

## MEAN FLUXES OF SOLAR RADIATION IN STRATUS CLOUDS WITH RANDOM UPPER BOUNDARY

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Received November 22, 1993*

*A model of continuous stratus clouds with stochastically inhomogeneous upper boundary is constructed based on the algorithm of simulating a Gaussian random field. This model considers such effects as optically thin regions, radiative interaction among individual billows, and mutual shadowing. The mean radiant fluxes are computed within the framework of the method of statistical simulation of cloud and radiation fields. Differences between the mean fluxes of solar radiation in continuous stratus clouds with irregular upper boundary and in plane cloud layer are maximum at small solar zenith angles and may reach a few tens of per cents.*

Real clouds are typically characterized by extreme spatial inhomogeneity of their optical properties resulting from their complex irregular geometry and macroscale fluctuations of water content, phase composition, and water droplet (ice crystal) size spectrum. Natural inhomogeneities occur simultaneously, and the geometric thickness and cloud extinction coefficient may vary considerably.<sup>1</sup> Visual observations of dark and light fragments at the lower boundary of continuous cloudiness give no way for tackling unambiguously the question about the contribution of macroscale fluctuations in the cloud optical parameters and cloud stochastic geometry to the formation of these fragments. In other words, a light fragment cannot be uniquely identified as a zone with a decreased value of the cloud extinction coefficient (or its geometric thickness). Reliable data with high spatial resolution on the vertical structure and statistical correlation among the cloud optical and geometric parameters are practically lacking now, thereby making the construction of adequate models of clouds difficult.

Since the radiative regime and brightness fields nonlinearly depend upon the cloud geometry and cloud optical characteristics, it is impossible to evaluate correctly the effect of fluctuating cloud optical characteristics on radiation field modulated by these fluctuations, using some mean parameters, e. g., mean optical thickness, in calculations. A number of models are available now considering the effect of stochastic geometric structure of cumulus cloud field on radiative regime and brightness field of the system atmosphere—underlying surface (see, e.g., Refs. 2, 3, and 4). As for continuous stratiform clouds, the model of a plane—parallel homogeneous layer is most widely used as before. For example, the model of a plane—parallel layer of vertically homogeneous turbid medium with horizontal continuous periodic variations of the scattering and absorption coefficients was used in Refs. 5 and 6 to study the effect of spatial inhomogeneity of cloud optical characteristics on radiation processes in the atmosphere.

The present paper uses the model developed in Ref. 2. This model is based on the algorithm of simulating a uniform isotropic Gaussian field<sup>7,8</sup> and considers the stochastic geometry of the upper boundary of stratus clouds with deterministic optical characteristics. The method of statistical simulation of

cloud and radiation fields<sup>9</sup> is used for studying the dependence of the mean fluxes of visible solar radiation on the parameters describing the random geometry of the cloud upper boundary.

### 1. MODEL OF STRATUS CLOUDS WITH RANDOM UPPER BOUNDARY

The idea of using the Gaussian random surfaces for simulation of cloud fields with random geometry was first suggested in Ref. 10. For a detailed mathematical description of the algorithm of constructing such surfaces and adjusting the model input parameters to the experimentally measured quantities, see Ref. 2. For clarity and integrity of presentation, we only briefly describe the model here. Let us assume that the cloudiness is bounded from bottom by the plane  $z = H_0$ , and its upper boundary  $z = \omega(x, y)$  is defined as

$$\omega(x, y) = H_0 + \max(v(x, y) + H, 0), \quad (1)$$

where  $H$  is the mean cloud layer thickness,  $v(x, y)$  is the uniform Gaussian field with zero mean, correlation function  $K(x, y)$ , and variance  $\sigma^2 = K(0, 0)$ . The model is uniquely determined by the parameters  $H$ ,  $\sigma$ , and correlation function  $K(x, y)$ . The realizations of field (1) are constructed numerically using a modification of the method of spectrum randomization borrowed from Ref. 7, where it was used to model isotropic field, since at the first stage of our study it makes sense to restrict ourselves to an examination of isotropic cloudiness. The correlation function for this field is of the form  $K(x, y) = K(r) = \sigma^2 J_0(\rho r)$ , where  $r^2 = x^2 + y^2$ , and  $J_0$  is the Bessel function. In this case the model formula is written as

