RECONSTRUCTION OF THE SPECTRA OF THE SURFACE WAVES FROM THE SPECTRA OF THEIR IMAGES WITH AN ACCOUNT OF THE NONLINEAR MODULATION OF THE BRIGHTNESS FIELD

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The brightness field of the "atmosphere–ocean" boundary is analyzed for the nonlinear transfer functions relating the slopes and the brightness of the image of the elements of the sea surface. A method is developed for estimating the spectra of rises in the sea surface. Some techniques are described for calculating the spatial–frequency filters reconstructing the spectra of the slopes of the waves. An analytical formula is derived to determine the orientations of the brightness gradients under certain conditions of illumination.

When constructing the models of the "atmosphere–ocean" system, we must estimate some characteristics of the water surface, in particular, the spatial spectra of this surface from the optical signals recorded with the use of the remote sensing instruments.[1] Though there are many works devoted to the investigation of the relations between the spectra of the image and the spectra of rises in the sea surface (SS), the proposed methods are either incapable of reconstructing the complete two–dimensional spectrum of rises in the SS[2-5] or operate on the linear expansion of the brightness field in the slopes of the SS.[6-8] The linear approximation significantly distorts estimates of the spectra of waves.[9] For this reason, the problem is urgent of developing the methods of reconstructing the spectra of the SS taking into account the nonlinear modulation of the brightness field by the slopes of the SS.

The brightness field of the wavy SS $B(x, y)$ recorded with the use of the optical remote sensing instruments is the sum of the following components:

$$B(x, y) = (B^{(1)}(x, y) + B^{(2)}(x, y) + B^{(3)}(x, y))\cdot \tau_a(x, y), \quad (1)$$

where $B^{(1)}(x, y)$ is the component of the brightness associated with scattering in the atmospheric layer between the surface and the receiver of light emitted by the radiation source, $B^{(2)}(x, y)$ is the component formed by the surface reflection of light emitted by the source and scattered in the atmosphere, $B^{(3)}(x, y)$ is the component formed by the refraction of light at the "atmosphere–ocean" boundary, $\tau_a(x, y)$ is the transmittance of the atmosphere for light arriving at the receiver from the point (x, y) located on the sea surface, and (x, y) are the Cartesian coordinates coinciding with the coordinates of the water surface without waves.

To study the possibilities of estimating the spatial spectrum of the SS from the recorded spectra of the brightness field, we will now consider the relation between the components of the field (1) and the characteristics of the surface. At small field-of-view angles of the recording instruments the intensity of light scattered in the atmosphere may be assumed independent of the viewing angle.[10] The value $B^{(1)}(x, y)$ may then be considered constant. Let us represent the components $B^{(2)}$ and $B^{(3)}$ in the form

$$(B^{(2)}(x, y) = \Gamma(\beta(x, y)) \cdot B_{\downarrow}(\mathbf{r}(x, y)); \quad (2)$$

$$B^{(3)}(x, y) = (1 - \Gamma(\beta(x, y))) \cdot B_{\uparrow}(\mathbf{r}^*(x, y)), \quad (3)$$

where $\Gamma(\beta)$ is the Fresnel reflectance, $\beta(x, y)$ is the angle of incidence on the SS of light falling within the receiver after reflection at point (x, y), $B_{\downarrow}(\mathbf{r})$ and $B_{\uparrow}(\mathbf{r}^*)$ are the brightness fields of radiation arriving at the SS from the upper and lower hemispheres, respectively, $\mathbf{r}(x, y)$ and $\mathbf{r}^*(x, y)$ are the unit vectors specifying the directions of arrival of light falling within the receiver after reflection and refraction at point (x, y) in upper and lower hemispheres (Fig. 1).

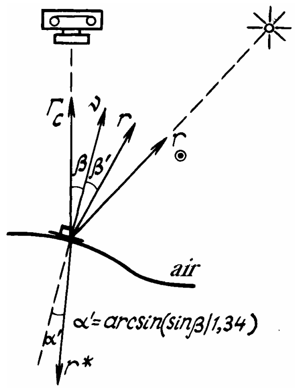

FIG. 1. The diagram of reflection and refraction of light at the "atmosphere–ocean" boundary.

