THE EFFECT OF SEA SURFACE ROUGHNESS ON THE IMAGE QUALITY OF AN UNDERWATER OBJECT FOR OBSERVATIONS THROUGH THE SEA SURFACE

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The effect of the wind–driven sea waves on the optical transfer function (OTF) and point spread function (PSF) of an optical system of observations through the air–water interface is analyzed theoretically. It is shown that with directional illumination the effects of radiation double passage through a random sea surface lead to specific distortions in the form of the point spread function in comparison with diffuse illumination.

The image of an underwater object obtained through the sea surface is random due to a random character of water roughness at the air–water interface. When this image is statistically averaged over realizations of the interface (when an optical signal is integrated over a long period of time) it is possible to obtain a regular image whose quality is determined by an optical transfer function (OTF) or a point spread function (PSF) of the image transfer channel including a randomly rough interface and a scattering water column. When the object is illuminated with a diffuse source (with scattered light of the sky), the through OTF of the image transfer channel is found by multiplying the optical transfer functions of the rough sea surface, scattering layer of the water, and receiver of the viewing system (VS).[1–3] When illumination is directional (the sun or artificial light source), the situation is much more complicated and interesting. Due to the effects of double passage of radiation through a rough sea surface, the through OTF acquires an integrated character and the PSF becomes asymmetric in contrast to the diffuse illumination.

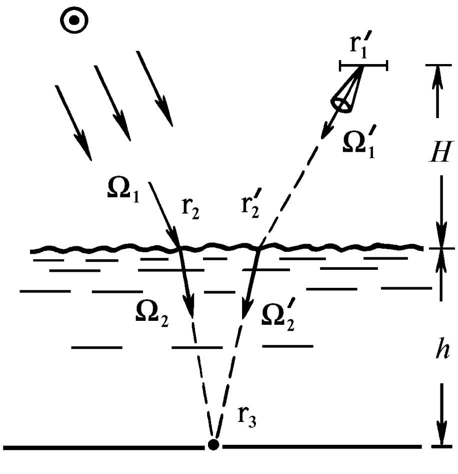

FIG. 1. Viewing geometry.

Some features of the OTF with directional illumination were studied elsewhere.[4] The aim of the present work is to analyze the PSF properties with solar illumination. The solution of this problem is based on geometric optical treatment of light propagation through the sea water with the use of a small–angle solution of the radiative transfer equation in turbid media. Let us consider the viewing geometry (Fig. 1). The sea surface specified by the function $\mathbf{q}(\mathbf{r}_2)$ ($\mathbf{q}$ is the vector gradient of the sea surface) is illuminated with a broad beam of solar rays. A diffusely reflecting surface with the reflectance distribution $R_0(\mathbf{r}_3)$ is located at the depth h. At the altitude H above the sea surface there is a photodetector of the viewing system (VS) oriented in the nadir.

The expression was derived in Ref. 5 for random realization of light power in a pixel (an instantaneous image was considered there)

$$P = P_0 \int_{-\infty}^{\infty}\int R_0(\mathbf{r}_3) E_s(\mathbf{r}_3) E_r(\mathbf{r}_3)\, d\mathbf{r}_3 , \qquad (1)$$

where P_0 is the light power received from the object surface with a uniform reflectance, E_s is the distribution of solar illumination at the depth h, E_r is the distribution of illumination from a unit–power source with the directional pattern identical to that of the receiver in the z_3 plane.

The relations for $E_{s,\,r}$ have the form

$$E_s(\mathbf{r}_3) = m^2 \int_{-\infty}^{\infty} \cdots \int D_s(\Omega_1 - \Omega_s) \times$$

$$\times\, \delta[\Omega_1 - m\Omega_2 - (m-1)\mathbf{q}(\mathbf{r}_2)]\, e_p(\mathbf{r}_3 - \mathbf{r}_2 - h\Omega_2)\, d\mathbf{r}_2 d\Omega_1 d\Omega_2 ;$$

$$E_r(\mathbf{r}_3) = m^2 \int_{-\infty}^{\infty} \cdots \int D_r(\Omega_1' - \Omega_r)\, \delta[\Omega_1' - m\Omega_2' -$$

