exp\left\{\frac{i\,k}{2\,(f_1+f_2+\mathrm{D})}\left[(x_2-x_3)^2+(y_2-y_3)^2\right]\right\} p_2(x_3, y_3) \times$$

$$\times \exp\left\{-i\left[\frac{i\,k}{2\,f_2}(x_3^2+y_3^2) - \varphi_2(x_3, y_3)\right]\right\} \times$$

$$\times \exp\left\{\frac{i\,k}{2l_2}\left[(x_3-x_4)^2+(y_3-y_4)^2\right]\right\} \mathrm{d}x_1 \mathrm{d}y_1\, \mathrm{d}x_2\, \mathrm{d}y_2\, \mathrm{d}x_3\, \mathrm{d}y_3\,, \quad (1)$$

where k is the wave number, $t(x_1, y_1)$ is the complex amplitude of a mat screen transparency, being the random function of coordinates, $p_1(x_2, y_2)$ is the generalized function of the pupil of lens L_1 (Ref. 5) with the focal length f_1, which allows for its axis wave aberrations, $p_2(x_3, y_3)\exp i\,\varphi(x_3, y_3)$ is the generalized function of the pupil of lens L_2 with the focal length f_2, Δ is the optical length of a microscope viewing hood, l_1 is the distance from the principal plane (x_2, y_2) of L_1 lens to the mat screen, l_2 is the distance from the principal plane (x_3, y_3) of L_2 lens to the photographic plate.

If the condition $1/R + 1/l_1 - N/l_1^2 = 0$, where $1/N = 1/l_1 - 1/f_1 + 1/(f_1 + f_2 + \Delta) - M/(f_1 + f_2 + \Delta)^2$ and $1/M = 1/(f_1 + f_2 + \Delta) - 1/f_2 + 1/l_2$, holds, the Eq. (1) can be reduced to the form

$$u(x_4, y_4) \sim \exp\left[\frac{i\,k}{2\,l_2}\,(x_4^2+y_4^2)\right]\left\{\exp\left[-\frac{i\,k\,M}{2\,l_2^2}\,\left(x_4^2+y_4^2\right)\right]\times\right.$$
$$\times\left\{\exp\left[\frac{-ikNM^2(x_4^2+y_4^2)}{2(f_1+f_2+\Delta)^2l_2^2}\right]F\left[\frac{kNMx_4}{(f_1+f_2+\Delta)l_1l_2},\frac{kNMy_4}{(f_1+f_2+\Delta)l_1l_2}\right]\otimes\right.$$
$$\left.\left.\otimes\,P_1(x_4, y_4)\right\}\otimes P_2(x_4, y_4)\right\}, \qquad (2)$$

where $\otimes$ denotes the operation of convolution;

$$F\left[\frac{k\,N\,M\,x_4}{(f_1 + f_2 + \Delta)\,l_1\,l_2},\frac{k\,N\,M\,y_4}{(f_1 + f_2 + \Delta)\,l_1\,l_2}\right] =$$
$$= \int\!\!\int_{-\infty}^{\infty} t(x_1, y_1)\exp\left[-\frac{ikNM}{(f_1+f_2+\Delta)\,l_1\,l_2}\left(x_1x_4+ y_1\,y_4\right)\right]\mathrm{d}\,x_1\,\mathrm{d}\,y_1\,,$$

$$P_1(x_4, y_4) = \int\!\!\int_{-\infty}^{\infty} p_1(x_2, y_2)\,\exp\,i\,\varphi_1(x_2, y_2)\times$$
$$\times\exp\left[-\frac{i\,k\,M}{(f_1 + f_2 + \Delta)l_2}\left(x_2\,x_4+ y_2\,y_4\right)\right]\mathrm{d}\,x_2\,\mathrm{d}\,y_2\,,$$

$$P_2(x_4, y_4) = \int\!\!\int_{-\infty}^{\infty} p_2(x_3, y_3)\,\exp\,i\,\varphi_2(x_3, y_3)\times$$
$$\times\exp\left[-\frac{i\,k}{l_2}\left(x_3\,x_4+ y_3\,y_4\right)\right]\mathrm{d}\,x_3\,\mathrm{d}\,y_3$$

are the Fourier transforms of corresponding functions.