According to the principles of geometric optics, the angle β and the vectors $\mathbf{r}$ and $\mathbf{r}^*$ are continuous functions of the angles of slope of the surface element at point (x, y).[11] With natural illumination the brightness fields $B_{\downarrow}$ and $B_{\uparrow}$ are continuous functions of $\mathbf{r}$ and $\mathbf{r}^*$ (Refs. 12 and 13). This makes it possible to represent the function $B((x, y)$ at each point (x, y) by the Taylor series

$$B(x, y) = C_0 + C_x\nabla_x\xi(x, y) + C_y\nabla_y\xi(x, y) +$$
$$+ N(x, y, \nabla_x\xi(x, y), \nabla_y\xi(x, y)) , \quad (4)$$

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where C_0, C_x, and C_y are the coefficients of the linear part of the expansion of $B(x, y)$ in terms of powers of $\nabla_x\xi$ and $\nabla_y\xi$, where $\xi(x, y)$ is the field of rises (the y—coordinates) in the surface, $\nabla_x\xi$ and $\nabla_y\xi$ are the slopes of the SS (the components of the gradient of the field of rises), $\nabla_\alpha\xi(x, y) = \frac{\partial\Sigma}{\partial\alpha}\,\xi(x, y)$, where $\alpha = x, y$ and $N(x, y, \nabla_x\xi, \nabla_y\xi)$ is the nonlinear component of brightness field, which includes the terms proportional to $(\nabla_x\xi)^2$, $(\nabla_y\xi)^2$, $\nabla_x\xi\nabla_y\xi$, and the terms of higher orders.

To study the relation between the energy spectrum (the spectral power density) of the image $S(\boldsymbol{k})$ and the spectrum $\Psi(\boldsymbol{k})$ of the field of rises $\xi(x, y)$, we will define field spectrum (4) as the square modulus of the Fourier transform.[14] Taking into account that the constant component C_0 can be eliminated by preliminary processing of the image and by employing the well—known theorems of the Fourier analysis, we obtain

$$S(\boldsymbol{k}) = (C_x\boldsymbol{k}_x + C_y\boldsymbol{k}_y)^2\Psi(\boldsymbol{k}) + S_N(\boldsymbol{k}) , \tag{5}$$

Here $S_N(\boldsymbol{k})$ is the term including the spectral power density of the nonlinear component of the brightness field and the cross spectral component of power density for linear and nonlinear components.[15]

For $N = 0$, the expression in the right side of Eq. (4) is proportional to the field of slopes of the SS

$$\beta_\theta(x, y) = \nabla_x\xi(x, y)\cdot\cos\theta + \nabla_y\xi(x, y)\cdot\sin\theta \tag{6}$$

in the direction $\theta = \arctan(C_x/C_y)$. The spectrum of field (6) has the form

$$\Phi_\theta(\boldsymbol{k}) = (\cos\theta k_x + \sin\theta k_y)^2\Psi(\boldsymbol{k}) . \tag{7}$$

Therefore, in the linear approximation, the spectrum of the image S(k) is proportional to the spectrum of slopes $\Phi_\theta(\boldsymbol{k})$

$$S(\boldsymbol{k}) = (C_x^2 + C_y^2)\Phi_\theta(\boldsymbol{k}).$$

One may neglect the nonlinear term N in Eq. (1) provided that $(\sigma/B_\downarrow)\partial B_\downarrow/\partial\boldsymbol{r} \ll 1$, and $(\sigma/B_\uparrow)\partial B_\uparrow/\partial\boldsymbol{r}^* \ll 1$, where σ is the rms slope of the SS. This condition is satisfied with an acceptable accuracy with diffuse illumination of the SS (continuous cloudiness) or far off the solar track on a cloudless day.[16] It is violated in the region of the solar track. At the points of specular reflection of the direct solar radiation the brightness of the SS B_d significantly exceeds the brightnesses B_s of the points that reflect the diffuse radiation of the sky. Variations in the brightness of the order of $B_\mathrm{d} - B_\mathrm{s} \gg B_\mathrm{s}$ are observed when the direction $\boldsymbol{r}$ is changed at the angles of the order of the angular size of the Sun $\delta_\odot \ll \sigma$. Strong overshoots (bright spots) of the brightness at specular points of the SS make the "slope—brightness" transformation significantly nonlinear. It then follows that the spectrum of image $S(\boldsymbol{k})$ is no longer proportional to the spectrum of slopes $\Phi_\theta(\boldsymbol{k})$. Under these conditions we must construct an operator $\mathbf{W}$ that allows us to reconstruct the spectrum of slopes $\Phi_\theta(\boldsymbol{k})$ from the spectrum of image $S(\boldsymbol{k})$

