COMPARISON OF THE DETECTION CAPABILITIES OF TELEVISION AND LIDAR SYSTEMS IN SCATTERING MEDIA

I.L. Katsev and E.P. Zege

Institute of Physics, Academy of Sciences of the Belorussian SSR, Minsk Received September 11, 1989

The potential detection capabilities of television and lidar systems are shown to be equal, provided the requirements on information content are the same for both. Availability of a priori information on the sensor-to-target direction significantly increases the detection range for the same radiation power of the source. Expressions are obtained for estimating the optimal receiver aperture angles, i.e., those which maximize the signal-to-noise ratio.

In Ref. 1 the problem of television versus visual target detection in scattering media (such as in the sea, in clouds, and in a turbid atmosphere) was analyzed. According to the concept suggested in Ref. 1, during both visual observations and television image analysis the observer performs similar optimal processing of the image: in a sense he selects an elementary area for spatial integration of the image Σ_{el}^{opt} such a way as to obtain a maximum signal-to-noise ratio. Within the framework of this concept relationships were obtained which make it possible to determine simultaneously and in closed form the maximum detection range for a given target, and the optimal element size for spatial integration Σ_{el}^{opt} .

One of the principal modern techniques for target detection is pulse location. Similar to television systems, location systems not only permit detection of the target, but also form its image. ^{2,3} Therefore it is natural to pose the question of the comparative capabilities of television and lidar location systems. To answer it most completely, let us first consider the relatively simple case in which the source-totarget direction is known a priori, so that one needs only to confirm the target presence and determine its distance from the observation point. Actually we are speaking then about simple ranging (as, for example, in the case of pulse location of the sea bottom). Apparently the location range is then at its maximum and may be taken as the upper limit of pulse location capabilities.

LOCATION IN A GIVEN DIRECTION (RANGING)

Let a short pulse of radiation be aimed at the target center. The receiver, its axis also aiming at the target center, is positioned close to the emitter, and registers the reflected signal power as a function of time. The presence of the target and the distance to it are estimated from the location signal "peak" against the background of backscattered interference (BSI). The quantity that determines the possibility of target detection is the signal-to-noise ratio:

$$\delta = \frac{\Delta W}{\sqrt{W_b/C + \alpha_2 W_b^2}} \,. \tag{1}$$

where ΔW is the energy of the locating signal, accumulated during the time $\Delta t_{\rm el}$; the latter is determined by the temporal resolution of the receiver; W_{bg} is the background energy (BSI), entering the receiver during the same time in the abscence of the target; *C* is the receiver constant, which determines its quantum efficiency, α_2 is a parameter which determines the internal noise of the receiver system and relates the threshold contrast of the system k_{th} to the threshold signal-to-noise ratio δ_{th} by the relation $k_{th} = \delta_{th} \sqrt{\alpha_2}$.¹

When a radiation pulse propagates through a scattering medium, its shape changes as a result of multiple scattering. Therefore, in general, the values of ΔW and W_{bg} depend not only on Δt_{el} and the initial pulse duration $\Delta t_{\rm Q}$, but also on the characteristic times Δt_1 and Δt_{BSI} for the locating signal and the BSI, respectively. The latter two determine the temporal spreading of an instantaneous pulse in the medium. Accurate calculation of Δt_1 and Δt_{BSJ} . and, respectively, of ΔW and $W_{\rm bg}$, requires the application of results from nonstationary light scattering theory.⁴ Here we shall restrict our treatment to the comparatively simple but practically important case most often encountered, in which such spreading of the locating pulse is not strong, so that $\Delta t_{\rm BSI} \gg \Delta t_0 \gg \Delta t_1$. Therefore, the optimal value $\Delta t_{\rm el} \approx \Delta t_1$,⁵ and one may apply the results of stationary radiation transfer theory to the calculation of the effective signal energy, ΔW . To calculate the BSI energy $W_{\rm bg}$, simple analytical solutions from the theory of nonstationary light scattering may be used, e.g., the quasi-single scattering approximation or

asymptotic temporal solutions.⁴ Essentially such an approach was applied in Ref. 5, where the optimal location range in sea water was calculated for certain fixed optical-geometrical parameters of the system.