Since the width of the function $P_1(x_4, y_4)$ is of the order of $\lambda(f_1 + f_2 + \Delta)l_2/(Md_1)$ (see Ref. 6), where λ is the wavelength of coherent radiation, used for hologram recording and reconstruction, d_1 is the diameter of pupil of lens L_1,we assume that within the domain of this function the phase change of a spherical wave with the curvature radius $(f_1 + f_2 + \Delta)^2l_2^2/NM^2$ is below π. Then, taking into account that the condition $1/l_1 + 1(f_1 + \Delta) = 1/f_1$ holds for a microscope, for the photographic plate area with the diameter $D_1 \le d_1f_1/(f_1 + \Delta)$ the factor $\exp\left[\frac{i\,k\,N\,M^2}{2(f_1 + f_2 + \Delta)^2}\,\frac{x_4^2 + y_4^2}{l_2^2}\right]$, characterizing the spherical wave phase distribution, can be factored outside the integral of convolution with the function $P_1(x_4, y_4)$ in Eq. (2). Thus we obtain

$$u_1(x_4, y_4) \sim \exp\left[\frac{i\,k}{2\,l_2}\,(x_4^2+y_4^2)\right]\left\{\exp\left[-\frac{i\,k}{2\,l_2}\left(x_4^2+y_4^2\right)\right]\times\right.$$
$$\left.\times\left\{F^{-1}\left[\frac{k\,x_4}{f},\frac{k\,y_4}{f}\right]\otimes P_1^{-1}(x_4, y_4)\right\}\otimes P_2(x_4, y_4)\right\}, \qquad (3)$$

where

$$F^{-1}\left[\frac{k\,x_4}{f},\frac{k\,y_4}{f}\right] = \int\!\!\int_{-\infty}^{\infty} t(x_1, y_1)\exp\left[i\,k(x_1x_4+y_1y_4)/f\right]\mathrm{d}x_1\mathrm{d}\,y_1$$

is the inverse Fourier transform of the mat screen transmission function, $f = f_1f_2/\Delta$ is the microscope focal length,

$$P_1^{-1}(x_4, y_4) = \int\!\!\int_{-\infty}^{\infty} p_1(x_2, y_2)\,\exp\,i\,\varphi_1(x_2, y_2)\times$$
$$\times\exp\left[\,i\,k\left(x_2\,x_4+ y_2\,y_4\right)/f\left(1 + \frac{f_1}{R} + \frac{f_2^2}{R\,\Delta}\right)\right]\mathrm{d}\,x_2\,\mathrm{d}\,y_2;$$

is the inverse Fourier transform of the generalized function of the objective pupil.

Since the width of function $P_2(x_4, y_4)$ is of the order of $\lambda l_2/d_2$, where d_2 is the diameter of L_2 lens pupil, we suppose that within its domain the phase of spherical wave with the curvature radius l_2 varies within π. Then for the photographic plate area with the diameter $D_2 \le d_2$ we remove the factor $\exp[-ik(x_4^2 + y_4^2)/2l_2]$ from the integral of convolution with the function $P_2(x_4, y_4)$ in Eq.(3) and thus we have

$$u_1(x_4, y_4) \sim F^{-1}\left[\frac{k\,x_4}{f},\frac{k\,y_4}{f}\right]\otimes P_1^{-1}(x_4, y_4)\otimes P_2(x_4, y_4). \qquad (4)$$

As follows from Eq. (4), if $d_2 \ge d_1f_1/(f_1 + \Delta)$ then within the photographic plate area with the diameter D_1 the Fourier transform of input function can be convoluted with the pulse response of microscope objective and ocular, i.e., each point of inverse Fourier transform of the mat screen in (x_4, y_4) plane is broadened to the size of a subjective speckle determined by the width of function $P_1^{-1}(x_4, y_4)$. In this case, as in the case of a single–component optical system being used for formation of the mat screen Fourier transform,[7] the Fourier transform is being scaled according to the focal length with the scale being independent of the wave front curvature radius of radiation used for the mat screen illumination. The position, $l_2 = f_2(1 + f_2/\Delta + ff_1/R\Delta)$, of (x_4, y_4) plane of transform only depends on the curvature radius, and, consequently, the width of optical system pulse response depends on it too.