$$\Phi_\theta(\boldsymbol{k}) = \mathbf{W}S(\boldsymbol{k}). \tag{8}$$

Since various physical processes in the atmosphere and ocean contribute to the formation of the nonlinear component of the brightness field N, the analytical calculations of the reconstruction operator are difficult and may be performed for significantly limited range of the parameters of illumination and viewing of the SS and for limited class of the spectra of the waves.[4] Thus, numerical methods for the correction of the nonlinear distortions of the spectra must be developed that make use of, for example, numerical modeling of random fields under certain conditions of their formation.[9,17] The procedure for construction on of such an operator is given below.

Formula (7) makes it impossible to reconstruct completely the spectrum of rises $\Psi(\boldsymbol{k})$ from only one spectrum of slopes $\Phi_\theta(\boldsymbol{k})$ because of the singularity that arises at $k_x = -k_y\tan\theta$. Several ($N \geq 2$) images (or several fragments of a single image) obtained under different conditions of illumination and viewing and having different orientations of the brightness gradient $\theta^{(n)}$, where $n = 1, \ldots, N$, may be employed for complete reconstruction of the spectrum $\Psi(\boldsymbol{k})$. Having determined the spectra of these images $S^{(n)}(\boldsymbol{k})$ and having reconstructed the corresponding spectra, of slopes $\Phi_\theta^{(n)}(\boldsymbol{k})$ based on Eq. (7), the spectrum of rises of the SS may be calculated with the use of the operators W, which depend on the conditions of image formation, on the basis of the formula

$$\Psi(\boldsymbol{k}) = \frac{\sum\limits_{n=1}^{N}\Phi_\theta^{(n)}(\mathrm{K})}{\sum\limits_{n=1}^{N}\left(k_x\cos\theta^{(n)} + k_y\sin\theta^{(n)}\right)^2} . \tag{9}$$

For the linear modulation of the brightness field by the slopes of the SS, the quantities $\theta^{(n)}$ in Eq. (9) can be calculated, e.g., by determining the directions of the minima and the maxima in the angular distributions of the energy in the image spectra[7] or by approximating the image by the linear functions of the coordinates x and y using the least—squares technique.[8] However, when the brightness field is nonlinearly modulated, these methods may introduce the undesirable systematic errors in determining the orientations $\theta^{(n)}$. Thus, it is necessary to develop methods of calculation of these orientations $\theta^{(n)}$ that would take into account the specific conditions of formation of the brightness field.

To estimate analytically the orientations $\theta^{(n)}$ under certain conditions, we note that, in accordance with the physical meaning, the quantity $\theta^{(n)}$ is defined by the direction of the normal to the isolines of average brightness within the observed section of the SS. In the region of the solar track the brightness of the points of specular reflection of the direct solar radiation exceeds by two orders of magnitude the brightness of the points reflecting the diffuse sky radiation.[12] Therefore, the isolines of the average brightness should closely follow the isolines of spatial density of the specular points given by the relation

$$\boldsymbol{r}_\odot + \boldsymbol{r}_c(x, y) = 2(\nu(x, y),\ \boldsymbol{r}_c(x, y))\nu(x, y),$$

where ν is the unit vector of the normal to the SS at point (x, y) and $\boldsymbol{r}_\odot$ and $\boldsymbol{r}_c$ are the unit vectors specifying the directions from this point toward the Sun and the receiver, respectively (see Fig. 1). Taking into account that the vector ν is related to the slopes of the SS

$$\mathbf{v} = (-\nabla_x\xi,\ -\nabla_y\xi,\ 1/\sqrt{1+s^2}), \tag{10}$$

where $s^2 = (\nabla_x\xi)^2 + (\nabla_y\xi)^2$, we derive the following equation for the isolines of the brightness:

$$(\sin\Theta_\odot - \tilde{x}/\omega)^2 + (\tilde{y}/\omega)^2 = 4\cos^2\Theta_\odot s^2 . \tag{11}$$

Here $\Theta_\odot$ is the zenith angle of the Sun, $\tilde{x} = x_c/H$, $\tilde{y} = y_c/H$, (x_c, y_c) are the coordinates of the image center, H is the imaging altitude, and $\omega = \sqrt{1 + \tilde{x}^2 + \tilde{y}^2}$.

