AN ALGORITHM OF PHASE FRONT RECONSTRUCTION OF AN INPUT BEAM FROM MEASURED INTENSITY OF ITS FOURIER TRANSFORM

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An algorithm of phase front reconstruction of an input beam with nonuniform intensity distribution over the beam aperture based on the analysis of Fourier spectra of a multiply amplitude–transformed optical field is derived.

In recent years an increased interest to the problems in reconstructing the phase characteristics of optical fields, in particular, the problems on reconstructing the field phase from the absolute value of its Fourier spectrum has been observed. In the two–dimensional case the problem has a wide practical application to the systems of adaptive optics with an atmospheric channel of radiation propagation.

The available iteration algorithms of reconstructing[1,2] possess very slow convergence. The rate of convergence, and sometimes the convergence of these algorithms itself, strongly depends on selection of the initial approximation for the image spectrum phase. Even for the algorithm[3] which takes into account the analytical relation between the amplitude and phase when forming the initial approximation the number of iterations for satisfactory reconstruction of the initial field is about several tens. It is clear that in this case realization of such algorithms in real time is too problematic.

The algorithms of reconstructing phase characteristics from measurements of intensities attributed to two and more field cross sections are quite interesting for practical applications.[1,4] However, if the measurement results are the values of partial derivatives of the phase distribution at some points of a receiving aperture, as is the case, e.g., in Refs. 4 and 5, then the time spent on the field phase reconstruction cannot be much shorter than that of the iteration algorithms.

In this connection, there arises the problem on searching alternatives for processing of the results of intensity measurements that would permit direct calculations of discrete values of the field phase. One of such methods is described in this paper.

Let us introduce designations for the complex amplitude distribution over the aperture of input optical beam

$$F(x, y) = A(x, y) \exp(-j\,\varphi(x, y)), \qquad (1)$$

where $A(*)$ and $\varphi(*)$ are distributions of the amplitude and phase, and for its image in the focal plane of a lens

$$G(u, \upsilon) = B(u, \upsilon) \exp(-j\,\xi(u, \upsilon)), \qquad (2)$$

where $B(*)$ and $\xi(*)$ are distributions of amplitude and phase.

Using known functional relationship between $F(x, y)$ and $G(u, \upsilon)$ it is possible to write

$$B(u, \upsilon) = \int\!\!\int_S A(x, y) \exp[-j\{\varphi(x, y) + (ux + \upsilon y) + \xi(u, \upsilon)\}]\, dx\, dy, \qquad (3)$$

where S is the area of the entrance aperture. Now we make formal differentiation of the right and left sides of expression (3) with respect to the parameter S. The result is

$$\frac{\partial B(u, \upsilon, x_i, y_i)}{\partial S} = A(x_i, y_i) \exp[-j\{\varphi(x_i, y_i) + (ux_i + \upsilon y_i) + \xi(u, \upsilon)\}], \qquad (4)$$

where x_i and y_i are the coordinates of an elementary area. By representing the left side of Eq. (4) in the form corresponding to the difference scheme and rejecting the imaginary terms we have

$$\frac{B(u, \upsilon) - B(u, \upsilon, x_i, \Delta x_i, y_i, \Delta y_i)}{\Delta S} = A(x_i, y_i) \cos[\varphi(x_i, y_i) + (ux_i + \upsilon y_i) + \xi(u, \upsilon)], \qquad (5)$$

where $\Delta S = \Delta x_i \Delta y_i$, Δx_i and Δy_i are the dimensions of an area element at a point with the coordinates x_i and y_i, $B(u, \upsilon, x_i, \Delta x_i, y_i, \Delta y_i)$ is the amplitude distribution over the lens focal plane provided that there is an opaque mask of the size Δx_i by Δy_i in the lens aperture at the point x_i and y_i.

In order to pass from amplitudes to intensities we have to multiply both parts of expression (5) by $\{B(u, \upsilon t) + B(u, \upsilon, x_i, \Delta x_i, y_i, \Delta y_i)\}$, to introduce the designations

$$[B(u, \upsilon)]^2 = I(u, \upsilon); \qquad (6)$$

$$[B(u, t, x_i, \Delta x_i, y_i, \Delta y_i)]^2 = I(u, t, x_i, \Delta x_i, y_i, \Delta y_i);$$

$$[A(x_i, y_i)]^2 = I(x_i, y_i), \qquad (7)$$

and to assume that

$$B(u, \upsilon) + B(u, \upsilon, x_i, \Delta x_i, y_i, \Delta y_i) \approx 2\,B(u, \upsilon)$$

because of small difference between these amplitudes when the condition that ΔS values are small is valid.