Distribution of the object field complex amplitude, corresponding to second exposure, over the photographic plate takes the form

$$u_2(x_4, y_4) \sim \exp(-i\,k\,a\,x_4/f)\,F^{-1}\left[\frac{k\,x_4}{f},\frac{k\,y_4}{f}\right]\otimes P_1^{-1}(x_4, y_4)\otimes$$
$$\otimes\,P_2(x_4, y_4). \qquad (5)$$

according to a known property of the Fourier transform.

In the approximation used the complex amplitudes of reference wave in the photographic plate can be presented in the form

$$u_{01}(x_4, y_4) \sim \exp i\left[\, k\, x_4 \sin\theta_1 + \varphi_3(x_4, y_4)\right];$$

$$u_{02}(x_4, y_4) \sim \exp i\left[\, k\, x_4 \sin\theta_2 + \varphi_3(x_4 + b, y_4)\right],$$

where $\varphi_3(x_4, y_4)$ is the deterministic function characterizing the phase distortions of the reference wave due to the wave aberrations of optical system forming it, b is the shift caused by the change of tilt angle of the reference wave front before the reexposure of the photographic plate.

Let us assume further that amplitude transmission of a hologram depends linearly on the intensity and that the hologram is illuminated by a copy of the reference wave corresponding, for example, to that used for first exposure. Then in the hologram plane the distribution of field of diffraction in the minus first order takes the form

$$u(x_4, y_4) \sim F^{-1}\left[\frac{k\, x_4}{f}, \frac{k\, y_4}{f}\right] \otimes P_1^{-1}(x_4, y_4) \otimes P_2(x_4, y_4) +$$

$$+ \exp i\left[\, k\, x_4 \sin\theta_1 - k\, x_4 \sin\theta_2 + \varphi_3(x_4, y_4) - \varphi_3(x_4 + b, y_4)\right] \times$$

$$\times \left\{ \exp(-ikax_4/f)\, F^{-1}\left[\frac{kx_4}{f}, \frac{ky_4}{f}\right] \otimes P_1^{-1}(x_4, y_4) \otimes P_2(x_4, y_4)\right\}. \quad (6)$$

If the condition $\sin\theta_1 - \sin\theta_2 - a/f = 0$ holds, the expression (6) reduces to the form:

$$u(x_4, y_4) \sim F^{-1}\left[\frac{k\, x_4}{f}, \frac{k\, y_4}{f}\right] \otimes P_1^{-1}(x_4, y_4) \otimes P_2(x_4, y_4) +$$

$$+ \exp i\left[\varphi_3(x_4, y_4) - \varphi_3(x_4 + b, y_4)\right] \left\{ F^{-1}\left[\frac{k\, x_4}{f}, \frac{k\, y_4}{f}\right] \otimes\right.$$

$$\left. \otimes \exp(ikax_4/f)\, P_1^{-1}(x_4, y_4) \otimes \exp(ikax_4/f)\, P_2(x_4, y_4)\right\}. \quad (7)$$

As follows from Eq.(7), the subjective speckle fields of both exposures coincide in the hologram plane at a relative tilt angle between them being $\alpha = a/f$, and information about the phase distortions introduced by the microscope objective and ocular is within an individual speckle. Consequently, the interference pattern caused by reference wave aberrations is in the hologram plane as has been shown in Ref.7. If the opaque screen p_3 (Fig. 1) with a hole at its center is installed in the hologram plane and the condition $\varphi_3(x_4, y_4) - \varphi_3(x_4 + b, y_4) \le \pi$ holds within this hole, i.e., the width of an interference band of the interference pattern located in its plane is smaller than the diameter of filtering aperture then the diffraction field in the filtration plane is determined by the equation

$$u(x_4, y_4) \sim p_3(x_4, y_4) \left\{ F^{-1}\left[\frac{k\, x_4}{f}, \frac{k\, y_4}{f}\right] \otimes\right.$$

$$\otimes \left[P_1^{-1}(x_4, y_4) \otimes P_2(x_4, y_4) +\right.$$

$$\left.\left. +\exp(\, i\, k\, a\, x_4/f) P_1^{-1}(x_4, y_4) \otimes \exp(\, ika\, x_4/f)\, P_2(x_4, y_4)\right]\right\}, \quad (8)$$

where $p_3(x_4, y_4)$ is the transmission function of the screen with a round hole.[8]

