REFLECTION OF ELECTROMAGNETIC WAVE FROM AN OSCILLATING MIRROR

V.I. Reshetskii

Pacific Institute of Oceanography,
Far–Eastern Branch of the Russian Academy of Sciences, Vladivostok
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The problem on electromagnetic wave reflection from an oscillating mirror has been studied theoretically. An exact solution for the reflected wave is obtained. This solution represents an infinite superposition of plane monochromatic waves at combination frequencies with proper amplitudes and reflection angles. Some possible applications of this effect, in particular, to frequency conversion of optical laser radiation into the UV and X–ray ranges are discussed.

1. INTRODUCTION

As is well known,[1] the exact solution of the problem on the monochromatic wave reflection from a mirror moving at a constant velocity has made for the appearance of special theory of relativity. A short time later the relativistic Doppler effect caused by reflection of radiation from moving interface attracted attention of researchers in connection with its practical applications. So in 1952 Landecker treated the normal reflection of electromagnetic wave from the leading front of relativistic electrons propagating counter to this wave. He found the conditions of considerable increase in frequency and amplitude of a reflected wave at the relativistic velocities of interface movement.[2] At that time, however, the attempts to observe this effect experimentally failed due to insufficient electron density in a beam.

Lampert[3] called his attention to the fact that relativistic effects caused by wave reflection from the moving interface could be obtained without relativistic velocities. To do this, he offered to use decelerating systems in which phase velocity was considerably lower than vacuum velocity.[3]

Totaro[4] theoretically investigated reflection and refraction of waves from moving interface between two media. In Refs. 5–14 further investigations and generalization of the above–mentioned results were made.

These theoretical investigations were experimentally tested. The Doppler shift of frequency of electromagnetic waves reflected from the front of a shock wave propagating in argon was experimentally observed in Ref. 15. The estimated relative change of frequency equalled 10^{-3}%. The experimental technique allowed Hey et al.[15] to measure the propagation velocity of the shock wave with high accuracy in spite of so small frequency change caused by reflection. Zagorodnov et al.[16] studied reflection and frequency shift of electromagnetic wave caused by reflection from plasma front moving at a velocity of about 10^7 m/s. The experiment was carried out in a decelerating system in the form of a spiral waveguide in which the wave velocity was about 1/200 of the velocity of light in vacuum. The relative frequency shift was equal to about 20%. However, the relativistic increase of wave amplitude caused by reflection was not observed due to insufficient plasma density.

In a number of experiments the frequency change caused by multiple reflection of waves from moving plasma was observed. Linhart and Ornstein[17] measured the frequency increase due to multiple reflection of waves from approaching walls of vacuum cavity created in plasma. In the experiments performed by Zagorodnov et al.[18] the frequency increase by a factor of 2.3 was obtained due to multiple reflection from plasma piston moving at a velocity of $2\cdot10^7$ m/s. Consequently, the electromagnetic wave exhibited about a thousand of reflections.

Sixfold frequency increase and twofold energy increase were experimentally obtained in Ref. 19 due to wave reflection from the front of an electron beam with a current of 2 kA and electron energy of 1 MeV. Analogous results were obtained by Buzzi et al.[20]

From the above–listed experimental results one can see that experiments failed to obtain a large increase of frequency and amplitude of wave due to reflection from the interface moving at a constant velocity. It is connected with insufficiently high velocity of the interface or with low density of reflecting medium in the form of relativistic electron beams or plasma fluxes. Here we are pinning our hopes on the progress of high–current electron and plasma accelerators.[1]

The above–mentioned investigations were connected with uniform movement of the interface. The studies of wave reflection from irregulary moving interfaces are the natural generalization of these results and provide more effective frequency conversion. The case of wave reflection from an oscillating interface[21–23] is most interesting in this respect, since any movement of the interface can be always represented in the form of the Fourier integral or Fourier series. This case can be simply realized experimentally using an electroacoustic transducer as an oscillating interface. This problem is considerably more complicated in theoretical aspect, because it deals with the problems of mathematical physics with variable boundary conditions depending on time. For this reason only approximate solutions were obtained using different methods.[21–23] These solutions are true under certain sufficiently limited conditions. The second difficulty is connected with the application of relativity theory to noninertial reference systems[24] to which we affix the oscillating interfaces. Probably, this is one of the reasons why the fundamental results were obtained for acoustic waves. However, as was shown in Ref. 24, relativity theory can also be applied to the noninertial reference systems. The mathematical techniques for electrodynamics of moving media[1] are direct confirmation of this fact. They allow us to treat the boundary value problems for a single observer at rest.

