STATISTICAL CHARACTERISTICS OF THE STREHL PARAMETER FLUCTUATIONS NEAR ITS THRESHOLD VALUE

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Some results of studying numerical characteristics of the rate of events when the Strehl parameter crosses the level of its threshold value are discussed. A first–order adaptive optical system has been used in this study. Principal regularities in the behavior of the mean rate of the Strehl parameter crossing its threshold value as well as its variance as a function of the threshold value and the position of the observation point with respect to the center of the illumination spot have been experimentally determined.

The performance of adaptive optical systems (AOS) can be conveniently characterised by the Strehl parameter K_{St}. Normally investigations of this parameter are limited by seeking for its first moment[1] while the optimization of the AOS also requires a knowledge of the time behavior of the value K_{St}. Some results of investigation of the statistical characteristics of the rate r_{th}, at which the Strehl parameter crosses its threshold level, as well as of distribution laws for the intervals between the crossings are discussed in this paper. This experimental study has been conducted using the first–order AOS.[2] In the experiments the principal mode of a pulse–periodic CO_2–laser radiation was directed through a transfer power meter onto a target where the radiation detectors (RD) connected to a computer were installed (see Fig. 1). The computer served the following functions: processing and documenting of the recorded intensity fluctu ations of an optical signal at fixed points. The sources of these fluctuations were atmospheric turbulence, inaccuracy of directing the radiation, and fluctuation effects occurring at the radiation generation. The radiation parameters were $\rho_c/a \approx 0.1$, $\Omega = ka^2/L = 25$, $k = 2\pi/\lambda$, $\lambda = 10.6$ μm, $L = 10^4$ m; $L/F = 10$, $a_s/a_d \le 10^{-2}$, where a, ρ_c, and F the radii of spatial coherence emitting aperture, and beam wave front curvature; λ is the wavelength; L is the path length; a_s and a_d are the radii of light spot at the target and light–sensitive area of the photodetector, respectively. The pulse repetition rate was 10 Hz, the duration of pulses was 30 μs and the length of realizations was 60 s. The experiments were conducted for a near–ground path in October during nighttime under clear sky conditions. The meteorological visual range exceeded 60 km and the structural characteristic of the air dielectric constant fluctuations did not exceed 10^{-16} m$^{-2/3}$.

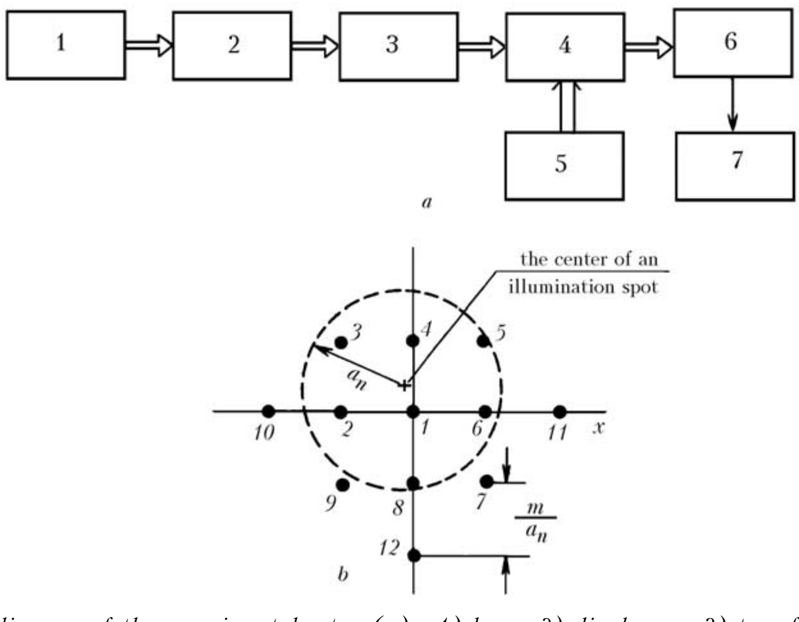

FIG. 1. Block diagram of the experimental setup (a) : 1) laser, 2) diaphragm, 3) transfer power meter, 4) directing mirror, 5) control unit for the mirror, 4, 6) target, and 7) computer. The scheme of RD positions (b) : 1–12 are the fixed points for recording the optical signal.

