THE REASON WHY INTERFERENCE BANDS ARE PRODUCED BY A BEAM OF REFRACTED GLANCING RAYS

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This paper presents a description of an interference pattern produced by refracted laser rays propagating along a transparent prism. The high contrast of bands in the pattern is explained. The refracted light rays are separated into the edge rays and rays that obey the ordinary laws of refraction. It is shown that the intensity of the edge rays is quite sufficient to produce the observed interference pattern in combination with the rays refracted at the critical refraction angle. Based on the interference of the edge rays and rays refracted at the critical angle, an empirical formula has been derived that describes well the experimentally observed interference patterns.

Present paper continues the study of refraction of glancing rays passing from air to transparent media,[1] which is the phenomenon unexpected for modern understanding of light refraction.

Experiments discussed here were based on the scheme described in Ref. 1 with the use of a glancing light beam of width 300 μm in the plane of the image S' of the slit S illuminated by a parallel monochromatic light beam. Divergence of glancing rays was within 0.32°. The image S' was formed in the plane of the front face of a glass prism and half of the image was covered by the prism to maximize the refracted beam intensity.

When a screen is incerted into the path of a refracted light beam, the interference pattern is formed on it analogous to the diffraction pattern from a screen. Figure 1 shows the intensity distribution J_r of the pattern formed by a He–Ne laser radiation when the distance from hypotenuse face of the prism to the plane of scanning slit S_s was equal to 112 mm, the scanning slit of the width 30 μm was perpendicular to the refracted beam, and the scanning step was equal to 25 μm.

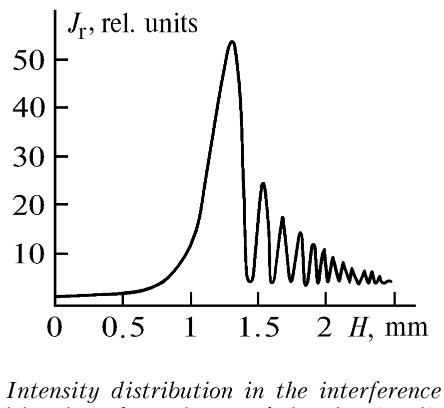

FIG. 1. Intensity distribution in the interference pattern produced by the refracted rays of the glancing light beam propagating along a prism.

A bright vertical line was seen by the naked eye (Fig. 2). Bands were seen on both sides of this line, which gradually weakened toward the periphery. We placed the screen E in front of the hypotenuse face at a distance of 3 mm and moved it from left to right. After a while, the right band began to shorten from its end to its origin until complete vanishing. In this case the line itself and its left band remained unchanged. As the screen was moved further, the left band began to shorten in the direction from the line to the end with the formation of broadening dark gap between its moving end and the line. At the same time the line brightness became progressively less, while its position remained unchanged. Clearly such behavior of the pattern seen by the naked eye testifies the propagation of a parallel beam coming from the main part of the face AB and being focused as a line onto the eye retina as well as the formation of the refracted edge rays in the region of the input edge A. The edge rays E_1 propagating at the angles smaller than the critical refraction angle β formed the right band in the field of vision, while the edge rays E_2 propagating at the angles larger than β formed the left band.

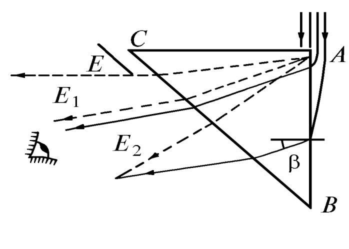

FIG. 2 Experimental scheme of detection of the edge rays in the refracted beam.

Because the edge rays E_2 and parallel rays are superposed, their interference is natural explanation of the interference pattern formed on the screen.

Fresnel[2] regarded the energy of the edge light to be insufficient for the formation of bands with the intensity recorded in the experiment. However, he was wrong (see Refs. 3 and 4).

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Let us replace the screen E by the slit S_h of width t_h = 56 μm (Fig. 3). When it moves along the hypotenuse face in the direction from the edge B to the edge C, it first intersects the region of the edge rays E_1. In doing so, the single gradually intensifying central maximum with the angular half–width λ/t_h was seen through the slit. This maximum was due to diffraction of light incident on the slit. When the slit had passed the region with maximum intensity of refracted light, the maximum began to broaden and then to separate into two gradually moving apart maxima max_{or1} and $max_{e2.1}$.