$$v(x, y) = \frac{\sigma}{\sqrt{I}} \sum_{i=1}^I \sqrt{-\ln(\alpha_i)} \cos((x \rho \cos \omega_i + y \rho \sin \omega_i) + 2\pi \beta_i), \quad (2)$$

where  $\rho$ ,  $\omega_i$  are the polar coordinates of spectrum points;  $\alpha_i$ ,  $\beta_i$  are independent random variables being uniformly distributed on the interval  $[0, 1]$ ; and,  $I = 10$  is the number of segments into which the spectral space is divided. This

model was employed in Ref. 2 to simulate cumulus cloud fields. Obviously, when  $\sigma \leq (1/3)H$ , field (1) can be used as a mathematical model of stratus clouds with random upper boundary. It is reasonable to characterize the mean vertical and horizontal extents of inhomogeneities with the rms deviation  $\sigma$  and correlation length  $r_c \approx 1.75/\rho$ , respectively.

**2. NUMERICAL RESULTS**

As has been noted above, radiation field nonlinearly depends on cloud optical and geometric characteristics, and the random geometry of stratus cloud upper boundary may have significant effect on solar radiative transfer. Let us consider factors governing the mean radiation fluxes. In contrast to a plane-parallel cloud layer, irregular upper boundary model has (Fig. 1):

- 1) on the average, optically thinner regions which, unless shadowed, transmit larger amount of direct radiation and, due to strongly forward-peaked scattering phase function, larger amount of scattered radiation;
- 2) on the average, optically thicker regions increasing the cloud albedo;
- 3) multiple re-reflections among cloud billows increasing the albedo.

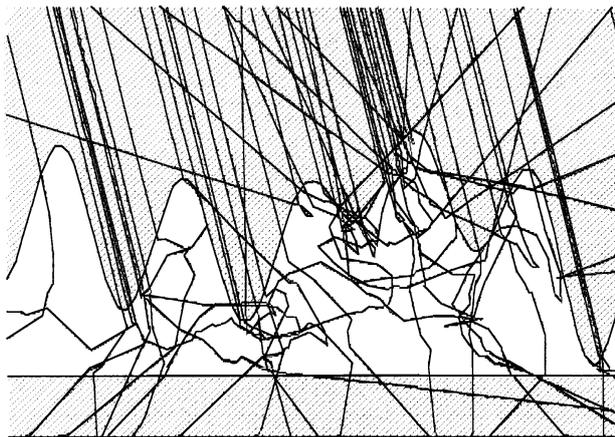


FIG. 1. Cross section through the cloudiness, constructed for model (1), by the vertical plane  $y = 0$ , and realizations of photon trajectories.

These are the main physical effects caused by the random geometry of cloud layer upper boundary and governing the radiative transfer. The last effect is determined by the solid angle within which one billow is seen from each point of the other billow; the larger is the angle, the higher is the probability for a photon to undergo additional scattering in nearby cloud billows. Thus, the larger is the ratio of the mean vertical to the mean horizontal extent of inhomogeneities, the stronger is the radiation interaction among individual cloud billows.

Below we give the results computed for the mean albedo  $\langle R \rangle$  as well as the mean direct  $\langle S \rangle$  and diffuse  $\langle Q \rangle$  transmitted radiation, with angular brackets denoting the ensemble averages over cloud field realizations. Clouds normally were assumed purely scattering media, the scattering phase function was for Deirmendjian's C1 cloud,<sup>11</sup> and the wavelength was  $\lambda = 0.69 \mu\text{m}$ . Scattering beyond the clouds and reflection from the underlying surface were not considered. Obviously, the limiting value  $\sigma = 0$  refers to the model of

a layer with plane-parallel boundaries. We denote by  $\delta R = (\langle R(0) \rangle - \langle R(\sigma) \rangle) / \langle R(\sigma) \rangle$  the relative deviation of the mean albedo  $\langle R(\sigma) \rangle$  from the mean albedo  $\langle R(0) \rangle$  of a plane-parallel cloud layer with mean thickness  $H$ . The same notation is used for the scattered transmitted radiation. The relative error in radiant flux computations was within 1%.