$$-(m-1)\mathbf{q}(\mathbf{r}_2')]\, \delta(\mathbf{r}_r - \mathbf{r}_2' + H\Omega_1')\, e_p(\mathbf{r}_3 - \mathbf{r}_2' - h\Omega_2') d\mathbf{r}_2' d\Omega_1' d\Omega_2', \qquad (2)$$

where $D_{s,\,r}$ are the directional patterns of the light source and the receiver of the VS, $\int\int D_{s,\,r}(\Omega) d\Omega = 1$, e_p is the distribution of illumination from a single–point unidirectional source of unit power at the distance h from it in a scattering medium, $\mathbf{r}_r$ is the coordinate of the center of the VS receiving aperture, $\Omega_{s,\,r}$ are the projections of unit vectors collinear with the axes of the directional patterns of the source and receiver to the plane z = const, and $m = 1.33$ is the refractive index of water.

By averaging Eq. (1) over realizations of the sea surface slopes on account of Eq. (2) after some transformations we obtain the expression for brightness distribution of the regular image of the underwater object

$$P(\mathbf{r}) = P_0(2\pi)^{-2} \int_{-\infty}^{\infty}\int F_0(\mathbf{k})\, \Phi(\mathbf{k};\, \Delta\rho)\, e^{i\mathbf{kr}} d\mathbf{k}\ , \qquad (3)$$

where F_0 is the Fourier transform of the object R_0, Φ is the OTF of the channel of image transfer, $\mathbf{r} = \mathbf{r}_r + L\mathbf{\Omega}_r$ is the coordinate of the observation point in the plane of the object, and $L = H + \frac{h}{m}$.

The expression for the OTF entering into Eq. (3) has the form

$$\Phi(\mathbf{k};\, \Delta\rho) = (2\pi)^{-2} \int_{-\infty}^{\infty} \cdots \int F_s(h_0\, \omega) F_r(h_0\, \omega + L\mathbf{k}) F_1(\omega)\, \times$$

$$\times\, F_1(\omega + \mathbf{k})\, \Theta_2(a\omega;\, -a(\omega + \mathbf{k});\, \rho)\, e^{i\omega(\rho - \Delta\rho)} d\omega d\rho\ , \qquad (4)$$

where $F_{s,\, r}$ are the Fourier transforms of the aperture functions of the source and receiver $D_{s,\, r}$, F_1 is the optical transfer function of a water layer, Θ_2 is a two–point characteristic probability function of the rough sea surface slopes, $\Delta\rho = h_0(\mathbf{\Omega}_s - \mathbf{\Omega}_r)$, $a = h_0(m - 1)$, and $h_0 = h/m$.

It follows from relation (3) that the image can be formed by scanning the object either in space ($\mathbf{r}_r$ = var) or angle ($\mathbf{\Omega}_r$ = var). Below we deal with the first method of forming the image.

Disregarding the correlations of the light beams entering the water and emanating from the water column, relation (4) is reduced to the form

$$\Phi(\mathbf{k}) = F_1(\mathbf{k})\Theta_1(a\mathbf{k})F_r(L\mathbf{k})\ , \qquad (5)$$

where Θ_1 is the single–point characteristic probability function of the sea surface slopes (or frequency–contrast characteristic of the rough surface).[2,3]

The OTF with diffuse illumination of the sea surface is described in the same way (it follows from Eq. (4) when $F_s(\cdot) \sim \delta(\cdot)$).

The subsequent analysis of the OTF requires that the functions entering into Eq. (4) be specified. Let us assign the OTF of the source and receiver in the Gaussian form

$$F_{s,\, r}(\mathbf{p}) = \exp\left(-\frac{\Delta_{s,\, r}}{4\pi}\, p^2\right), \qquad (6)$$

where $\Delta_{s,\, r}$ are the solid angles of emission and reception (of a pixel).