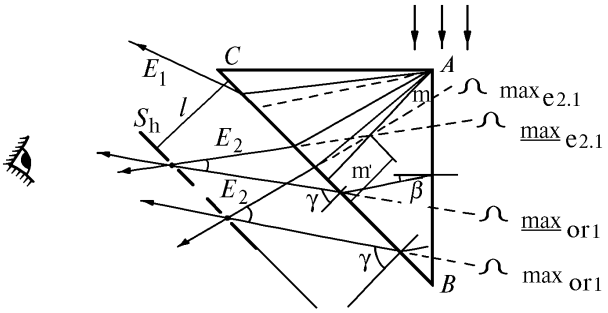

FIG. 3. Scheme of separation of a refracted beam into the edge rays and rays refracted at the critical angle.

Judging from the insignificant mobility, the more intense maximum max_{or1} is formed by ordinary refracted rays incident on the slit at the angle γ from the part of the face corresponding to the location of the slit. The maximum $max_{e2.1}$, which was shifted towards the left of that maximum, was formed by the edge rays incident at larger and larger angles with respect to the parallel beam as the slit S_h moved. The above–considered method of separation of the edge rays E_2 and rays propagating at the critical angle allowed us to measure the intensities of these rays. Their values J obtained in experiments with green light (λ = 0.53 μm) for various positions of S_h are shown in Fig. 4 as functions of $l_{h.f}$, where $l_{h.f}$ is the distance from the point of incidence of rays on the hypotenuse face, when the slit S_h is adjacent to the center of the first maximum of the interference pattern being formed on the slit face, to the point of ray incidence on the face for subsequent positions of S_h. For ordinary refracted rays the value of $l_{h.f}$ is equal to the shift of S_h from the center of the first maximum.

If we extend trajectories of the edge rays propagating through the air towards the prism, they will intersect within the prism in some point spaced at the distance $m' = m/n$ from the hypotenuse face because of the small value of γ (see Fig. 3), where m is the height of the prism being equal to 7.02 mm and n is the refractive index of the K–8 glass being equal to 1.51927. In this case $l_{h.f}$ for the edge rays is given by the relation $l_{h.f} = \Delta_s\, m/n(l + m/n)$, where l is the distance between the slit S_h and the prism, Δ_s is the amount of displacement of the slit from the first maximum of the interference pattern to the point at which the edge rays of appropriate intensity J fall within the slit.

Curve *1* illustrates the intensity of ordinary refracted rays, and curve *2* –– the intensity of the edge light. Coming from different points of the hypotenuse face, the edge and ordinary refracted rays converge to the maximum max_1 on the slit S_h, thus both curves have common origin on the $l_{h.f}$ axis.

These curves were constructed with the use of experimental data to the point D. For shorter $l_{h.f}$, the edge and ordinary refracted rays partially mix. As a result, the maximum intensities in the points max_{or1} and $max_{e2.1}$ differ from the real values of J for these rays. The curves were extended until they cut the J axis to find the values of J for ordinary refracted and edge rays at the point max_1. The first curve was extended on the basis of previous behavior of its curvature, and the second curve – on the basis of linear dependence of the edge ray amplitude on $l_{h.f}$ (see curve *3*).

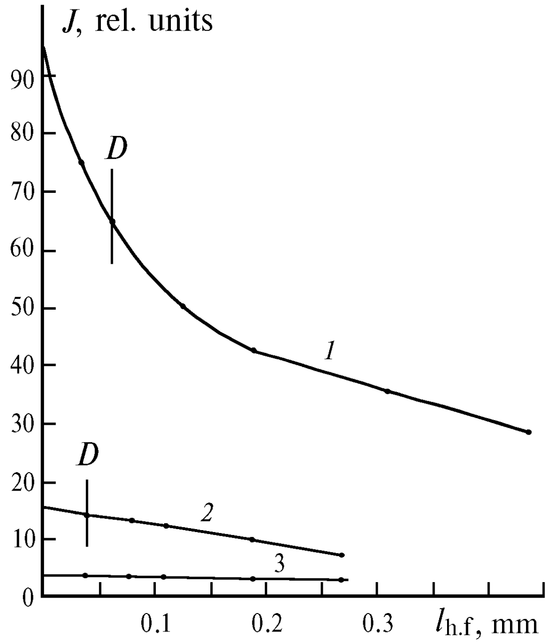

FIG. 4. Intensity variations for the edge rays and rays refracted at critical angle vs. the length of the hypothenuse face.