The larger is the variance  $\sigma^2$ , the smaller is the minimum geometric thickness  $H_{\text{min}}$  of cloud layer, and hence smaller is the optical thickness of the region localized around  $H_{\text{min}}$ . Due to strong forward peaking of the cloud scattering phase function, major portion of radiation interacting with this region will pass through the cloud layer. Therefore, the mean albedo will decrease while the mean transmission increase, in accordance with the results of computations shown in Fig. 2. Here and below  $\xi_{\oplus}$  is the solar zenith angle, and  $\Sigma$  is the extinction coefficient. As the correlation length decreases, an average "period" of fluctuations and the fraction of optically thin regions, on the average, increase. This is the reason for  $\langle R(\sigma) \rangle$  to decrease while for  $\langle Q_s(\sigma) \rangle$  to increase with  $r_c$  decrease. This variable behavior of  $\langle R(\sigma) \rangle$  and  $\langle Q_s(\sigma) \rangle$  attendant to variations in horizontal extents of inhomogeneities is in qualitative agreement with the results of Ref. 5 obtained for a plane-parallel cloud layer with horizontally variable coefficients of layer scattering and absorption.

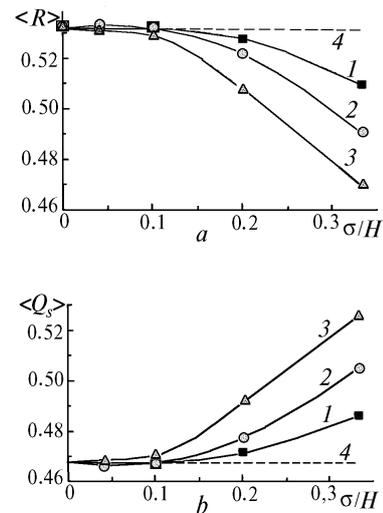


FIG. 2. The dependence of  $\langle R \rangle$  (a) and  $\langle Q_s \rangle$  (b) on the dimensionless parameter  $\sigma/H$  at  $\xi_{\oplus} = 0^\circ$ ,  $\Sigma = 30 \text{ km}^{-1}$ , and  $H = 0.5 \text{ km}$  for different correlation lengths  $r_c = 0.233$  (1),  $0.117$  (2), and  $0.05 \text{ km}$  (3). Curve 4 is for a plane layer.

Obviously, as  $\Sigma \rightarrow 0$  and  $\Sigma \rightarrow \infty$ , the mean fluxes become insensitive to the fluctuations in the upper boundary of cloud layer, with  $\delta R$  and  $\delta Q_s$  vanishing (Fig. 3). The mean albedo depends most strongly on the variance of the upper boundary at intermediate optical thickness  $\langle \tau \rangle \sim 20-30$ , when  $\delta R$  attains its maximum. When  $\Sigma \leq 120 \text{ km}^{-1}$ ,  $|\delta Q_s|$  monotonically increases with increasing  $\Sigma$  up to a maximum of  $\sim 30\%$ . The last fact is of prime importance when reflection from the underlying surface does not follow the Lambert law and depends strongly on the angular structure of an incident light.

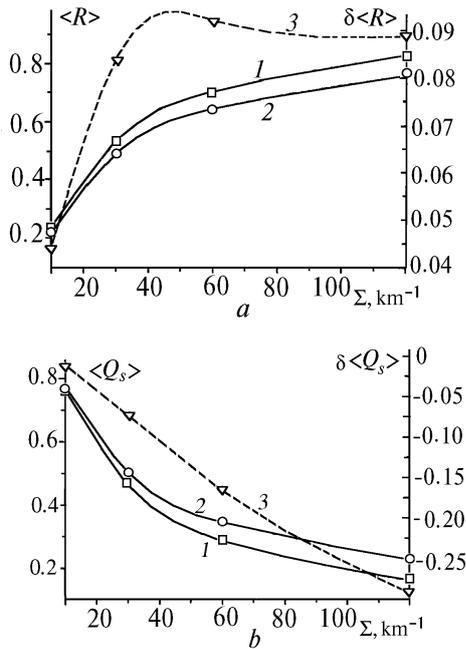


FIG. 3. The dependence of  $\langle R \rangle$  (a) and  $\langle Q_s \rangle$  (b) on the extinction coefficient at  $\xi_s = 0^\circ$ ,  $H = 0.5$  km,  $r_c = 0.117$  km, and  $\sigma/H = 0$  (1) and  $1/3$  (2). Dashed curves are for  $\delta R$  and  $\delta Q_s$  at  $\sigma/H = 1/3$ .