As is known from Ref. 6, the energy of the effective signal reaching the receiver is equal to

$$\Delta W = \frac{\frac{W_{o} \Sigma_{rec} \Omega_{rec}}{\pi}}{\pi} \int_{-\infty}^{\infty} A(\vec{r}) E_{s}(\vec{r} - \vec{r}_{o}) E_{rec}(\vec{r} - \vec{r}_{o}) d\vec{r}, \qquad (2)$$

Here W_0 is the source pulse energy; $E_{\rm rec}$ and $\Omega_{\rm rec}$ are the entrance pupil area and the receiver solid angle of view; $A(\vec{r})$ is the target albedo distribution given as a function of the coordinate \vec{r} in the plane $z = z_0 = {\rm const}$, perpendicular to the source axis; $E_s(\vec{r})$ the illumination distribution produced in the target plane $z = z_0$ by a unit power source with directional diagram identical to that of the true source; $E_{\rm rec}(\vec{r})$ is the respective distribution for a fictious source, with directional diagram identical to that of the sensitivity diagram of the actual receiver; $\vec{r_0}$ is the point in the target plane, at which both the emitter and the receiver axes are directed.

Assume for simplicity that the functions $A(\vec{r})$, $E_s(\vec{r})$, and $E_{rec}(\vec{r})$ are azimuthally symmetrical, so that they may be approximated by a Caussian distribution. Then we have from Eq. (2) for $r_0 = 0$:

$$\Delta W = \frac{W_0 \sum_{rec} \langle A \rangle S^2(z_0)}{\pi z_0^2} \frac{D_{rec} D_{tar}}{D_{tar} + D_a} \frac{1}{D_s + D_{rec} + 2D_r}(3)$$
$$D_a = \frac{(D_s + D_r)(D_{rec} + D_r)}{D_s + D_{rec} + 2D_r}, \qquad (4)$$

where $S(z_0)$ is the zero spatial moment of the point spread function (PSF) of the medium, i.e., the transmittance of the medium layer of thickness z_0 ; Σ_{tar} and $\langle A \rangle$ are the area and average albedo of the target; $D_{\mathrm{tar}} = \Sigma_{\mathrm{tar}}/2\pi, \qquad D_{\mathrm{rec}} = \Omega_{\mathrm{rec}} z_0^2 / 2\pi,$ $D_s = \Omega_s z_0^2 / 2\pi$, Ω_s is the solid angle of the source; D_r is the variance of the PSF of the medium; D_a is the variance of the function $a(\vec{r}) = E_s(\vec{r})E_{rec}(\vec{r})$. When $D_{\rm rec} \gg D_s + 2D_r$ (or $D_{\rm tar} \rightarrow \infty$ and $D_s \gg D_{\rm rec} + 2D_r$, relations (3) and (4) yield the well-known relations^{4,6} for the effective signal transfer coefficient $\eta_{BSI} = \Delta W / W_0 = \langle A \rangle$, for observational systems of "narrow-wide" and "wide-narrow" type, respectively.

When calculating W_{bg} we shall assume, as is done in the quasi-single scattering approximation, that at the time $t = 2z_0/v$ (v is the speed of light in the medium) corresponding to the arrival of the locating signal, the BSI is produced by radiation scattered from a layer of thickness $v\Delta t_{\rm el}/2$, located at the depth z_0 . Therefore an approximate expression for $W_{\rm bg}$ may be obtained directly from relation (3), replacing $\langle A \rangle$ by the brightness coefficient of the radiation backscattered by this layer, i.e., by the value $\sigma \chi(\pi) v \Delta t_{\rm el}/2$ (σ is the scattering coefficient, χ is the scattering indicatrix in the backward direction). Assuming that $D_{\rm tar} \rightarrow \infty$, we then obtain

$$W_{bg} = \frac{W_{0}\Sigma_{rec}S^{2}(z_{0})}{\pi z_{0}^{2}} \frac{\sigma \chi(\pi) \ \upsilon \Delta t_{e1}}{2} \frac{D_{rec}}{D_{s} + D_{rec} + 2D_{r}}(5)$$

Relations (2)–(5) are convenient for analyzing the locating system capabilities. Let us, for example, consider the problem of the optimal choice of the receiver solid angle $\Omega_{\rm rec}$. This solid angle is considered to be optimal if the signal-to-noise ratio for it is maximal. Therefore the value $D_{\rm rec}^{\rm opt}$ (and, respectively, the value $\Omega_{\rm rec}^{\rm opt}$) may be found from the condition

$$d\delta/dD_{rec} = 0. (6)$$

A quite simple analysis demonstrates that the very existence of optimal values of $D_{\rm rec}$ and $D_{\rm rec}^{\rm opt}$ depends on the type of noise (either shot noise or internal noise) prevalent in the locating system, in other words — on the relationship between the value 1/C and $a_2W_{\rm bg}$. It is generally rather easy to calculate $D_{\rm rec}^{\rm opt}$ from relations (1) and (6) taking account of relations (3)–(5); however such calculations need to account for all the optical-geometrical and energetic parameters of the system. Therefore we shall limit the discussion to two important limiting cases, which can be analyzed without stating explicitly any given parameters of the system.