As a result one obtains

$$\frac{I(u, \upsilon) - I(u, \upsilon, x_i, \Delta x_i, y_i, \Delta y_i)}{\Delta S} = 2[I(x_i, y_i)\, I(u, \upsilon)]^{1/2} \cos[\varphi(x_i, y_i) + (ux_i + \upsilon y_i) + \xi(u, \upsilon)]. \qquad (8)$$

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From Eq. (8) one easily obtains the expression for the field phase at the points x_i and y_i

$$\varphi(x_i, y_i) = \arccos\left\{\frac{I(u, \upsilon) - I(u, \upsilon, x_i, \Delta x_i, y_i, \Delta y_i)}{2\Delta S\,[I(x_i, y_i)\,I(u, \upsilon)]^{1/2}}\right\} -$$

$$- (ux_i + \upsilon y_i) - \xi(u, \upsilon)\,. \tag{9}$$

In view of an ambiguity produced by the second and the third terms of this expression it cannot be used for calculations. To resolve this problem let us integrate both sides of Eq. (9) over the variables u and t taking into account symmetry of the measuring scheme. It is obvious that the integral of the second term of the right side of expression (9) equals zero and the integration of the last term yields a constant.

It should be noted that in the phase measurements the reference point can be chosen arbitrary since only the relative state of phases is meaningful in this case. Hence the calculational relation in a continuous form is

$$\varphi(x_i, y_i) = \frac{1}{4\,u_1\,\upsilon_1} \times$$

$$\times \int\limits_{-u_1}^{u_1}\int\limits_{-\upsilon_1}^{\upsilon_1} \arccos\left\{\frac{I(u, \upsilon) - I(u, \upsilon, x_i, \Delta x_i, y_i, \Delta y_i)}{2\Delta S\,[I(x_i, y_i)\,I(u, \upsilon)]^{1/2}}\right\} \mathrm{d}u\,\mathrm{d}\upsilon. \tag{10}$$

As a result the procedure of measuring the field phase is as follows:

– First the intensity distribution of the reference field spectrum $I(u, \upsilon)$ is recorded.

– Then the area of the entrance aperture is divided into N area elements ΔS with the centers assigned by the coordinates x_i, y_i, $i = \overline{1, N}$, and the field intensity incident on each of the area elements $I(x_i, y_i)$ is measured.

– Opaque masks are successively placed onto each of the area elements and the intensities $I(u, \upsilon, x_i, \Delta x_i, y_i, \Delta y_i)$ are recorded in the focal plane of the lens.

– Unknown values of phases are calculated by formula (8).

The aforementioned steps can be performed either successively in time or simultaneously provided that the optical beams are coming in parallel.

Example. Consider the simplest case when the amplitude distribution of the input beam is uniform and the phase amplitude is given by the slope $L = (2\pi/\lambda)\,(\Delta\lambda/a)$, i.e.,

$$F(x) = A\,\exp\,(-\,jL\,x)\,. \tag{11}$$

The image in the focal plane of the lens has the form

$$I(u) = \frac{4\,A^2}{(u + L)^2}\left|\sin\frac{(u + L)\,a}{2}\right|^2, \tag{12}$$

where a is the size of the aperture.

Since the intensity $I(u)$ is assigned analytically, it is senseless to pass to a difference scheme in this case. It is quite sufficient to differentiate Eq. (10) with respect to the parameter a making it a variable, i.e.,

$$\left.\frac{\mathrm{d}I(u, x)}{\mathrm{d}x}\right|_{x = x_i} = \frac{4\,A^2}{(u + L)}\left|\sin\frac{(u + L)\,x_i}{2}\cos\frac{(u + L)\,x_i}{2}\right|. \tag{13}$$

By substituting the derived expression into Eq. (8) and making simple transformations we obtain

$$\varphi(x_i) = \frac{1}{2\,u_1}\int\limits_{-u_1}^{u_1} \arccos\left\{\left|\frac{\sin\frac{(u + L)\,x_i}{2}\cos\frac{(u + L)\,x_i}{2}}{\sin\frac{(u + L)\,a}{2}}\right|\right\}\mathrm{d}u.$$

(14)

It is clear that for $x_i = a$, after making cancellations and integration, we have

$$\varphi(a) = \pm\,\frac{aL}{2}\,. \tag{15}$$

A plot of $\varphi(x_i)$ with the other values of x_i is shown in the figure.

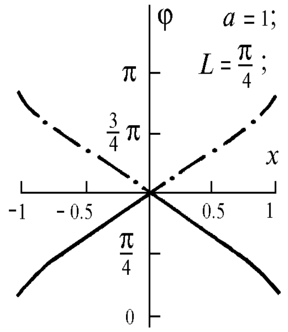

FIG. 1. Reconstructed phase distribution.

CONCLUSIONS

Thus the proposed algorithm, in contrast to the known ones, is insensitive to nonuniformity of the distribution of a beam intensity over the entrance aperture. Its practical performance in real time could be done using optical transparencies. It is obvious that the example considered in the paper is too simple. However it enables one to show the sequence of operations during realization of the developed algorithm. We have obtained the above results without taking into account the size of the mask applied to the entrance aperture and without recording the background. These are the problems for further studies.

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