To determine the statistical characteristics of the Strehl parameter crossings of its threshold level the recorded signals were processed in the following way. Based on the data obtained with the help of the transfer power meter we calculated the maximum value of signal intensity in vacuum in the target plane

$$I_{max} = \frac{P_e}{\pi a_{so}^2},$$

$$a_{so}^2 = a^2\left[\left(1 - \frac{L}{F}\right)^2 + \Omega^{-2}\left(1 + \frac{a^2}{\rho_s^2}\right)\right],$$

where P_e is the emitted power. Subsequently each recorded realization was normalized by the value I_{max}. A number of the events when K_{St} exceeded the level r_{th} was calculated for ten equal intervals within each realization. Based on these values the number of crossings $\bar{\mathbf{N}}_j$ and its variance σ_j^2 were determined

$$\bar{\mathbf{N}}_j = \frac{1}{n}\sum_{i=1}^{n} f_{ij}\ ,\quad \sigma_j^2 = \frac{1}{n-1}\sum_{i=1}^{n}(f_{ij} - \bar{\mathbf{N}}_j)^2\ ,$$

$$f_{ij} = M_{ij}/t\ ,$$

where M_{ij} is the number of events when the parameter K_{St} exceeded the r_{th} level in the ith interval of the jth signal realization and t is the length of an interval.

Shown in Fig. 2 are the results of studying the mean rate of the Strehl parameter crossings of the threshold level from below $\bar{\mathbf{N}}_{+j}$ and its relative variance $\sigma_{+j}^2/\bar{\mathbf{N}}_{+j}$ as a function of the threshold value and of the displacement of the detector sensitive area from the center of the radiation spot m/a_s.

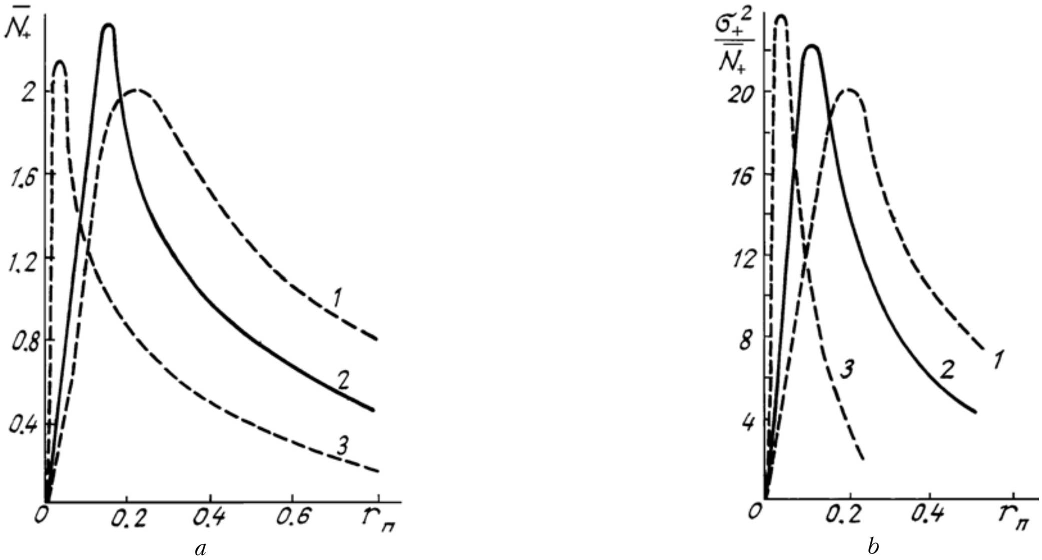

FIG. 2. Statistical characteristics of the rate of the Strehl parameter crossings of the threshold level. 1) RD No. 1, 2) RD No. 2, and 3) RD No. 10.