The experimental value of $J_{max\,1}$ was equal to 194.3 relative units. The values of J for ordinary refracted and edge rays, derived from these curves at $l_{h.f}$ = 0, were equal to 98.7 and 16.0 relative units, and their sum was $(\sqrt{98.7} + \sqrt{16})^2 = 194$ relative units, i.e., was equal to the experimentally observed intensity at the point max_1.

Hence, the observed interference pattern is really formed due to interference of ordinary refracted and edge rays.

Let us adjust the slit S_s of width 0.15 mm to the first maximum of the interference pattern, then insert the slit S_h of width 50 μm in front of the hypotenuse face (l = 4.05 mm) and move it so that the maximum light flux comes from S_s as before. When the slit S_h was displaced from this position to the right at the distance R = 1.03 mm (λ = 0.53 μm and the distance between S_s and S_h along the ordinary refracted ray was L' = 102.4 mm), the light flux coming from S_s progressively vanished. Because R turned out to be equal to half–width of the maximum formed as a result of diffraction of refracted rays by the slit S_h

($\lambda L'/t_h$ = 1.05 mm), the rays from the left side of these maximum will fall within the slit S_h, in other way the light flux coming from S_s would disappear earlier. Therefore, refracted rays from each point of refracting face AB propagate within the scattering angles being smaller than 57.3°R/nL' = 0.38°, i.e., points of refracting surface are not sources of light oscillations propagating in all directions, as Huygens' principle postulates, and refracted light is not due to radiation of secondary waves by electrons oscillating upon exposure to the incident light wave. Otherwise, light oscillations of elementary sources propagating in all directions would fall within the slit S_s at arbitrary positions of S_h relative to its starting position. But this is not the case.

If incident light had produced the secondary waves on the refracting side, each elementary source would have formed its own diffraction pattern from the slit S_h in the field of view. Because light oscillations from elementary sources, distributed along the entire length of the face, enter the slit at different angles, the elementary diffraction patterns would be shifted from each other and a long band should be observed. However, a single central maximum is observed in the direction of propagation of the refracted light instead. Therefore, the secondary waves are absent, and refracted light is the light that initially passed through the first medium and then changed the direction of its propagation in the second medium. One of the reasons, if not the only, for changing the direction of propagation is the deflection of light rays within the deflection zone that exists, as experiments have shown, on both sides of the interface. In accordance with curve *1* (Fig. 4), the intensity of ordinary refracted rays reduces gradually in the direction from the input edge of the prism with especially sharp decrease at the origin of the edge. This fact allows the conclusion to be made that the efficiency of light deflection within the deflection zone degrades along the refracting edge and in the direction from this edge towards the outside boundary (top) of the zone. In this case deflected glancing rays falling within the initial part of the edge is to be deflected in the effective region of the zone and, therefore, will have larger angles of incidence in comparison with the rays incident on the side at longer distances from the input edge. In this case the edge will have higher transmissivity. Thus the intensity of the refracted flux coming from the origin of the edge must reach its maximum. This is the case.

The fact that the deflection zone is more effective near the edges is also confirmed by the increasing intensity of the edge rays coming from the screen when the screen is thick and right–angle. There are the following arguments in favour of this assumption. Judging by propagation of edge rays on both sides of the initial direction of light incidence, the zone deflects the rays not only towards the screen, but also in the opposite direction. When a right–angle plate or prism is a screen, rays deflected within the zone into the shadow fall within this screen and undergo reflection. If the zone had the same efficiency along its whole length, reflected rays could not leave it, because they should be deflected again towards the screen. If the zone is most effective near edges, light rays are deflected within its effective region before reflection and then propagate within its less effective region incapable to reflect them toward the edge. For this reason they leave this zone in the direction of propagation of rays primarily deflected from the screen, and interfere with these rays. Due to the loss of half wavelength as a result of reflection, both fluxes, which were initially out of phase,[5] turned out to be in phase and, therefore, reinforced each other.