Let us present the light field in the rear focal plane of lens L_3 (Fig. 1) with the focal length f_3 in the form of a Fourier integral of light field in the plane of spatial filtration. Then using the property of Fourier transform we obtain

$$u(x_5, y_5) \sim t(\mu_1\, x_5, \mu_1\, y_5)\left\{p_1(\mu_2\, x_5, \mu_2\, y_5), p_2(-\mu_3\, x_5, -\mu_3\, y_5) \times\right.$$

$$\times \exp i\left[\varphi_1(\mu_2 x_5, \mu_2\, y_5) + \varphi_2(-\mu_3 x_5, -\mu_3\, y_5)\right] + p_1(\mu_2\, x_5 - a_1, \mu_2\, y_5) \times$$

$$\times\, p_2(-\mu_3\, x_5 + a_1, -\mu_3\, y_5)\, \exp i\left[\varphi_1(\mu_2\, x_5 - a_1, \mu_2\, y_5) +\right.$$

$$\left.\left. + \varphi_2(-\mu_3\, x_5 + a_2, -\mu_3\, y_5)\right]\right\} \otimes P_3(x_5, y_5), \quad (9)$$

where $\mu_1 = f/f_3$, $\mu_2 = f(1 + f_1/R + f_1^2/R\, \Delta)/f_3$, and $\mu_3 = f_2(1 + f_2/\Delta + f\, f_1/R\, \Delta)/f_3$ are the scale factors of image transformation, $a_1 = a(1 + f_1/R + f_1^2/R\, \Delta)$, and $a_2 = af_2(1 + f_2/\Delta + f\, f_1^2/R\, \Delta)/f$ are introduced for brevity,

$$P_3(x_5, y_5) = \int\int_{-\infty}^{\infty} p_3(x_4, y_4) \exp\left[-\, i\, k(x_4\, x_5 + y_4\, y_5)/f_3\right] \mathrm{d}\, x_4\, \mathrm{d}\, y_4$$

is the Fourier transform of the transmission function of filtering screen.

It follows from Eq. (9) that if the diameter D_0 of the illuminated area of a mat screen satisfies the condition $D_0 \ge d_1\, f_2/[f_2 + f(f_1 + \Delta)/R]$, then within the area of overlapping of the functions $p_1(\mu_2\, x_5, \mu_2\, y_5)\, p_2(-\mu_3\, x_5, -\mu_3\, y_5)$ and $p_1(\mu_2\, x_5 - a_1, \mu_2\, y_5)\, p_2(-\mu_3\, x_5 + a_2, -\mu_3\, y_5)$ identical subjective speckles coincide. Assuming that in the recording plane *4* (Fig. 1) the size of speckle determined by the function $P_3(x_5, y_5)$ width is small in comparison with the period of modulation of the speckle field phase, we can remove the function $\exp i\left[\varphi_1(\mu_2 x_5, \mu_2 y_5) + \varphi_2(-\mu_3 x_5, -\mu_3 y_5)\right] +$ $+ \exp i\left[\varphi_1(\mu_2\, x_5 - a_1, \mu_2 y_5) + \varphi_2(-\mu_3 x_5 + a_2, -\mu_3 y_5)\right]$ from the convolution integral in Eq. (9). Then a superposition of correlating speckle fields yields the following distribution of the irradiance

$$I(x_5, y_5) \sim \left\{1 + \cos\left[\varphi_1(\mu_2\, x_5, \mu_2\, y_5) + \varphi_2(-\mu_3\, x_5, -\mu_3\, y_5) -\right.\right.$$

$$\left.\left. -\, \varphi_1(\mu_2\, x_5 - a_1, \mu_2\, y_5) - \varphi_2(-\mu_3\, x_5 + a_2, -\, \mu_3\, y_5)\right]\right\} \times$$

$$\times\, |t(\mu_1\, x_5, \mu_1\, y_5) \otimes P_3(x_5, y_5)|^2. \quad (10)$$

Expression (10) describes the speckle structure modulated by the interference bands. Interference pattern looks like a lateral shear interferogram in the bands of infinite width, which characterizes the axial wave aberrations of a two–component optical system. This is because the information about the phase distortions introduced into light wave by the microscope objective and ocular is within a separate speckle in the hologram plane.

