ANALYSIS OF THE QUALITY OF IMAGE FORMATION IN AN ADAPTIVE SYSTEM WITH COMPENSATION OF RANDOM WAVEFRONT TILTS ON SEGMENTS OF THE APERTURE

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An adaptive optical system with independent compensation of random local wavefront tilts on four segments (sectors) of the aperture was modeled. The tilts were estimated from the shifts in the centers of gravity of the images formed by the corresponding segments of the aperture. The dependence of the resolution of the system on the ratio of the aperture diameter to the size of the region of correlation of the atmospheric distortions (Fried's parameter) is derived. It is shown that compensation of four local tilts makes it possible to obtain 1.9 *times higher resolution than with the compensation of the overall wavefront tilt on the entire aperture.*

When small telescopes, whose aperture diameter D is only insignificantly greater than the size of the region of correlation of the atmospheric distortions of the optical radiation field (the value of Fried's parameter) r_0 $(D/r_0 \le 4)$, are used each short-exposure realization of the phase distortions can be approximated quite accurately by the aperture-averaged constant phase shift and tilt of the phase front $\vec{a}$. It is well known that the variance of the error in such an approximation is equal to $0.14\,(D/r_0)^{5/3}$ and virtually never exceeds 1 rad^2 right up to $D = 3.5\,r_0$. Since constant phase shifts do not affect the image quality and the wavefront tilts lead to a simple displacement of the image, in this case adaptive compensation, which makes it possible to follow the displacement of the image formed and thereby combine the short-exposure images as they are accumulated in the course of a long exposure, is effective.[1] There are a number of theoretical and experimental works devoted to such systems.[2–4] It has been shown that compensation of random tilts cam appreciably increase (by up to a factor 4) the resolution of the system. However the efficiency of such a system drops sharply when the diameter of the aperture is increased up to values exceeding $4r_0$. One of the obvious ways to extend the range of applicability of this system is to determine and compensate the partial wavefront tilts on separate segments of the aperture. The simplest implementation of this approach is to divide the aperture into four equal sectors (segments). If the maximum size of the segments is 3–4 r_0, it can be expected that the quality of the image formed by such a system with aperture diameters right up to 6–$8r_0$ will be significantly improved. In this paper we investigate an adaptive system with compensation of four local tilts, determined from the shifts of the centers of gravity of short-exposure images.

To determine the potential dependence of the average resolution R on the ratio D/r_0 of a system with compensation of four subtilts on four segments of the aperture and to estimate the expected gain over previously studied systems we simulated mathematically the processes involved in the formation of the average (long-exposure) image of a point source in the following systems.

1) The standard system without any compensation.

2) A system with adaptive compensation of the overall tilt $\vec{a}'$ of the wavefront, estimated from the shift of the center of gravity of the corresponding short-exposure image, over the aperture.

3) A system with compensation of the tilts $\vec{a}_1$ determined from the shifts of the centers of gravity of the short-exposure images formed by the corresponding segments of the aperture.

4) The system 3 with additional compensation of the constant phase shifts γ_1, averaged over the segments and determined from the condition that the "irregularity" of the compensating wavefront be minimum:

$$\int d\vec{v}\, W_1(\vec{v}) \left[\vec{a}_0\vec{v} - \gamma_1 - \vec{a}_1\vec{v}\right]^2 = \min, \tag{1}$$

where $\vec{v}$ are the coordinates in the plane of the aperture and $W_1(\vec{v})$ is the aperture function of the ith segment. The tilt $\vec{a}_0$ was chosen from the condition (1).

If is useful to note that $\vec{a}_0$ is equal to $\vec{a}'$. To prove this it is sufficient to recall that the tilts of the wavefront $\vec{a}$ are related with the distribution of the short-exposure realization of the phase distortions $\varphi(\vec{v})$ by the equalities[5]

$$\vec{a} = S^{-1}\int d\vec{v}\, W(\vec{v})\ \mathrm{grad}\varphi(\vec{v});$$

$$\vec{a}_1 = S_1^{-1}\int d\vec{v}\, W_1(\vec{v})\ \mathrm{grad}\varphi(\vec{v}),$$

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where $W(\vec{v}) = \sum_1 W_1(\vec{v})$ is the total aperture function, $S = \int d\vec{v} W(\vec{v})$ and $S_1 = \int d\vec{v} W_1(\vec{v}) = S/4$, and the minimum in Eq. (1) is obtained for $\vec{a}_0 = \frac{1}{4}\sum_1 \vec{a}_1$.

5) An "optical" four-segment adaptive system, in which the local values of the tilts and displacements γ_1 are determined directly from each realization of the phase distortions $\varphi(\vec{v})$ in accordance with a condition of the form

$$\int d\vec{v}\, W_1(\vec{v}) \left[\varphi(\vec{v}) - \gamma_1 - \vec{a}_1\vec{v}\right]^2 = \min. \qquad (2)$$

6) An adaptive system in which the local wavefront tilts are determined from the displacements of the centers of gravity of the short-exposure images and the constant phase shifts are determined from the condition (2) for realization of the phase distortions.

Looking ahead, we note that the characteristics of an adaptive system in which only the wavefront tilts determined from the realizations of the phase distortions are compensated are virtually identical to those of the system (3), so that this system is not studied here.