Really, edge light from a thick screen of the above–mentioned shape has the intensity several times higher than that from a thin screen.[6]

The interference pattern discussed above has higher contrast of bands in comparison with the diffraction pattern from a thin screen.[3]

Such a peculiarity is due to higher value of the ratio of the edge light intensity to the intensity of ordinary refracted rays in comparison with the ratio of the intensity of the edge light to the intensity of the incident light in the diffraction pattern from a thin screen. One reason of this phenomenon is that the refracted edge rays are closer to the edge A than ordinary refracted rays. Thus transmissivity is higher for the edge rays than for ordinary refracted rays.

Glancing rays deflected in more effective and less effective regions of the zone are incident on the prism face at different angles. In spite of this, the divergence of the ordinary refracted beam is small because sinus of the angles close to 90° remains constant.

From the above reasoning, the sharp increase in the intensity of refracted light seen from curves *4* and *5* in Fig. 2 (see Ref. 1) is due to the increase in the intensity of rays refracted at the origin of the face and due to interference of these rays with the edge rays, and $J_{\text{opt max}}$ is the light intensity at the center of the first interference maximum. Absence of higher–order maxima is caused by large width of the scanning slit.

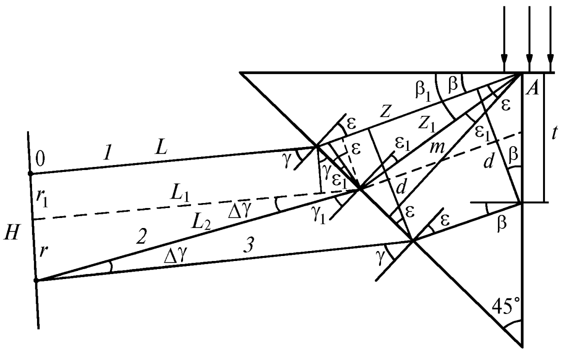

FIG. 5. Scheme of interference of the edge rays with the rays refracted at the critical angle.

Based on the foregoing, let us derive the formula for the position of the bands in the interference pattern. To do this, we use the scheme shown in Fig. 5, where H is the distance from the point O to the interference band. The point O is the point of incidence of ray *1* coming from the edge region A, that obeys the ordinary laws of refraction, rays *2* and *3* are the edge and ordinary refracted rays, which interfere on the screen, t is the distance between the points of their incidence, m is the prism height, ε_1 and ε are the angles of incidence of the edge and ordinary refracted rays on the hypotenuse face, γ_1 and γ are the corresponding angles of refraction, z and z_1 are the path length of rays *1* and *2* in the prism, L and L_2 are that in air up to the plane of the interference pattern, and $\Delta\gamma$ is the angle between the interfering rays.

As is seen from the figure, the path difference between rays *2* and *3* is equal to

$$\Delta = (n z_1 + L_2) - (n z + L) = (L_2 - L) - n (z - z_1) =$$
$$= (L_1 + \Delta_2 - L) - n \Delta_1 = (L + r_1 \tan\gamma + \Delta_2 - L) - n \Delta_1 =$$
$$= \Delta_2 + r_1 \tan\gamma - n \Delta_1.$$

A little manipulation yields

$$\Delta = \frac{r^2 + 2\, r_1\, r}{2\, L},$$

where

$$r = \frac{H\, n\, L\, \cos^3\varepsilon}{m\, \cos^2\gamma + n\, L\, \cos^3\varepsilon},$$

$$r_1 = \frac{H\, \cos\gamma\, [\, m^2 \cos^3\gamma + m \cos\gamma\, n\, L\, \cos^3\varepsilon - n\, L\, \cos^4\varepsilon\, \sin\varepsilon]}{(\, m\, \cos^2\gamma + n\, L\, \cos^3\varepsilon)^2}.$$

Replacing r and r_1 by their corresponding expressions, we derive

$$H = (\, m\, \cos^2\gamma + n\, L\, \cos^3\varepsilon)\times$$

$$\times \sqrt{\frac{k\lambda}{n\, \cos^3\varepsilon\, [\, n\, L\, \cos^3\varepsilon + 2\, m\, \cos^2\gamma - 2\, \cos\gamma\, \cos\varepsilon\, \sin\varepsilon\,]}}.$$