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There is no need for the repeated use of the Lorentz transformations in going from one coordinate system to another.

2. PROBLEM FORMULATION

Let an electromagnetic wave with electric field strength

$$\mathbf{E} = \frac{1}{2}\,\mathbf{A}\exp[i(\kappa_x x + \kappa_z z - \omega t)] + \text{c.c.} \quad (1)$$

be incident on a mirror oscillating along its normal being parallel to the z axis

$$z = z_0 = D\sin(\Omega t + \varphi)\ , \quad (2)$$

where D is the amplitude of displacement of the mirror. The coordinate plane (x, z) coincides with the plane of the wave incidence. Proposed theory can be experimentally tested by spray of a specular layer on a piezoelectric transducer with the resonant frequency Ω. The given problem is most simply solved in the laboratory coordinate system in which expressions (1) and (2) have been written down. The boundary conditions in this coordinate system have the following form[1]:

$$[\mathbf{n},\ \mathbf{E} + \mathbf{E}^{(r)}] = \frac{v_n}{c}\,(\mathbf{B} + \mathbf{B}^{(r)})\ , \quad (3)$$

where $\mathbf{n}$ is the normal to the surface of the mirror; v_n is the instantaneous velocity of the mirror along its normal; c is the speed of light in vacuum; $\mathbf{B}$ is the magnetic field strength of incident wave; $\mathbf{E}^{(r)}$ and $\mathbf{B}^{(r)}$ are the electric and magnetic field strengths of the reflected wave, respectively. Obviously, refracted waves are absent and this is taken into account in boundary condition (3).

For brevity let us restrict our consideration to the case in which wave (1) is polarized perpendicularly to the incidence plane, e.g., to $\mathbf{E} = (0, E_y, 0)$. Then from Eq. (3) it follows

$$B_z + B_z^{(r)} = 0 \text{ at } z = z_0\ . \quad (4)$$

We note that boundary conditions (3) and (4) refer to the moving interface. However, the corresponding fields are treated in the laboratory coordinate system in which the reflected wave parameters are measured.

3. SOLUTION TO THE PROBLEM WITH TIME–DEPENDENT BOUNDARY CONDITIONS

To use boundary condition (4), we must define the magnetic field strength of incident wave. Using Maxwell's equation

$$\frac{\partial \mathbf{B}}{\partial t} = -\,c\ \text{rot}\mathbf{E}$$

and expression (1), we find

$$\mathbf{B} = \frac{1}{2}\left[\frac{c\mathbf{k}}{\omega},\ \mathbf{A}\right]\exp[i(\kappa_x x + \kappa_z z - \omega t)] + \text{c.c.}\ ,$$

By substituting Eq. (2) into this expression, we find the expression for magnetic field strength near the oscillating interface when the observer is at rest

$$\mathbf{B}(z_0) = \frac{1}{2}\left[\frac{c\mathbf{k}}{\omega},\ \mathbf{A}\right]e^{i\kappa_x x}\sum_{n=-\infty}^{\infty}\Gamma_n(\kappa_z D)\times$$

$$\times\exp[i(n(\Omega t + \varphi) - \omega t] + \text{c.c.}\ , \quad (5)$$

In the derivation we made use of the Fourier series expansion in the Bessel functions[25]

$$\exp(i\xi\sin\alpha) = \sum_{n=-\infty}^{\infty}\Gamma_n(\xi)\exp(i\,n\alpha)\ . \quad (6)$$

Here $\Gamma_n(\xi)$ is the nth order Bessel function of the first kind. It is seen from Eq. (5) that the field at the interface is described by the product of two periodic functions having two fundamental periods $2\pi/\omega$ and $2\pi/\Omega$. Consequently, in the general case the solution for the reflected wave must be found as a product of two periodic functions having the same fundamental periods $2\pi/\omega$ and $2\pi/\Omega$. Taking into account the above–mentioned, we represent the solution for the electric field strength of the reflected wave in the form

$$\mathbf{E}^{(r)} = \frac{1}{2}\sum_{m=-\infty}^{\infty}\mathbf{A}_m\exp\left[i\left(\kappa_x^{(m)}x - \kappa_z^{(m)}z - (\omega - m\Omega)\,t\right)\right] + \text{c.c.}\ , \quad (7)$$

where $\mathbf{A}_m$ and $\mathbf{k}^{(m)}$ are unknown values that must be determined. We note that certain of the amplitudes $\mathbf{A}_m$ may be equal to zero.