Thus, spatial filtration, when being done on the optical axis in the hologram plane, enables one to isolate, from the spatial spectrum of waves scattered by a mat screen, a narrow interval of spatial frequencies about the direction along the optical axis of the microscope. Displacement of filtering diaphragm along x axis in the hologram plane leads to the formation of lateral shear interferogram in the bands of infinite width, which characterizes the combination of on–axis and off–axis wave aberrations of the microscope objective and ocular.

In this case the filtering hole isolates a narrow interval of spatial frequencies near the direction along spatial frequency $x_{40}/\lambda f$, where, x_{40} is the coordinate of the center of filtering aperture. Assuming that the displacement is small in comparison with the diameter of pupils of lenses L_1 and L_2, it can be shown that because of limitations of beams in the microscope optical system the range 2ω of wave aberrations control over field is determined by the value $\tan 2\omega = 2(x_{40})_{\max}/f = \; d_1(R\Delta + f_1^2)/f_1\left[R\Delta + f_1(/f_1 + \Delta)\right] - $ $- d_2(R\Delta + f_1^2)/\left[R\Delta\,(f_2 + \Delta) + f_1^2 f_2\right]$.

To record the interference pattern located in the hologram plane, the spatial filtering of diffraction field on optical axis in the frequency plane of a collimating optical system of two positive lenses installed behind the hologram is needed as in Ref. 9. In this case the extent of interference pattern in space is limited by the size of the mat screen Fourier transform in space. To record it in the whole area of $D \geq d_1 f_2 (f_1 + \Delta)$, where the subject field exists in the hologram plane, we will consider the spatial filtering of diffraction field on the optical axis as shown in Fig. 2.

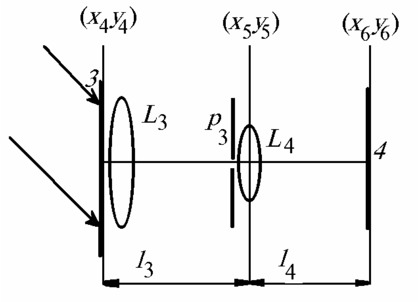


FIG 2. The optical scheme of recording the interferogram located in the hologram plane.

Field distribution in the hologram plane takes the following form in this area:

$$u_1(x_4, y_4) \sim \exp\left[\frac{i\,k\,(x_4^2+y_4^2)}{2\,l_2}\right]\left\{\exp\left[-\frac{i\,k\,\left(x_4^2+y_4^2\right)}{2\,l_2}\right]\times\right.$$
$$\times\left\{F^{-1}\left[\frac{k\,x_4}{f}, \frac{k\,y_4}{f}\right]\otimes P_1^{-1}(x_4, y_4)\right\}\otimes P_2(x_4, y_4)\Bigg\} +$$
$$+ \exp i\left[\varphi_3(x_4, y_4) - \varphi_3(x_4 + b, y_4)\right] \times$$
$$\times\exp\left[\frac{i\,k\,(x_4^2+y_4^2)}{2\,l_2}\right]\left\{\exp\left[-\frac{i\,k\,\left(x_4^2+y_4^2\right)}{2\,l_2}\right]\times\right.$$
$$\times\left\{F^{-1}\left[\frac{k\,x_4}{f}, \frac{k\,y_4}{f}\right]\otimes\exp(i\,k\,z_4/f)\;P_1^{-1}(x_4, y_4)\right\}\otimes$$
$$\left.\otimes\exp(i\,k\,z_4/f)\;P_2(x_4, y_4)\right\} \qquad (11)$$