In the simulation 128 random realizations of the phase distortions $\varphi(\vec{v})$ with a 5/3-law structure function were accumulated for ratios D/r_0 from 1 to 15. After this, for each system with a fixed value of D/r_0 the corresponding 128 short-exposure images were averaged and the resolution R was estimated as the ratio of the intensity at the center (maximum) of the average image to its total energy. We emphasize that for all six systems there exists a common limit of the resolution, which is reached as $D/r_0 - \omega$ and is equal to[5]

$$R_\infty = \frac{\pi}{4}\left(\frac{r_0}{\lambda F}\right)^2,$$

where λ is the wavelength and F is the focal length.

Figure 1 shows the curves of R_k/R_∞ versus the ratio D/r_0 for the six systems listed above.

The following basic conclusions can be drawn based on the results obtained.

1. The maximum gain in resolution of the adaptive system with compensation of the overall wavefront tilt, as compared with the system without any compensation, is equal to 3.7 and is achieved for $D/r_0 = 3.5$. The maximum resolution of the adaptive system with compensation of the overall wavefront tilt is reached for $D/r_0 = 3.5$ and is equal to $3.3\,R_\infty$. In previous investigations the maximum resolution of such a system reached $3.5\,R_\infty$, while the increase in resolution of such a system as compared with a system without compensation was equal to 4. The difference of the values of the resolution and the gain of this system can be explained by the fact that in preceding works the wavefront tilts were estimated directly from the realizations of the phase distortions, and this is a more accurate method (but, as a rule, not realizable in practice) than the method of estimating the wavefront tilt from the shift in the center of gravity of the image.

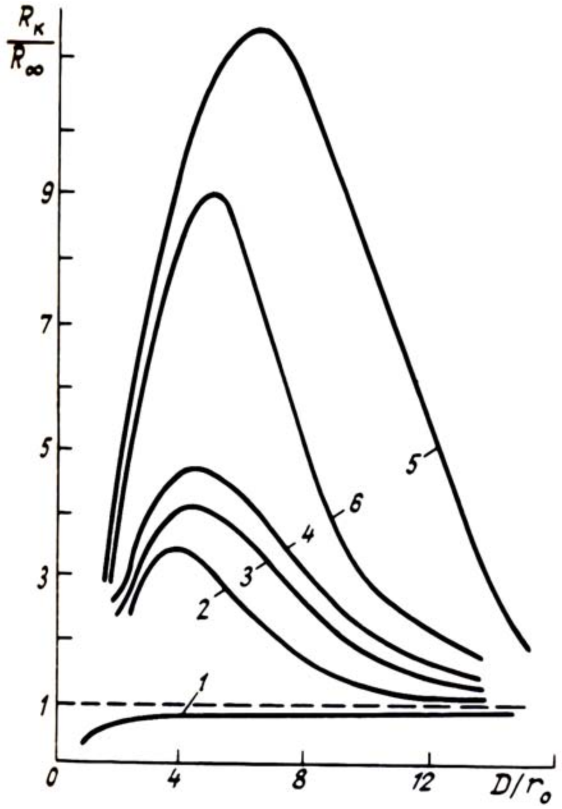

FIG. 1. The resolution R_k/R_∞ versus the ratio D/r_0 for the systems 1–6.

2. The system studied here with compensation of the four subtilts of the wavefront from separate segments of the aperture, determined from the shift in the centers of gravity of the short-exposure images, is more efficient that the system with compensation of the overall wavefront tilt. The maximum gain of this system is equal to 1.7 and is obtained with $D/r_0 = 6$. The ratio D/r_0 for which the resolution of the system reaches the maximum value $3.95\,R_\infty$ is equal to 4.5.

3. The range of values of D/r_0 for which the system studied is more efficient with compensation of the overall wavefront tilt is 3 to 10.

4. When additional compensation is made for the longitudinal displacements in order to eliminate the discontinuities of the wavefront at the boundaries of the segments the quality of the image formed improves. The maximum resolution of such a system reaches $4.5R_\infty$ with $D/r_0 = 4.5$. The maximum gain of such a system, with respect to the system studied, is equal to 1.14.

5. As one can see from the figure, however, the potential maximum resolution of the system with

compensation of the four tilts and constant phase shifts is equal to 11.45 R_∞ and is reached for $D/r_0 = 6$, which is 2.5 times greater than the resolution obtained with the system 4.

6. It is obvious from the analysis of the dependence of the resolution of the system with compensation of four wavefront subtilts determined from the shifts of the centers of gravity of the images and the constant phase shifts, calculated directly from the phase distortions, taking into account the fact that the characteristics of systems with compensation of only the wavefront tilts determined from the phase distortions and from the shifts of the centers of gravity of the images are virtually identical, that the most important aspect of the solution of the problem of further improving the characteristics of the system studied is the determination of the longitudinal displacements of the segments.

Thus we arrive at the general conclusion that although a system with compensation of tilts on separate segments of the aperture has better resolution (1.7 times higher than for the previously studied system 2) it does not give the potentially possible resolution. To eliminate this drawback of the system it is necessary to develop a method for determining independently the close to optimal values of the phase shifts γ_1, since estimation of the values of γ_1 from the measured tilts is not effective.

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