Using solution (7) and Maxwell's equations, we find the corresponding expression for the magnetic field strength of reflected wave

$$\mathbf{B}^{(r)} = \frac{1}{2}\sum_{m=-\infty}^{\infty}\frac{c[\mathbf{k}^{(m)},\ \mathbf{A}_m]}{\omega - m\Omega}\exp\left[i\,\kappa^{(m)}r - (\omega - m\Omega)\,t)\right] + \text{c.c.} \quad (8)$$

To make use of boundary condition (4), we determine the magnetic field at the interface, more precisely, near it. Substitution of Eq. (2) into Eq. (8) with allowance for expansion (6) yields

$$\mathbf{B}^{(r)}(z_0) = \sum_{m=-\infty}^{\infty}\ \sum_{p=-\infty}^{\infty}\frac{c[\mathbf{k}^{(m)},\ \mathbf{A}_m]}{2(\omega - m\Omega)}\,\Gamma_p\left(-\,\kappa_z^{(m)}D\right)\times$$

$$\times\exp\left[i\left(\kappa_x^{(m)}x + (m + p)\Omega t - \omega t + p\varphi\right)\right] + \text{c.c.}\ , \quad (9)$$

Further we make use of the technique of variable separation after substitution of Eqs. (5) and (9) into Eq. (4), e.g., equate the functions of the independent variables x and t. As a result, we obtain:

$$\kappa_x = \kappa_x^{(m)}\ ,\quad \sin\theta_m = \frac{\omega}{\omega - m\Omega}\sin\theta\ , \quad (10)$$

where θ is the incidence angle of wave (1) and θ_m is the reflection angle of combination wave denoted by the subscript m in solutions (7) and (8). From Eq. (10) it is seen that the reflection angle depends on the subscript m, e.g., each combination wave with the subscript m has its proper reflection angle which differs from the other. Consequently, Snell's reflection law is violated in the case of reflection from the oscillating interface. The singular case $\omega - m\Omega = 0$ is not difficult because the corresponding amplitude of combination wave is equal to zero, as is seen from further treatment.

By equating the functions of the independent variable t, we obtain

$$\Gamma_n(\kappa_z D) = -\sum_{m=-\infty}^{\infty} \frac{\omega[\mathbf{k}^{(m)}, \mathbf{A}_m]_z \exp(-im\varphi)}{[\mathbf{k}, \mathbf{A}]_z (\omega - m\Omega)} \Gamma_{n-m}\left(-\kappa_z^{(m)} D\right). \quad (11)$$

In the derivation of this equation the necessary equality for exponential coefficients of the Fourier basis $n = m + p$ was taken into account.

Expression (11) represents the infinite system of equations for unknown amplitudes $\mathbf{A}_m$. To solve this system, we make use of the addition theorem for the Bessel functions,[25] according to which

$$\Gamma_n(a + b) = \sum_{m=-\infty}^{\infty} \Gamma_m(a)\, \Gamma_{n-m}(b) .$$

The similarity of equation (11) to this theorem becomes obvious if we take

$$-\frac{\omega[\mathbf{k}^{(m)}, \mathbf{A}_m]_z \exp(-im\varphi)}{[\mathbf{k}, \mathbf{A}]_z (\omega - m\Omega)} = \Gamma_m\left(\kappa_z D + \kappa_z^{(m)} D\right),$$

from which we find the solution for the amplitudes

$$A_y^{(m)} = -\exp(im\varphi)\, \Gamma_m\left(\kappa_z D + \kappa_z^{(m)} D\right) \frac{\omega - m\Omega}{\omega} A_y . \quad (12)$$

From Eq. (12) it is seen that really $A_m = 0$ when $\omega - m\Omega = 0$ as was mentioned above.

The reflected waves must satisfy not only to the boundary conditions but also to the corresponding dispersion relations in vacuum

$$\frac{(\omega - m\Omega)^2}{c^2} = \left(\kappa_x^{(m)}\right)^2 + \left(\kappa_z^{(m)}\right)^2 ,$$

where the component of the wave vector $\kappa_z^{(m)}$ is unknown. From the dispersion relations we find

$$\kappa_z^{(m)} = \frac{\Omega}{c}\left(m^2 - 2\frac{\omega}{\Omega} m + \frac{\omega^2 \cos^2\theta}{\Omega^2}\right)^{\frac{1}{2}} . \quad (13)$$

Treating the radicand in the right side of Eq. (13) as a quadratic equation for m, we find its roots

$$m_1 = \frac{\omega}{\Omega}(1 - \sin\theta) , \; m_2 = \frac{\omega}{\Omega}(1 + \sin\theta) . \quad (14)$$

It is clear that the attenuating combination waves correspond to $m_1 < m < m_2$ and propagating waves correspond to $m < m_1$ and $m > m_2$. We note that two waves propagating along the interface in the opposite directions for which $\sin\theta_{m_1} = 1$ and $\sin\theta_{m_2} = -1$ correspond to $m = m_1$ and $m = m_2$ in accordance with Eq. (9).