If the lens L_3 in Fig. 2 is in the hologram plane and the condition $1/l_2 + 1/l_3 = 1/f_3$ holds, the complex amplitude of the field of two exposures in (x_5, y_5) plane is determined by the expression

$$u(x_5, y_5) \sim \exp\left[\frac{i\,k\,(x_5^2+y_5^2)}{2\,l_3}\right]\left\{\left\{\exp\left[-\frac{i\,k\,\left(x_5^2+y_5^2\right)\,l_2}{2\,l_3^2}\right]\otimes\right.\right.$$
$$\left.\otimes\left[t(\mu_4\,x_5, \mu_4\,y_5)\;p_1(\mu_5\,x_5, \mu_5\,y_5)\;\exp i\,\varphi_1(\mu_5\,x_5, \mu_5\,y_5)\right]\right\}\times$$
$$\left.\times p_2(-\mu_6\,x_5, -\mu_6 y_5)\;\exp i\varphi_2(-\mu_6 x_5, -\mu_6 y_5)\right\} + \exp\left[\frac{ik(x_5^2+y_5^2)}{2l_3}\right]\times$$
$$\times\left\{t\;\;(x_5, y_5)\otimes\left\{\exp\left[-\frac{i\,k\,\left(x_5^2+y_5^2\right)\,l_2}{2\,l_3^2}\right]\otimes\right.\right.$$
$$\left.\otimes\left[t(\mu_4\,x_5, \mu_4\,y_5)\,p_1(\mu_5\,x_5 - a_1, \mu_5\,y_5)\exp i\varphi_1(\mu_5\,x_5 - a_1, \mu_5\,y_5)\right]\right\}\times$$
$$\left.\times p_2(-\mu_6\,x_5 + a_2, -\mu_6\,y_5)\;\exp\,i\varphi_2(-\mu_6\,x_5 + a_2, -\mu_6\,y_5)\right\}, \qquad (12)$$

where $\mu_4 = f/l_3$, $\mu_5 = f(1 + f_1/R + f_1^2/R\,\Delta)/l_3$, and $\mu_6 = f_2(1 + f_2/\Delta + f\,f_1/R\,\Delta)/l_3$ are the scale factors of image transformation, $t\;\;(x_5, y_5) = \int\limits_{-\infty}^{\infty}\!\!\int \exp i\left[\varphi_3(x_4, y_4) - \varphi_3(x_4 + b, y_4)\right]\times$ $\times\exp\left[-\,i\,k\left[\left(x_4\,x_5 + y_4\,y_5\right)/l_3\right]\right]\mathrm{d}x_4\,\mathrm{d}y_4$ is the Fourier transform of the corresponding function.

Since $t\;\;(x_5, y_5) \simeq \delta(x_5, y_5)$, where $\delta(x_5, y_5)$ is the Dirac delta function, if the filtering diaphragm p_3 (Fig. 2) is installed in (x_5, y_5) plane, within whose aperture the condition $\varphi_2(-\mu_6\,x_5, -\mu_6\,y_5) - \varphi_2(-\mu_6\,x_5 + a_2, -\mu_6\,y_5) \leq \pi$ holds, the complex amplitude in its output is determined as

$$u(x_5, y_5) \sim \exp\left[\frac{i\,k\,(x_5^2+y_5^2)}{2\,l_3}\right]\left\{\exp\left[-\frac{i\,k\,\left(x_5^2+y_5^2\right)\,l_2}{2\,l_3^2}\right]\otimes\right.$$
$$\otimes\left[t(\mu_4\,x_5, \mu_4\,y_5)\;p_1(\mu_5\,x_5, \mu_5\,y_5)\;\exp\,i\,\varphi_1(\mu_5\,x_5, \mu_5\,y_5)\right] +$$
$$+\;t\;\;(x_5, y_5)\otimes\left\{\exp\left[\frac{i\,k\,\left(x_5^2+y_5^2\right)\,l_2}{2\,l_3^2}\right]\otimes\left[t(\mu_4\,x_5, \mu_4\,y_5)\times\right.\right.$$
$$\left.\left.\left.\times p_1(\mu_5\,x_5 - a_1, \mu_5\,y_5)\exp i\,\varphi_1(\mu_5 x_5 - a_1, \mu_5\,y_5)\right]\right\}\right\}p_3(x_5, y_5). \quad (13)$$