The desired formula for orientation follows from Eq. (11) and is given by

$$\Theta^{(n)} = \arctan\left(\frac{\pi}{2} + \frac{\mathrm{T}}{\mathrm{U}}\right), \tag{12}$$

$$\mathrm{T} = \tilde{y}^{(n)}(1 + (\tilde{x}^{(n)})^2) + \tilde{x}^{(n)}\tilde{y}^{(n)}(\omega\sin\Theta_\odot - \tilde{x}^{(n)}),$$

$$\mathrm{U} = (\tilde{x}^{(n)} - \omega\sin\Theta_\odot)((\tilde{x}^{(n)})^2 - \omega^2) + \tilde{x}^{(n)}(\tilde{y}^{(n)})^2$$

This makes it possible to find θ^n for the given conditions of illumination.

Below we describe the procedure of construction of the reconstruction operator W for the images of the SS obtained under given conditions of illumination.

1. We prescribe a model spectrum of rises of the SS $G(\mathbf{k})$ employing the well-known approximations of the spectra of the waves, for example, the JONSWAP formulas.[18] The driving wind velocity, which is the parameter of this approximation, can be determined from the wave number of the maximum in the spectrum of the image.[1]

2. We synthesize the two–dimensional discrete fields of slopes of the SS based on the formula[17,19]

$$\nabla_\alpha\xi(l_x\Delta_x,\ l_y\Delta_y) = F_D^{-1} \times$$
$$\times\left[2\pi k_\alpha \frac{N_xN_yG(\mathbf{k}_D)}{\Delta_x\Delta_y}\exp(if(\mathbf{k}_D))\right](l_x,\ l_y), \tag{13}$$

where F_D^{-1} is the operator of the inverse discrete Fourier transform, Δ_x and Δ_y are the discretization steps determined by the spatial resolution of the instruments, N_x and N_y are the numbers of the image elements along the Ox and Oy axes (N_x and N_y are determined by the size of the initial image), $\mathbf{k}_D$ is the discrete wave vector, l_x and l_y are integer indices, and f is the phase spectrum, which is an $N_x \times N_y$ matrix of the random uncorrelated numbers uniformly distributed in the interval (0.2π).

3. We now model the brightness field of synthesized realization of the SS based on formulas (1)-(3). We calculate the vectors $\mathbf{r}$ and $\mathbf{r}^*$ according to the formulas $\mathbf{r} = 2(\mathbf{r}_c, \mathbf{v}) - \mathbf{r}_c$ and

$$\mathbf{r}^* = ((\mathbf{r}_c, \mathbf{v})/m - \sqrt{1 - (1 - (\mathbf{r}_c, \mathbf{v})^2)/m^2})\mathbf{v} - (1/m)\mathbf{r}_c ,$$

where $m = 1.34$ is the refractive index of water. The functions $B_\downarrow(\mathbf{r})$ and $B_\uparrow(\mathbf{r}^*)$ can be either measured experimentally or be assumed in the form of one of the well–known approximations.

$$B_\downarrow(\mathbf{r}) = B_0\left(\frac{1 + (\mathbf{r},\ \mathbf{r}_\odot)^2}{(1 - \mathbf{r},\ \mathbf{r}_\odot)} + K\right)(1 - \exp(-0.32/\cos\Theta_\odot)), \tag{14}$$

where $B_0 = \mathrm{const}$ and $K = 0, \ldots 5$ is the parameter determined by the second–order scattering of light in the atmosphere. Under the given conditions of illumination and with the well–known hydrooptical characteristics the function $B_\uparrow(\mathbf{r}^*)$ may be calculated numerically.[13,22] To a first approximation $B_\uparrow(\mathbf{r}^*)$ may be estimated using the formula $B_\uparrow(\mathbf{r}^*) = \rho_D E_\downarrow$, where ρ_D is the diffuse reflectance of the water depth and $E_\downarrow$ is the total downwelling light flux.[12] A model image of the SS, obtained according to the above-described procedure is shown in Fig. 2. The coordinate axes x and y are drawn in such a way that the point $x = y = 0$ corresponds to the observation made in the nadir direction, while the Ox axis coincides with the direction of wind and the azimuth of the Sun. In modeling the following parameters were hosen: $\Delta_x = \Delta_y = 1$ m, $N_x = N_y = 512$, $H = 1000$ m, $\theta_\odot = 30°$, and $\rho_D = 0.01$.