Obviously, $\cos\theta_m$ is transformed into hyperbolic cosine which vanishes at infinity in the case of attenuating waves.

By substituting the amplitudes given by Eq. (12) into Eqs. (7) and (8), we obtain the exact solution for reflected wave in the form of linear superposition of the combination waves

$$\mathbf{E}^{(r)} = -\frac{1}{2}\sum_{m=-\infty}^{\infty} \exp(im\varphi)\, \Gamma_m\left(\kappa_z D + \kappa_z^{(m)} D\right) \frac{\omega_m}{\omega} \overline{\mathbf{A}} \times$$

$$\times \exp\left[i\left(\kappa_x^{(m)} x - \kappa_z^{(m)} z - \omega_m t\right)\right] + \text{c.c.} , \quad (15)$$

where $\omega_m = \omega - m\Omega$ is the frequency of corresponding combination wave. We also obtain

$$\mathbf{B}^{(r)} = -\frac{1}{2}\sum_{m=-\infty}^{\infty} \exp(im\varphi)\, \Gamma_m\left(\kappa_z D + \kappa_z^{(m)} D\right) \frac{c}{\omega} [\mathbf{k}^{(m)}, \overline{\mathbf{A}}] \times$$

$$\times \exp\left[i\left(\kappa_x^{(m)} x - \kappa_z^{(m)} z - \omega_m t\right)\right] + \text{c.c.} , \quad (16)$$

It is easy to verify that the specific case of reflection from the immobile mirror follows from the obtained solution.

4. ANALYSIS AND DISCUSSION OF THE OBTAINED RESULTS

Solutions (15) and (16) can be simplified in a number of cases which are encountered in practice. For convenience we write down a single combination wave from solution (15) in general form

$$\mathbf{E}^{(m)} = -\frac{\exp(im\varphi)}{2} \Gamma_m\left(\kappa_z D + \kappa_z^{(m)} D\right) \frac{\omega_m}{\omega} \mathbf{A} \times$$

$$\times \exp\left[i\omega_m\left(\frac{x}{c}\sin\theta_m - \frac{z}{c}\cos\theta_m - t\right)\right] + \text{c.c.} \quad (17)$$

It should be noted that the argument of the Bessel function in Eq. (17) depends on its order

$$\Gamma_m\left(\kappa_z D + \kappa_z^{(m)} D\right) = \Gamma_m\left(2\kappa_z D - m\frac{\Omega D}{c}\cos\theta_m\right) , \quad (18)$$

where we took into account that $\kappa_z^{(m)} = \omega_m \cos\theta_m / c = (\omega - m\Omega)\cos\theta_m / c$.

Further taking into account that $\Omega D = v$ is the amplitude of the interface velocity, expression (18) assumes the form

$$\Gamma_m\left(\kappa_z D + \kappa_z^{(m)} D\right) = \Gamma_m\left(2\alpha\cos\theta - m\beta\cos\theta_m\right) , \quad (19)$$

where $\alpha = \omega D/c$ and $\beta = v/c = \Omega D/c$. We note that $\beta \le 1$ always, whereas α can be as large as is wished since the amplitude of displacement D is unlimited, if we keep in mind the low–frequency oscillations of the interface. As is well known, the mth order Bessel function has maximum when its argument is of the same order. This allows us to control the spectral composition of the combination waves through change of the amplitude D or the incidence angle θ. Using the identities[25]

$$\Gamma_{-n}(\alpha) \equiv (-1)^n\, \Gamma_n(\alpha) \equiv \Gamma_n(-\alpha) ,$$

we analyze expression (19). For $m < 0$ we have

$$\Gamma_m\left(\kappa_z D + \kappa_z^{(m)} D\right) = (-1)^m\, \Gamma_{|m|}(2\alpha\cos\theta + |m|\beta\cos\theta_m). \quad (20)$$

For $m > 0$ two cases are possible. In the first case the argument of the Bessel function remains positive for $m < 2\omega\cos\theta/\Omega\cos\theta_m$. In the second case for $m > 2\omega\cos\theta/\Omega\cos\theta_m$ the argument becomes negative. Consequently, the Bessel functions are asymmetric with respect to the change in sign of the function order m in Eq. (19). This results in the increase of the spectral power density of high–frequency components. The last circumstance favours the conversion of laser radiation frequency into the UV and X–ray ranges. As this takes place, the spatiotemporal coherence conserves.