Let the condition $1/f_3 = 1/l_3 + 1/l_4$ be satisfied for the focal length of L_4 lens (Fig. 2), for which p_3 is the aperture diaphragm. Then the diffraction field distribution in the recording plane *4* takes the form

$$u(x_6, y_6) \sim \exp\left[\frac{i\,k\,(x_6^2+y_6^2)}{2\,l_4}\right]\left\{\left\{\exp\left[-\frac{i\,k\,\left(x_6^2+y_6^2\right)\,\mu_7^2}{2\,l_2}\right]\times\right.\right.$$
$$\times\left[F(x_6, y_6)\otimes\,P_1(x_6, y_6)\right] + \exp i\left[\varphi_3(-\mu_7 x_6, -\,\mu_7\,y_6) - \right.$$
$$\left.-\;\varphi_3(-\mu_7\,x_6 + b, -\mu_7 y_6)\right]\left\{\exp\left[-\frac{i\,k\,\left(x_6^2+y_6^2\right)\,\mu_7^2}{2\,l_2}\right]\times\right.$$
$$\left.\left.\left.\times\left[F(x_6, y_6)\otimes\exp(-ikax_6/\mu_5 l_4)\,P_1(x_6, y_6)\right]\right\}\right\}\otimes P_3(x_6, y_6)\right\} \quad (14)$$

where $\mu_7 = l_3/l_4$ is the scale factor of image transformation,

$$F(x_6,y_6)=\int\int_{-\infty}^{\infty} t(\mu_4\, x_5,\ \mu_4\, y_5)\exp\left[-ik(x_5x_6+y_5\, y_6)/l_4\right]\mathrm{d}\, x_5\mathrm{d}\, y_5,$$

$$P_1(x_6,\ y_6)=\int\int_{-\infty}^{\infty} p_1(\mu_5\, x_5\ ,\ \mu_5\, y_5)\ \exp\ i\ \varphi_1(\mu_5\, x_5,\ \mu_5\, y_5)\ \times$$

$$\times\ \exp\left[-\ i\ k(x_5\, x_6+\ y_5\, y_6)/l_4\right]\ \mathrm{d}x_5\ \mathrm{d}y_5\ ,$$

$$P_3(x_6,\ y_6)=\int\int_{-\infty}^{\infty} p_3(x_5,\ y_5)\exp\left[-\ ik(x_5\, x_6+\ y_5\, y_6)/l_4\right]\mathrm{d}x_5\mathrm{d}y_5,$$

are the Fourier transforms of corresponding functions.

Let us now write down the expression for light intensity distribution in the recording plane. In so doing and in order to exclude the speckle effect from consideration, we introduce averaging over the coordinates, assuming that the averaging area is far greater than a speckle while, at the same time, the phase factor $\exp\ i\ [\ \varphi_3(-\mu_7\, x_6,\ -\mu_7\, y_6) - \varphi_3(-\mu_7\, x_6+\ b,\ -\mu_7\, y_6)]$ remains constant within this area. Moreover, we also assume that the phase distortions introduced into the wave by the objective are caused only by defocusing, as in Ref. 10, and the value of d_1/μ_5 is assumed to be larger than the diameter of filtering aperture. Then for the case of equal average values of field intensities, corresponding to the first and second exposures, we obtain

$$\langle I(x_6,\ y_6)\rangle \sim 1\ +\ |V|\cos\left[\varphi_3(-\mu_7\, x_6,\ -\mu_7\, y_6)\ -\right.$$

$$\left.-\ \varphi_3(-\mu_7\, x_6+\ b,\ -\mu_7\, y_6)\ +\ \psi\right], \qquad (15)$$

where

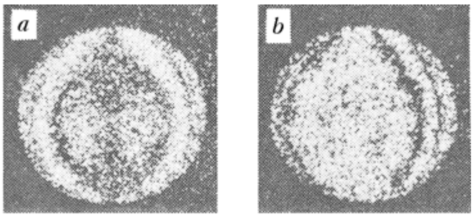
$$V\ =\ \exp\left(-i\ k\ \mu_5\mu_7\ E\ a_1\, x_6\, l_3/l_2\right)\ \int\int_{-\infty}^{\infty} p_3(x_5\ ,\ y_5)\ \times$$

$$\times\, p_3(x_5 - \mu_5\mu_7^2\ E\ a_1\ l_4^2/l_2,\ y_5\Big)\ \times$$

$$\times\exp\left(ik\mu_5 E\, a_1 x_5\right)\mathrm{d}\ x_5\ \mathrm{d}\ y_5\Big/\ \int\int_{-\infty}^{\infty}|p_3(x_5\ ,\ y_5)|^2\mathrm{d}\ x_5\ \mathrm{d}\ y_5 \qquad (16)$$