The characteristics of the Strehl parameter crossings of the level r_{th} from above are similar to those shown in Fig. 2. The extremal behavior of these dependences is explained as follows. When $r_{th} = 0$ the realization K_{St} is over the threshold and no crossings occur. The crossings appear with increasing r_{th}. When $r_{th} \ll 1$ they are seldom, and such events are independent and for $\sigma_{+j}^2/\bar{\mathbf{N}}_{+j} \ll 1$ they can be treated as a stationary ordinary sequence of events without any consequences. The same situation exists as $r_{th} \to 1$ when a realization of the parameter $\bar{K}_{St}$ lies below the threshold. The number of crossings is maximum for the values r_{th} close to $\bar{K}_{St}$. It is interesting that with increase of m/a_s, $\bar{\mathbf{N}}_{+max}$ first increases and then decreases. An increase of $\bar{\mathbf{N}}_{+max}$ with m/a_s is explained by an enhancement of intensity fluctuations of the optical signal when the point of observation is displaced from the center of the spot to its edge. At large values of m/a_s the probability of the RD area overlapping with the illumination spot decreases due to its wandering. This results in reduction of K_{St} oscillations around r_{th} and, consequently, in the $\bar{\mathbf{N}}_{+max}$ decrease.

To determine the laws of the Strehl parameter crossings distribution over duration for each value r_{th} we have constructed grouped statistical sets of the values τ_+ and τ_- based on which we subsequently obtained histograms, τ_+ and τ_- being the time intervals during which the parameter K_{St} was over or below the threshold, respectively. The histograms were then smoothed using the conventional distribution laws. The likelyhood of the hypotheses was verified using the χ^2 criterion. The following pecualiarities in behavior of the distribution laws for the values τ_+ and τ_- as functions of the value r_{th} have been revealed in our study. The results are presented only for the RD No. 1 since for the remaining RD the observed peculiarities are the same. In the case of small values of r_{th} ($r_{th} \ll 1$) when the K_{St} crosses the threshold very seldom and such events can be considered to be independent, the probability density of the intervals τ_+ is well and approximated them by the exponential distribution (see Fig. 3*a*). This well agrees with the well-known fact that the probability density of overshoots of a stationary random process exponentially decreases with increasing r_{th}.[3] At the same time, for r_{th} n 1 in the region of small τ_- values the time intervals follow the Rayleigh distribution (Fig. 3*b*). In the region of large τ values it yields underestimation.