By way of example, let us consider the possible increase of the laser radiation frequency by an order of magnitude due to reflection from the oscillating mirror. Let the oscillation frequency Ω be equal to the frequency $\omega = 3\cdot 10^{14}$ s^{-1} of laser radiation incident on the oscillating mirror. Oscillations at so high optical frequency can be excited by the second laser with the same characteristics. Laser beam is incident on a piezoelectric film transducer whose second surface is specular. We note that the excitation of hypersonic waves is not required, the oscillations of specular surface are sufficient. We are interested in the combination wave at the frequency $\omega_{-9} = \omega + 9\omega = 10\omega$. For simplicity, we restrict our consideration to the case of normal incidence $\theta = 0$. Then $\theta_m = 0$ for any m. From tables published in Ref. 25 we find the maximum of the ninth order Bessel function. It is equal to 0.31 when the argument varies in the range 10.6–10.8, i.e., $\Gamma_9(10.8) = 0.31$.

According to Eq. (20) we have

$$\Gamma_{-9}(\kappa D + \kappa^{(m)}D) = (-1)^9\,\Gamma_9(2\omega D/c + 9\Omega D/c) = -\,\Gamma_9(11\omega D/c).$$

Substituting in this relation $c = 3\cdot 10^{10}$ cm/s and $\omega = 3\cdot 10^{14}$ s^{-1}, we obtain $\Gamma_9(11\omega D/c) = \Gamma_9(1.1\cdot 10^5 D) = 0.31$. This is possible only when $1.1\cdot 10^5 D = 10.8$. That is, we need the amplitude of displacement being equal to $9.8\cdot 10^{-5}$ cm. We note that in practice the higher is the oscillations frequency, the more difficult is to obtain sufficiently large amplitude of displacement. Thus it turns out that the frequencies ω and Ω must be best decreased while the ratio of the frequencies must be increased resulting in sufficiently intense combination wave at the frequency $m\omega$. For instance,[26] the Bessel function $\Gamma_{16}(18) \simeq 0.26$ differs slightly in magnitude from $\Gamma_9(10.8) \simeq 0.31$, that is, the maxima of the Bessel functions vary slowly with increase of their order. We find for this case using Eq. (19)

$$\Gamma_{-16}(2\omega\, D/c + 16\Omega\, D/c) = \Gamma_{16}[D(2\omega + 16\omega)/c] =$$

$$= \Gamma_{16}(18\omega\, D/c) = 0.26 = \Gamma_{16}(18)\,,$$

where $\omega_{-16} = \omega + 16\omega = 17\omega$, from which we find $D = c/\omega = 10^{-4}$ cm.

It is seen from the comparison of these two numerical examples that in the first case the amplitude of displacement $D = 9.8\cdot 10^{-5}$ cm is required for tenfold increase of the frequency. In the second case the small increase of the amplitude of displacement up to 10^{-4} cm is required for 17–fold increase of the frequency. Using solutions (15) and (16), we determine the efficiency of wave conversion due to reflection from the oscillating interface in terms of the ratio of the intensities of combination waves to the intensity of the incident wave

$$R_m \equiv \frac{I_m}{I} = \left[\frac{\omega - m\Omega}{\omega}\,\Gamma_m\!\left(\kappa_z D + \kappa_z^{(m)} D\right)\right]^2 =$$

$$= \left(1 - m\,\frac{\Omega}{\omega}\right)^2 \Gamma_m^2\!\left(\kappa_z D + \kappa_z^{(m)} D\right),$$

where I is the intensity of the incident wave and I_m is the intensity of the combination wave of the mth order. We have $R_{-9} = 100\cdot 0.31^2 = 9.61$ and $R_{-16} = (1 + 16)^2\cdot 0.26^2 = 41$ for the aforementioned examples, i.e., the intensities of these combination waves exceed the intensity of the incident wave. We note that increase of the intensity was also observed in the case of reflection of electromagnetic waves from the mirror moving with constant velocity in the direction counter to the direction of wave propagation.[1] However, as was noted in Introduction, this gain was insignificant due to small velocities. Obviously, the increase of wave intensity in the case of reflection from the moving mirror is caused by the work done by the mirror on the wave under radiation pressure.[21]

Let us make several comments about the relation between the perfection of the mirror surface and ultrahigh frequency. As is well known, the principal difficulties emerge as the wavelength decreases when we try to manufacture the perfect mirror surface intended for reflection of such waves. However, in the given case such a problem does not emerge since the real Doppler shift of the frequency is small: $\Delta\omega = \omega\Omega D/c \sim \omega$.