An analysis shows that for the right–angle prism elimination of the terms containing cosγ and cosε from the above formula has practically no effect for wide range of variation of the parameters n and L. Thus $H = (m + n\, L) \times \sqrt{k\, \lambda / n\, (2\, m + n\, L)}$, where k is the number of half wavelengths being present along the length Δ. The formula correctly gives the distance between the interference bands when it is written in the form

$$H = (m + n\, L) \sqrt{\frac{(k_0 + k')\, 1}{n\, (2\, m + n\, L)}}.$$

In this case maxima are placed at $k' = 0, 2, 4, 6, \ldots$, minima – at $k' = 1, 3, 5, \ldots$, and the parameter k_0 is determined from the experimental distance between the first and second maxima

$$H_{21} = H_{\max 2} - H_{\max 1} = \left(\sqrt{k_0 + 2} - \sqrt{k_0}\right) \sqrt{\frac{(m + n\, L)^2\, \lambda}{n\, (2\, m + n\, L)}}.$$

In its turn the known value of k_0 is used to find the start of the count of H in terms of $H_{\max 1}$.

A comparison between the experimental and calculated results is summarized in Table I, where H_{exp} and H_{cal} are the experimentally measured and calculated distances from the bands to the point O and $\Delta H = H_{\text{cal}} - H_{\text{exp}}$. In the experiment, $\lambda = 0.6328$ μm, $L = 111.7$ μm, $n = 1.514555$, $\gamma = 5°33'$, $m = 7.071$ mm, $\varepsilon = 3°40'$, and the electric vector of laser radiation was parallel to the refracting face.

The nonzero value of k_0 ($k_0 = 0.354$) shows that at the instant of formation of the edge rays they experience a phase advance of $k_0\, \lambda/2$ with respect to the ordinary refracted rays.[5] Hence the maxima are formed at the points at which Δ exceeds the number of waves by $k_0\, \lambda/2$.

TABLE I.

Band	J_r, rel.units	H_{exp}, mm	k	H_{cal}, mm	ΔH, μm
	30.9	0	–	0	0
$\max_1$	56	0.158	0.354	0.158	0
$\min_1$	7.1	0.318	1.354	0.310	–8
$\max_2$	27.2	0.408	2.354	0.408	0
$\min_2$	6.9	0.488	3.354	0.487	–1
$\max_3$	20.6	0.553	4.354	0.555	2
$\min_3$	7.8	0.618	5.354	0.616	–2
$\max_4$	17.2	0.671	6.354	0.671	0
$\min_4$	6.7	0.723	7.354	0.722	–1
$\max_5$	15	0.773	8.354	0.769	4
$\min_5$	9.12	0.816	9.354	0.814	–2
$\max_6$	14	0.853	10.354	0.856	3
$\min_6$	8.6	0.893	11.354	0.897	4
$\max_7$	12.9	0.928	12.354	0.935	7
$\min_7$	8	0.968	13.354	0.972	4
$\max_8$	11.5	1.003	14.354	1.008	5
$\min_8$	8	1.041	15.354	1.043	2
$\max_9$	10.4	1.078	16.354	1.076	–2
$\min_9$	7	1.113	17.354	1.109	–4
$\max_{10}$	9.7	1.148	18.354	1.140	–8
$\min_{10}$	7.4	1.178	19.354	1.171	–7
$\max_{11}$	9.9	1.203	20.354	1.200	–3
$\min_{11}$	7.3	1.228	21.354	1.230	2
$\max_{12}$	8.5	1.258	22.354	1.258	0
$\min_{12}$	7.5	1.293	23.354	1.286	–7
$\max_{13}$	8	1.328	24.354	1.313	–15
$\min_{13}$	7.6	1.353	25.354	1.340	–13
$\max_{14}$	8	1.376	26.354	1.366	–10
$\min_{14}$	7.6	1.403	27.354	1.392	–11
$\max_{15}$	8	1.428	28.354	1.417	–11

In the considered interference scheme

$$\sin \Delta\gamma = \frac{n\, t\, \cos\beta}{n\, L\, \cos\varepsilon + m\, \cos\gamma}, \quad t = \frac{H\, \cos\varepsilon}{\cos\gamma\, \cos\beta}.$$

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