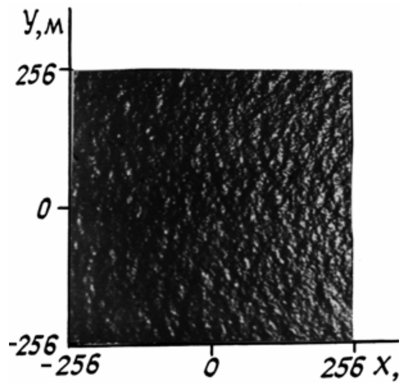

FIG. 2. Model image of the sea surface. (The explanation is given in the text).

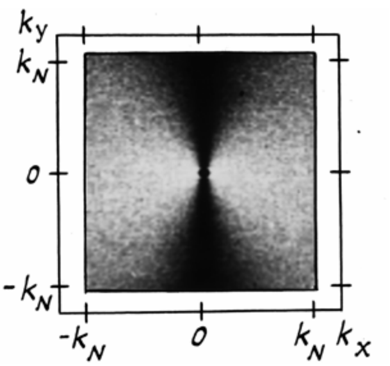

FIG. 3. The spatial–frequency filter reconstructing the spectra of the slopes of the sea surface ($\mathbf{k}_N$ is the Nyquist frequency).

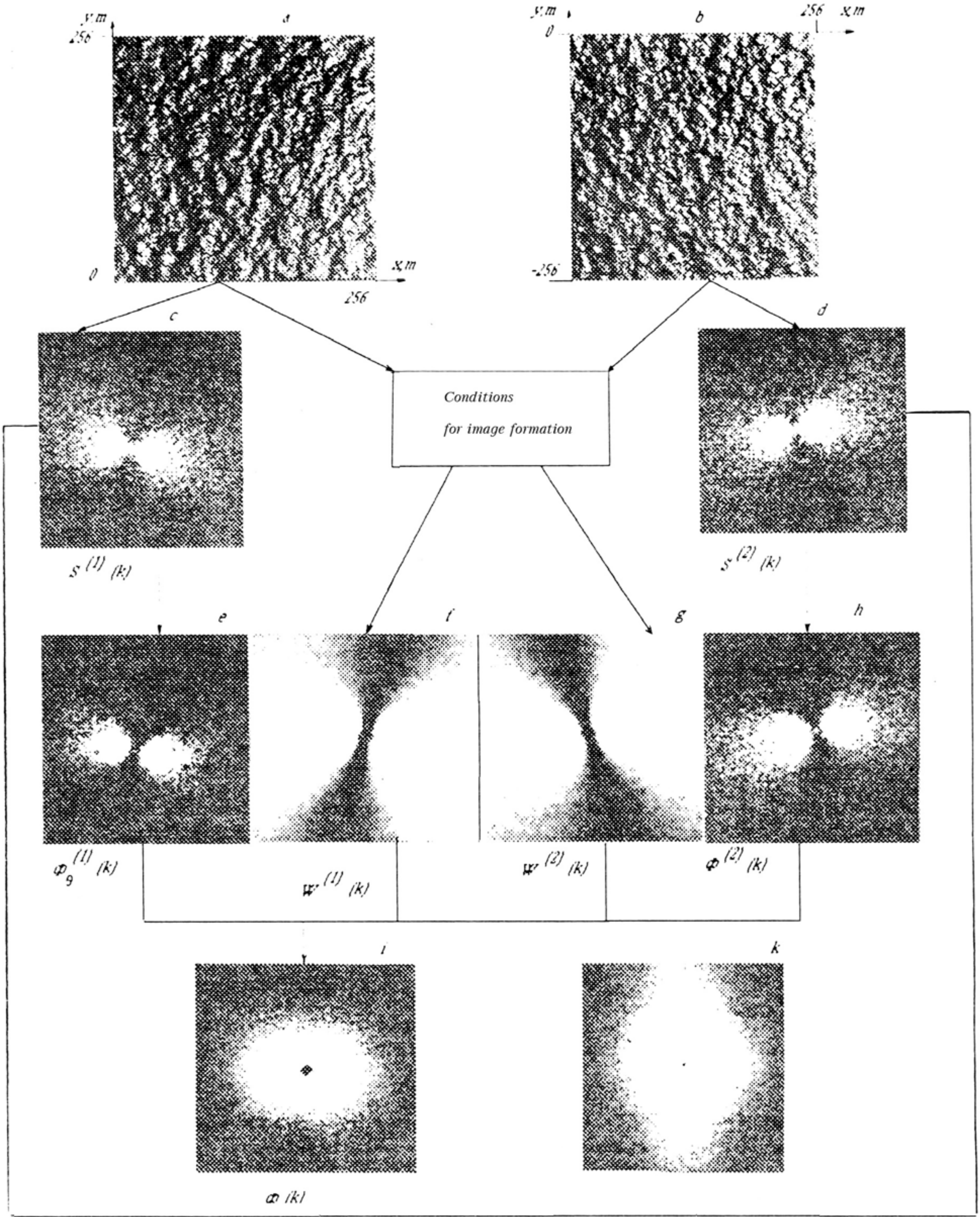

FIG. 4. The example of reconstructing the spectra of slopes and rises of the sea surface. (The explanation is given in the text).

4. Now we calculate the spectrum of the model image $S(\boldsymbol{k})$ and form the reconstructing operator $\hat{\boldsymbol{W}}$ as the spatial–frequency filter of the form

$$W(\boldsymbol{k}) = k_x^2 G(\boldsymbol{k})/S(\boldsymbol{k}) , \qquad (15)$$

where $k_x^2 G(\boldsymbol{k})$ is the spectrum of the model field of slopes $\nabla_x \xi(x, y)$. The form of this filter is shown in Fig. 3, where the light–colored sections correspond to the regions of the spatial–frequency transmission and the dark-colored sections—to the regions of suppression of the components of the image spectrum with the filter $W(\boldsymbol{k})$.

An example of reconstructed spatial spectra of the SS according to the above–described procedure is given in Fig. 4, where a and b show the images of the sections of the SS with the 256×256 m^2 size (the parameters of imaging were chosen as follows: $\Theta_{\odot} = 30°$, $H = 1000$ m, $\overset{(1)}{x_c} = \overset{(2)}{x_c} = 128$ m, $\overset{(1)}{y_c} = 128$ m, and $\overset{(2)}{x_c} = -128$ m) obtained with solar illumination c and d show the spectra of images $S^{(n)}(\boldsymbol{k})$, e and f show the reconstructing filters $W^{(n)}(\boldsymbol{k})$ (k) obtained by means of rotating the filter