The foregoing examples are of theoretical interest rather than of practical one, since the amplitude of the velocity close to the speed of light is required to realize them. However the high efficiency of wave conversion due to the coefficient $(1 - m\Omega/\omega)^2$ provides a means for obtaining the waves at ultrahigh frequencies for $m = 10^2$, 10^3, 10^4, etc, when the amplitude is not large ($\Omega\, D \ll c$). But the effective speed $m\Omega\, D$ can be always increased due to the factor m in Eq. (19) so that it becomes close in value to the speed of light and can even exceed it. Taking into account that the mth order Bessel function reaches extremum when its argument becomes comparable to its order m, we find using Eq. (19)

$$|m| \simeq \frac{2\omega D}{c - \Omega D}\,. \tag{21}$$

Since the argument of the Bessel function given by Eq. (19) depends on its order, in the general case we must use the addition theorem for Eq. (19) in the form

$$\Gamma_n(2\,\alpha\cos\theta - n\,\beta\cos\theta_n) = \sum_{m=-\infty}^{\infty} \Gamma_m(2\,\alpha\cos\theta)\,\times$$

$$\times\Gamma_{n-m}(n\,\beta\cos\theta_n)\,(-1)^{n-m}\,. \tag{22}$$

After this the principal terms of asymptotic formulas[25] can be used as $n \to \infty$ when the other variables are fixed

$$\Gamma_n(n\,\mathrm{sech}\gamma) \sim \frac{\exp[n(\tanh\gamma - \gamma)]}{\sqrt{2\pi n \tan\eta\gamma}}\,,\ \Gamma_n(\gamma) \sim \frac{1}{\sqrt{2\pi n}}\left(\frac{e\gamma}{2n}\right)^n.$$

For the fixed order of the function as $2\alpha\cos\theta \to \infty$ the asymptote is valid

$$\Gamma_m(2\,\alpha\cos\theta) \sim \sqrt{\frac{2}{2\pi\alpha\cos\theta}}\cos\left(2\,\alpha\cos\theta - \frac{1}{2}\,m\pi - \frac{1}{4}\,\pi\right). \quad (23)$$

For small arguments $|2\alpha\cos\theta - n\beta\cos\theta_n|$ we have

$$\Gamma_n(2\,\alpha\cos\theta - n\,\beta\cos\theta_n) \approx \frac{(2\,\alpha\cos\theta - n\,\beta\cos\theta_n)^n}{2^n\,n!}\,. \quad (24)$$

This approximation is valid only for sufficiently small order n.

One important comment about the interpretation of the obtained solution and correct choice of the boundary conditions at infinity should be made. As is seen from obtained solution (15), the value of the summation index $m \geq \omega/\Omega$ can be always found, starting from which the combination wave frequencies become "negative". Obviously, in these cases every expression for combination waves in the form of Eq. (17) must be represented in the form

$$\overline{E}^{(n)} = \frac{+e^{in\varphi}}{2}\,\Gamma_n^*\left(\kappa_z D + \kappa_z^{(n)} D\right)\frac{\omega_{-n}}{\omega}\,\overline{A}^* \times$$
$$\times \exp\left[i\omega_{-n}\left(\frac{x}{c}\sin\theta_n - \frac{z}{c}\cos\theta_n - t\right)\right] + \text{c.c.}\,,$$

where the complex conjugate terms are transformed. The kinematics of propagation of these waves corresponds to the effective reversion of time. In particular, it is not difficult to find the conditions of wavefront conjugation and reversion of the dynamics of dispersive coherent pulse.

5. CONCLUSION

The physical pattern of reflection resembles Raman scattering described phenomenologically by consideration of the nonlinear terms. In our case the frequency change is caused by the Doppler shift, whereas the physical mechanism of nonlinear interaction remains unknown. It should be noted that nonlinear interactions of waves manifest itself most effectively in piezoelectrics $LiNbO_3$, in which the Doppler mechanism of nonlinearity contributes undoubtedly.

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