To simplify the problem we restrict ourselves to one–dimensional wind–driven sea waves. The expression for a two–point characteristic function of the rough sea surface has the form[2]

$$\Theta_2(\mathbf{k}_1;\, \mathbf{k}_2;\, \rho) = \exp\left[-\frac{\sigma_q^2}{2}\left(k_{1x}^2 + k_{2x}^2 + 2R_q(\rho_x)\, k_{1x}\, k_{2x}\right)\right], \quad (7)$$

where σ_q^2 is the variance of slopes, R_q is the normalized correlation function of the sea surface slopes.

Since the main purpose of this paper is to estimate the effect of roughness on the image quality, we neglect scattering of optical radiation in the depth of water (though it is not so important), i.e., we set $F_1(\cdot) \equiv 1$.

Relations (4), (6), and (7) can be analyzed by numerical integration but we want to derive the basic results in an analytical form. To this end, the characteristic function of slopes is approximated by the following dependence:

$$\Theta_2(a\omega_x;\, -a(\omega_x + k_x);\, \rho_x) = \Theta_1(ak_x)\cdot\tilde{\mathbf{Q}}_2(\omega_x;\, k_x;\, \rho_x)\ ,$$

where

$$\Theta_1(\cdot) = \exp\left(-\frac{\Sigma_q}{4\pi}\, k_x^2\right); \qquad (8)$$

$$\tilde{\mathbf{Q}}_2(\cdot) = \exp\left(-\frac{\pi\rho^2}{S_q}\right) + \exp\left(-\frac{\Sigma_q}{4\pi}\,\omega_x(\omega_x + k_x)\right) \times$$

$$\times\left(1 - \exp\left(-\frac{\pi\rho^2}{S_q}\right)\right),$$

$2\pi a^2\sigma_q^2 = \Sigma_q$ is the characteristic area of beam spreading in the plane z_3 due to refraction at the rough air–water interface, $S_q = \pi\rho_q^2$ is the characteristic correlation circle of the interface slopes (ρ_q is the correlation length).

Let us substitute relations (6) and (8) into expression (4) and perform successive integration over the variables ρ_y, ω_y, ρ_x, and ω_x. As a result, after transformations we find the relation for the OTF of the channel of image transfer through the one–dimensional randomly rough interface in the analytical form

$$\Phi(\mathbf{k};\, \Delta\rho) = \Phi_0(\mathbf{k})\cdot\tilde{\mathrm{F}}(k_x;\, \Delta\rho)\ , \qquad (9)$$

where

$$\Phi_0(\mathbf{k}) = \exp\left[-\frac{1}{4\pi}\left(k_x^2 S_0 + k_y^2 S_r\right)\right],$$

$$\tilde{\mathrm{F}}(k_x;\, \Delta\rho) = 1 + \Phi_1(k_x;\, \Delta\rho) - \Phi_2(k_x;\, \Delta\rho)\ ,$$

$$\Phi_j(\cdot) = \sqrt{\frac{S_q}{S_j}}\, \exp\left[\frac{\pi}{S_j}\left(i\Delta\rho + \frac{\Sigma_j}{2\pi}\, k_x\right)^2\right], \qquad (j = 1,\, 2)\ ,$$

$$S_1 = (\Delta_s + \Delta_r)\, h_0^2 + S_q\ ,\ \ S_2 = S_1 + 2\Sigma_q\ ,$$

$$\Sigma_1 = \Delta_r\, Lh_0\ ,\ \ \Sigma_2 = \Sigma_1 + \Sigma_q\ ,$$

$$S_r = \Delta_r\, L^2\ ,\ \ S_0 = S_r + \Sigma_q\ ,\ \ \Delta\rho = h_0(\Omega_{sx} - \Omega_{rx})\ .$$

The function F_0 describes the OTF of the system of viewing through the rough surface with diffuse illumination (as $\Delta_s \to \infty$), the factor $\tilde{\mathrm{F}}$ determines a correction for directional character of solar illumination. It follows from Eq. (9) that in the case of sufficiently large misalignment of the directions of illumination and observation ($\Delta\rho \to \infty$) the quantity $\tilde{\mathrm{F}}(\cdot) \equiv 1$, i.e., finally we obtain a filter in the form of Eq. (5) as in the case of the diffuse source.

In general, the OTF given by Eq. (9) is complex and only when $\Delta\rho = 0$ or as $\Delta\rho \to \infty$ it is real (see Ref.4).