shown in Fig. 3 at the angles $\theta^{(1)} = 19°$ and $\theta^{(2)} = -19°$ and calculated according to formula (12), g and h show the spectra of slopes of the SS $\Phi_{\theta}^{(n)}(\boldsymbol{k})$ reconstructed according to formula (8), where $n = 1, 2$, and i shows the spectrum of rises of the SS $\Psi(\boldsymbol{k})$ reconstructed according to formula (9). The spectrum of rises $\Psi(\boldsymbol{k})$ obtained for $W^{(n)} = \text{const}$, i.e., when neglecting the nonlinear components of the brightness field, is shown in Fig. 4. It can be seen distinctly from the figure that neglecting the nonlinear terms of the expansion of the brightness field in the slopes of the SS substantially distorts the shape of the spectrum of rises of the SS.

Thus, the proposed technique makes it possible to reconstruct the two–dimensional spatial spectra of rises of the SS from the images obtained with the nonlinear modulation of the brightness field by the slopes of the SS. To implement this technique in practice, we only need the data on the parameters of formation of the images. Such parameters can be easily measured in outdoor experiments.

The results that have been presented here are based on model images and demonstrate the feasibility of reconstruction of the two–dimensional spectra of rises of the SS. To check experimentally the adequacy of the proposed method, we must compare the spectra of the SS reconstructed by this method with the spectra obtained by other methods, for example, by contact measurements or by stereoscopic imaging of the SS. The procedure of this comparison and the discussion of the results must be the subject of future work.

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