At larger solar zenith angle optically thinner regions are shadowed by surrounding cloud billows, thereby smoothing out the difference between the fluxes computed for a plane-parallel cloud layer and a layer with irregular upper boundary (Fig. 4). At  $\xi_s = 0^\circ$ ,  $\delta R$  and  $\delta Q_s$  are as great as 8–10%. At solar zenith angles larger than  $60^\circ$  these deviations are within the relative computational error ( $\leq 1\%$ ) and may be neglected.

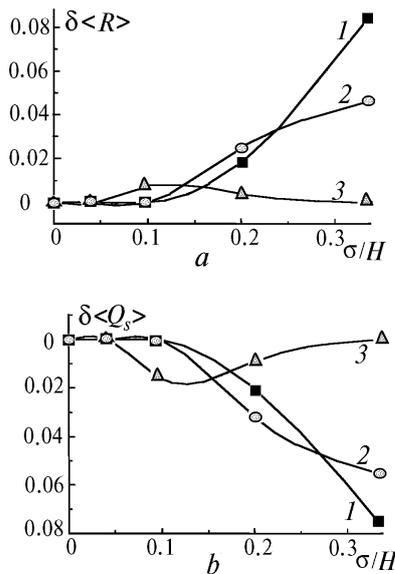


FIG. 4. Mean fluxes for  $\Sigma = 30 \text{ km}^{-1}$ ,  $H = 0.5$  km,  $r_c = 0.117$  km, and different solar zenith angles  $\xi_s = 0$  (1),  $30$  (2), and  $60^\circ$  (3).

As is well known a solution of the radiative transfer equation for a horizontally homogeneous plane layer of turbid medium off the absorption bands of atmospheric gases is invariant against the optical thickness. To preserve the invariance of cloud layers and photon trajectories for the case of a cloud layer with random upper boundary, the following parameters must be kept unchanged:

- mean optical thickness  $\langle \tau \rangle = \Sigma H$ ,
- ratio of the mean vertical to mean horizontal extent  $\sigma/r_c$ ,
- ratio  $\sigma/H$ .

With these parameters fixed, the mean fluxes practically coincide and differ by no more than the relative computational error (Fig. 5).

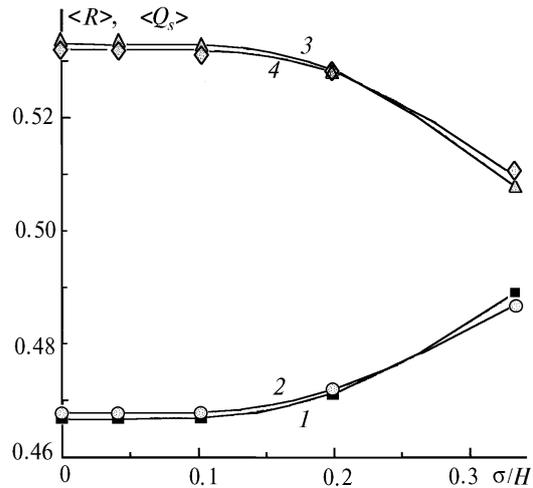


FIG. 5. Mean fluxes  $\langle Q_s \rangle$  (1, 2) and  $\langle R \rangle$  (3, 4) as functions of the dimensionless parameter  $\sigma/H$  at  $\xi_s = 0^\circ$ :  $\Sigma = 60 \text{ km}^{-1}$ ,  $H = 0.25$  km, and  $r_c = 0.117$  km (1 and 3);  $\Sigma = 30 \text{ km}^{-1}$ ,  $H = 0.5$  km, and  $r_c = 0.233$  km (2 and 4).

The requirements for the accuracy of solar flux computations become more stringent; therefore, the stochastic geometry of stratus cloud top must be considered when developing the models of radiation clouds as part of numerical models of the global circulation of the atmosphere. In this regard we note that the mean fluxes are most sensitive to cloud top fluctuations at small solar zenith angles, when a considerable portion of radiation may pass through optically thin regions located around a minimum optical thickness of stratus clouds.

**ACKNOWLEDGMENT**

This work was partly sponsored by the US Department of Energy (Grant No. NDE-FG02-91ER61128).

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