Let us analyze the case of $\Delta\rho = 0$. Assuming, for simplicity, the receiver to be ideal ($\Delta_r = 0$), we reduce Eq. (9) to the form

$$\Phi(\mathbf{k};\, 0) = \exp\left(-\frac{\Sigma_q}{4\pi}\, k_x^2\right) \times$$

$$\times\left(1 + \frac{1}{\sqrt{1+\alpha}} - \frac{1}{\sqrt{1+\alpha+2\gamma}}\exp\left(+\frac{\Sigma_q}{4\pi}\,\frac{\gamma\, k_x^2}{1+\alpha+2\gamma}\right)\right), \qquad (10)$$

where $\gamma = \Sigma_q/S_q$ and $\alpha = \Delta_s h_0^2/S_q$. Here γ is the parameter determining the focusing properties of the rough water surface[6] and α characterizes the relative area of beam caustic.

Based on Eq. (10) it is possible to find spatial frequency k_0 at which the OTF vanishes

$$k_0^2 = \frac{4}{\gamma\, \rho_q^2}\,\frac{1+\alpha+2\gamma}{\gamma}\,\ln\left(\sqrt{1+\alpha+2\gamma} + \sqrt{\frac{1+\alpha+2\gamma}{1+\alpha}}\right).$$

The value of k_0 normalized to the bandwidth of the filter Φ_0 being equal to $4/\gamma\rho_0^2$ depends on the depth of the object location nonmonotonically: at small ($\gamma \ll 1$) and great ($\gamma \gg 1$) depths k_0 is large, the minimum value of k_0 is attained at $\gamma \simeq 1$, i.e., when the object is located at a depth of maximum focusing[6]; in this case the correction $\tilde{\Phi}$ for Φ_0 is largest.

We now turn to the analysis of the PSF of the channel of image transfer through the sea surface. The PSF is defined as the Fourier transform of the OTF given by Eq. (9). After some simple transformations we obtain the following relation for the PSF:

$$Q(\mathbf{r};\, \Delta\rho) = Q_0(\mathbf{r})\tilde{Q}(x;\, \Delta\rho)\,,$$

$$Q_0(\mathbf{r}) = \frac{1}{\sqrt{S_0 S_r}}\exp\left[-\pi\left(\frac{x^2}{S_0} + \frac{y^2}{S_r}\right)\right]; \qquad (11)$$

where

$$\tilde{Q}(x;\, \Delta\rho) = 1 + Q_1(x;\, \Delta\rho) - Q_2(x;\, \Delta\rho)\,;$$

$$Q_j(\cdot) = \sqrt{\frac{S_0 S_q}{S_0 S_j - \Sigma_j^2}}\exp\left[-\,p\left(\Delta\rho + x\frac{\Sigma_j}{S_0}\right)^2 \frac{S_0}{S_0 S_j - \Sigma_j^2}\right],$$

$$(j = 1,\, 2)\,.$$

The function Q_0 describes the PSF of the system of viewing through the sea surface with diffuse illumination, and the correction factor $\tilde{Q}$ takes into account the specific character of directional solar radiation.

We now study analytically relation (11). To do this, we set $\Delta_s = \Delta_r = 0$ in it (narrow directional patterns of the source and receiver). Moreover, disregarding a two–dimensional character of the PSF, we will consider its part which depends only on the x coordinate. After transforming to dimensionless coordinates $\xi = x/\rho_q$ and $\sigma = \Delta\rho/\rho_q$ we write down the relation for the PSF in the form

$$Q(\xi;\, \delta) = \frac{1}{\sqrt{\Sigma_q}}\exp\left(-\frac{\xi^2}{\gamma}\right) \times$$

$$\times\left(1 + e^{-\delta^2} - \frac{1}{\sqrt{1+\gamma}}\exp\left(-\frac{(\xi+\delta)^2}{1+\gamma}\right)\right). \qquad (12)$$

The effect of correlation on the form of the PSF can be neglected when the condition $\sigma^2 \gg 1 + \gamma$ is fulfilled. This condition for the object location at a depth greater than 1 m is formulated in a simpler manner: $\Delta\Theta \gg \sigma_q$, where $\Delta\Theta = \Omega_{s,x} - \Omega_{r,x}$ is the angle between the directions of illumination and observation. When this condition is satisfied $Q(\cdot) \equiv Q_0(\cdot)$.