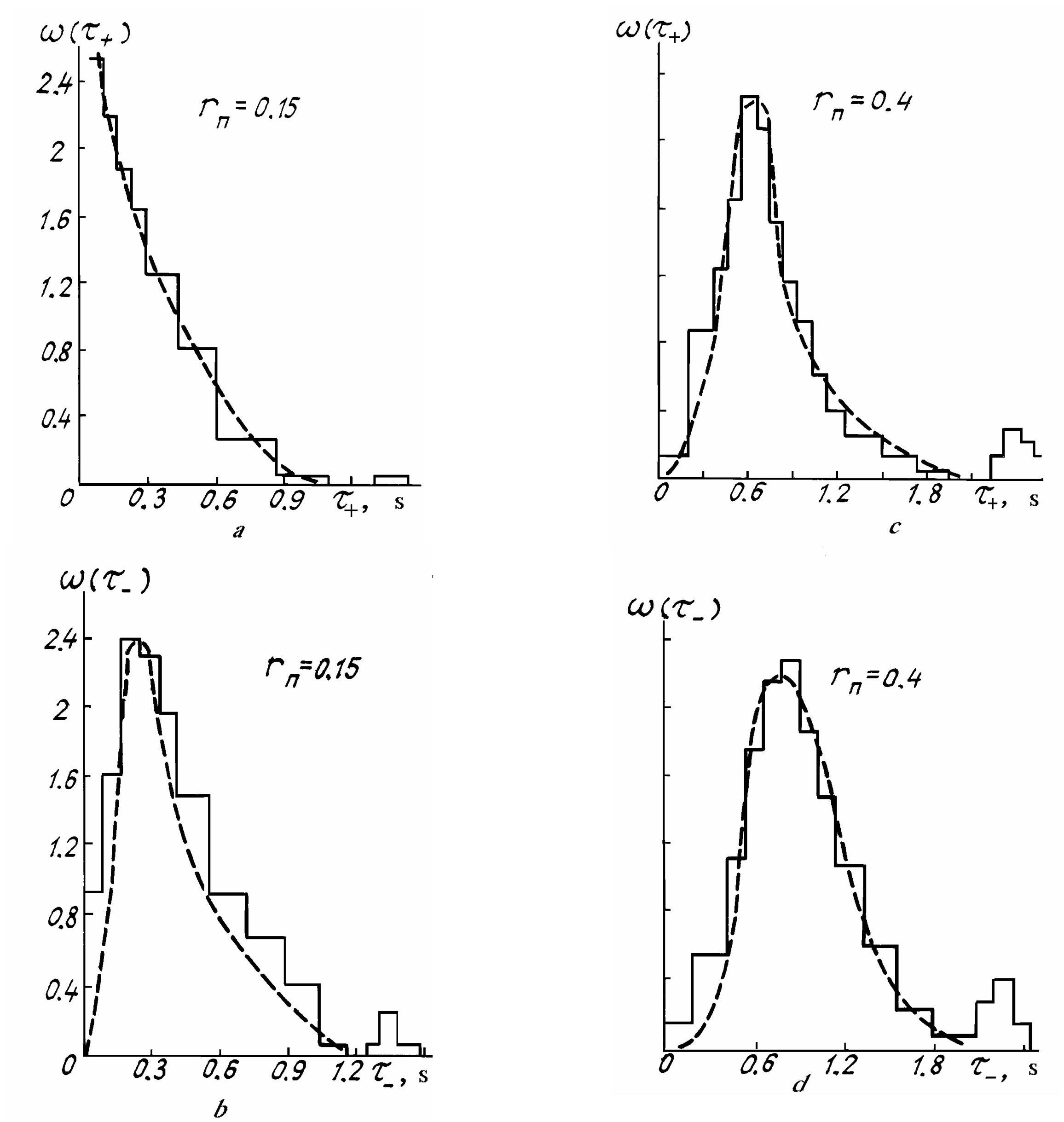

FIG. 3. The probability density for intervals of the Strehl parameter crossings of the threshold value: a—exponential distribution, b—Rayleigh distribution, c, d—lognormal distribution.

The rate of events when the Strehl parameter crosses the threshold level increases with ρ_{th} and when $\bar{N} \gg 1$ the probability densities of the above–mentioned intervals τ_+ and τ_- are satisfactorily approximated by the lognormal distributions (Fig. 3*c* and *d*). It should be noted that when $P(K_{St} \geq r_{th}) = P(K_{St} \leq r_{th})$ these distributions have equal parameters. Here $P(\cdot)$ is the probability of the event when $K_{St} \geq r_{th}$ or $K_{St} \leq r_{th}$. Further increase of r_{th} is accompanied by a decrease of the number of K_{St} overshoots with respect to the threshold level and the distribution of probability density for intervals τ_- is reduced to the exponential distribution while that for intervals τ_+ in the region of small τ values is approximated by the Rayleigh distribution. The absence of unimodality in the described distributions can be explained by a strong contributions into the K_{St} spectrum coming from high and low frequency fluctuations. Random amplitude of the recorded optical signal can be characterized by four time scales. The first of them is related to the coherence time of the laser radiation (10^{-3}–10^{-12} s), the second one is described by the characteristic time of the atmospheric state variation (10^{-1}–10^{-3}, Ref. 4). The third and fourth scales are determined by the correlation time of the error of directing the radiation (~ 0.35 s) and the RD time constant (~ 10^{-5} s), respectively. On the basis of this time scales, it is possible to state that since the global maxima in the given probability density distributions occur in the region of small τ, the oscillations of K_{St} around ρ_{th} are mainly determined by the high–frequency components of the spectrum which are caused by atmospheric turbulence and coherent properties of the laser source.

Unfortunately, the instrumentation available in our experiments made it impossible to separate out the relative contributions of the atmospheric turbulence and of the laser emission fluctuations into the processes yielding the above–described observational results.

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