is the normalized correlation function, $\varphi = \arg V$, E is the factor characterizing defocusing. As follows from Eq. (16), the portion of $(x_5,\ y_5)$ plane, where the integrand of Eq. (16) is nonzero, has the area equal to the total area of the filtering diaphragm and that of the same aperture but displaced relative to the first one by the distance $\mu_5\mu_7^2\, E\ a_1\ l_4^2/l_2$. When these two areas do not ovelap, V equals zero. Moreover, for $V \neq 0$ the view of interference pattern changes because of the factor $\exp\left(-i\ k\ \mu_5\mu_7\ Ea_1x_6l_3/l_2\right)$, characterizing the phase distortions introduced into light wave by the objective. For $\mu_5\mu_7^2 Ea_1 l_4^2/l_2 << d$, where d is the diameter of filtering diaphragm, and $d \leq \lambda l_2/\mu_5\mu_7 Ea_1 l_3$, the interference pattern can be recorded with a sufficiently high contrast, within the domain of existence of the object field in the photographic plate plane. This pattern characterizes the reference wave front phase distortions due to wave aberrations of the optical system forming it.

In experiment double exposure holograms were recorded on photographic plates of the type Mikrat VRL using He–Ne laser radiation at the wavelength of 0.63 μm. Fourier transform of mat screen being illuminated by a diverging spherical wave with R = 180 mm was formed using a microscope with the focal length of the objective f_1 = 35 mm and that of ocular, f_2 = 50 mm, and the diameters of pupils respectively d_1 = 8 and d_2 = 12 mm. Optical length of the microscope viewing hood was equal to Δ = 80 mm.



a *b*

FIG. 3. Lateral shear interferograms located in far diffraction zone and recorded with on–axis (a) and off–axis (b) spatial filtering.

As an example, Fig. 3*a* shows the interferogram recorded with spatial filtering on optical axis in the hologram plane as reconstructed with a small aperture laser beam. To this end, the laser radiation was focused with a long–focus lens of f_0= 500 mm. Interference pattern characterizes the spherical aberration with the defocusing of the microscope optical system being behind its focus. Prior to second exposure recording the mat screen was displaced in the direction normal to the optical axis by the distance a = (0.17 ± 0.002) mm and the reference beam tilt angle was changed by the value $\Delta\theta$ = 28 ± 10 .

The interferogram presented in Fig.3*b* was recorded with a spatial filtering by way of displacement of the hologram by 1.5 mm with respect to the beam used for its reconstruction. It characterizes the combination of on–axis and off–axis wave aberrations of the microscope optical system and corresponds to the case of diffraction of a plane wave, whose direction makes an angle ω with the microscope optical axis, at objective pupil. Further displacement of the hologram relative to laser beam reconstructing it leads to the recording of interference pattern, the spatial extent of which decreases due to limitations of beams in the microscope optical system.

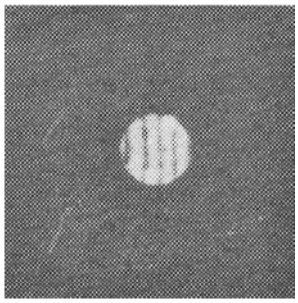


FIG. 4. Lateral shear interferogram located in the hologram plane.

Lateral shear interferogram in the bands of infinite width shown in Fig. 4 characterizes the phase distortions of the part of reference wave front due to wave aberrations introduced by the optical system forming it. It was recorded with the spatial filtering on optical axis according to scheme shown in Fig. 2. The extent of interference pattern limited by the area of the object field existence in the hologram plane was 5 mm that corresponds to the calculated value.

The theoretical and experimental results presented show that the method of double exposure recording of a Fourier hologram with a microscope provides formation of the lateral shear interferograms in bands of infinite width with diffusely scattered light. As this takes place, the interference pattern characterizing the wave aberrations of the microscope optical system is located in the far diffraction zone and for its recording the spatial filtering should be done in the hologram plane. Moreover, the spatial filtering allows the interference patterns, which correspond to the spatial frequencies of waves scattered by the mat screen, to be be separated out thus providing for a control of the wave aberrations over the field.

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