It follows from relation (12) that in general the PSF of the system of observation through the rough surface is not symmetric (even function) with respect to the point $x = 0$. To estimate the effect of roughness and conditions of illumination on the PSF form we may employ the integral moments of function (12):

$$m_n = \rho_q^{n+1}\int\limits_{-\infty}^{\infty} Q(\xi,\, \delta)\, \xi^n\, d\xi\,.$$

Based on these moments it is possible to determine the coordinate of the PSF centroid

$$C = m_1/m_0\,.$$

Since the quantity m_0 varies within small limits ($1 \le m_0 \le 2$), we may roughly assume that

$$C \equiv m_1 = \rho_q\,\frac{\gamma\delta}{(1+2\gamma)^{3/2}}\exp\left(-\frac{\delta^2}{1+2\gamma}\right). \qquad (13)$$

Hence it follows that the dependence $C(\delta)$ is nonmonotonic and central symmetric with respect to the point $\delta = 0$. The maximum of this dependence is

$$C_{max} = \frac{\rho_q}{\sqrt{2e}}\,\frac{\gamma}{1+2\gamma}\,.$$

It is attained at the misalignment of the axes of emission and reception patterns being equal to

$$\delta = \sqrt{\frac{1+2\gamma}{2}}\,.$$

It is also of interest to estimate the PSF width in the case of coaxial illumination and observation ($\Delta\rho = 0$). The rms width of the PSF given by Eq. (12) and normalized to the PSF width with diffuse illumination is found from the relation

$$\Delta x_{norm} = \sqrt{\left(2 - \frac{1+\gamma}{(1+2\gamma)^{3/2}}\right)\Bigg/\left(2 - \frac{1}{(1+2\gamma)^{3/2}}\right)}. \quad (14)$$

It then follows that the dependence $\Delta x_{norm}(\gamma)$ is nonmonotonic: it reaches a maximum of 1.07 at $\gamma = 0.71$. It can be concluded that when $\Delta\rho = 0$ the function $Q(\mathbf{r};\, 0)$

differs most strongly from $Q_0(\mathbf{r})$ in the case in which the object is located at the depth of maximum focusing $h_{max} = 4R_{cur}$ (R_{cur} is the rms value of the curvature radius of the sea surface).

The results of analytical studies are supported by exact calculations from formula (11) for the following values of the basic parameters: $H = 5$ m, $\Theta_s = 5$ mrad, $\Theta_r = 1$ mrad ($\Theta_{s,r}$ are the beam widths of the source and receiver), $\sigma_q^2 = 0.021$, and $\rho_q = 0.16$ m. They correspond to the wind–driven sea waves at a wind velocity of 4 m/s.

Figure 2 depicts a plot of coordinates of the PSF centroid vs the angle between the directions of illumination and observation (The numbers adjacent to the curves indicate the depth in meters). It should be noted that these dependences were normalized to the specific PSF width with diffuse illumination which equals to $\sqrt{S_0/2\pi}$. The same normalization was made for the functions shown in the other figures. As can be seen from Fig. 2, the function $C_n(\Delta\theta)$ is nonmonotonic. The maximum in this dependence shifts toward smaller values of $\Delta\Theta$ as the depth of object location increases, and its value first increases and then decreases at depths smaller than h_{max}. The PSF centroid shifts away from the sun.

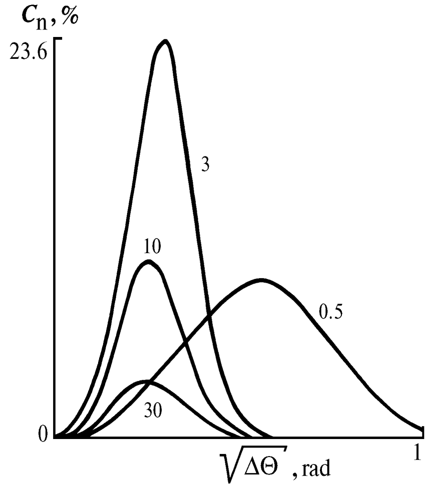

FIG. 2.

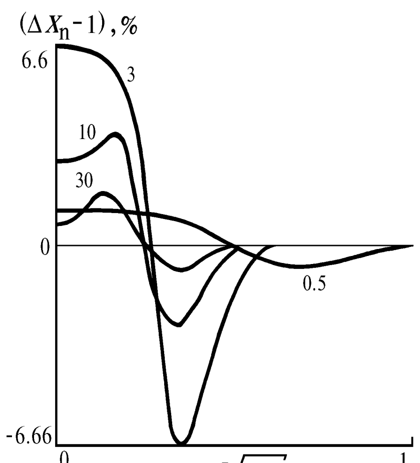

FIG. 3.

Depicted in Fig. 3 are the normalized dependences of the PSF width on the angle between the direction of illumination and observation calculated from the formula

$$D = \sqrt{\frac{m_2}{m_0} - \left(\frac{m_1}{m_0}\right)^2} \Bigg/ \sqrt{\frac{S_0}{2\pi}} - 1 ,$$

where m_n are the moments of spread function (11) (The numbers adjacent to the curves indicate the depth in meters).

These dependences have a complicated oscillating character. At the same time, at small $\Delta\Theta$ the PSF width $Q(x, \Delta\rho)$ is larger than the width $Q_0(x)$ (in the vicinity of focusing depth h_{max} this difference is maximum), whereas the inverse relation exists at sufficiently large $\Delta\Theta$.

The normalized values of C and D are plotted in Figs. 4 and 5 vs the object depth at different angles (numbers adjacent to the curves) between the directions of illumination and observation. These functions are nonmonotonic that testifies to the selectivity of the effect of the PSF form distortion not only vs the angle $\Delta\Theta$, but also vs the depth h.

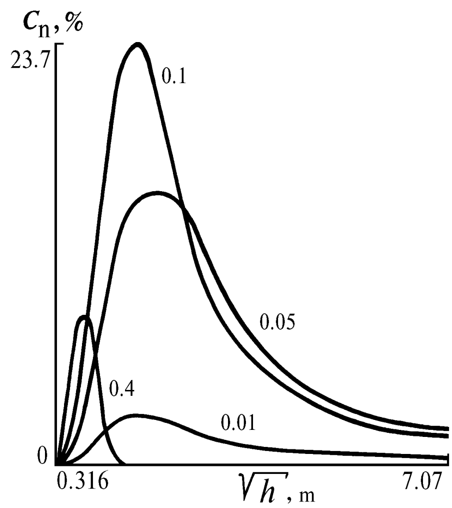

FIG. 4.

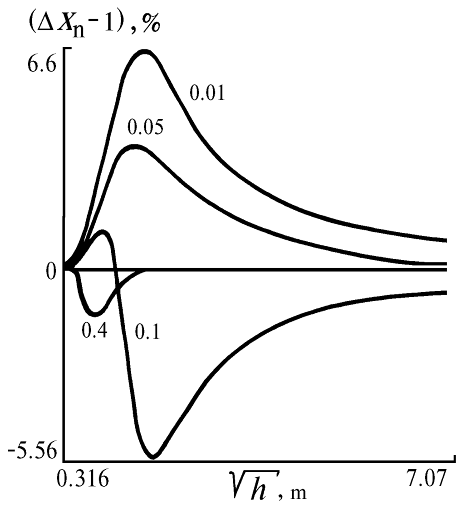

FIG. 5.

The obtained results are in agreement with the estimates obtained on the basis of approximate formulas (13) and (14). It follows from these results that at a wind velocity of 4 m/s the PSF $Q(x, \Delta\rho)$ differs from $Q_0(x)$ more or less strongly at the depths down to 10–15 m. At higher wind velocities this, generally speaking, tentative boundary rises to the surface.

Light scattering in turbid water results in smoothing of the effects under study. Their amplitude decreases and the dependences of position of the PSF centroid and PSF width on the angle $\Delta\Theta$ and depth h broaden. These problems need a more careful analysis and are beyond the scope of this paper.

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