1. Let $a_2CW_{\rm bg} \gg 1$. This case corresponds to a very high source energy. Then the location range is limited by the internal noise of the system and is determined by the condition $k = \Delta W/W_{\rm bg} > k_{\rm th}$. It is easy to show that no optimum exists then in $D_{\rm rec}$, and the signal-to-noise ratio increases as $D_{\rm rec}$ increases. Note here that a decrease in $D_{\rm rec}$ is possible only down to a certain limit, since as $D_{\rm rec} \rightarrow 0$ the value of $W_{\rm bg} \rightarrow 0$, and our initial condition is then violated.

2. As a rule we have a different situation, when $a_2 C W_{\text{bg}} \ll 1$ and the location range is limited 2 by the shot noise of the receiver. In that case we have

$$D_{\rm rec}^{\rm opt} = \frac{D_{\rm tar} (D_{\rm s} + 2D_{\rm r}) + D_{\rm r} (D_{\rm s} + D_{\rm r})}{D_{\rm s} (D_{\rm s} + D_{\rm r}) / (D_{\rm s} + 2D_{\rm r}) - D_{\rm tar}}.$$
 (7)

It may be seen from Eq. (7) that for an optimum in $D_{\rm rec}$ to exist it is necessary to satisfy the following condition:

$$D_{tar} < D_{s} (D_{s} + D_{r}) / (D_{s} + 2D_{r}).$$
 (8)

Note that the necessary (but not sufficient) condition

$$D_{\text{tar}} < D_{\text{s}}.$$
 (9)

follows from condition (8).

Therefore if the spot of unscattered radiation in the target plane is smaller than the target area, no optimal receiving angle exists and the signal-to-noise ratio increases as $D_{\rm rec}$ increases. At the same time it is clear that for a fixed source energy the value δ increases as D_s decreases. Therefore the maximum signalto-noise ratio in the considered case is attained as $D_s \rightarrow 0$, $D_{\rm rec} \rightarrow \infty$.

Let us now consider the more general location problem of detecting a target, the direction to which is unknown a priori. To determine the position of the target in that case, one needs to scan a certain search zone with a series of short pulses. Technical realizations of such a system are described in Refs. 2 and 3. Now the return signal from each emitted pulse is divided into a series of intervals, each $\Delta t_{\rm el}$ long, and is stored, where the *i*th interval corresponds to the radiation returned from that segment of the medium a distance $z_1 = vt_1/2$ from the lidar. Then a set of images of layers is formed as a result of scanning through the search zone with a narrow beam, each layer $v\Delta t_{\rm el}/2$ thick and located at the distance \boldsymbol{z}_1 from the lidar. It is clear that such a system, which we shall call an image-forming lidar, is a complete analogue of the "narrow-wide" pulse observation system (the system of type 1). As a rule, modern television systems are implemented as "wide-narrow" systems (type 2). Therefore, the question of comparing location and television systems reduces to comparing observation systems of types 1 and 2, which employ temporal strobing of the signal. We shall consequently index all the lidar system characteristics by 1, and the television system characteristics by 2.

Assume that the solid angle $\Omega_{rec}^{(1)} = \Omega_s^{(2)}$ is sufficiently wide (i.e., is considerably wider than the angular size of the spread function), and that the solid angle $\Omega_s^{(1)} = \Omega_{rec}^{(2)}$ is sufficiently small. Also let the source energy spent on forming an image of the same from be equal for both types of systems. This means that if the lidar has to send m pulses to view a frame of $\Omega_{c}^{(2)} z_{0}^{2}$ area, the corresponding television system "2" would have to accumulate $m = \Omega_s^{(2)} / \Omega_s^{(1)}$ pulses to form the same image. Let us also assume that the strobing time intervals $\Delta t_{\rm el}^{(2)} = \Delta t_{\rm el}^{(1)}$, and that the receiver sensitivities, the characteristics of their internal noise, and all the other analogous parameters in both systems are identical. Then using the results of the analysis given in Refs. 7 and 9 we arrive at the important conclusion that the signal-to-noise ratio and the target detection range for both systems is the same.

We assumed the source solid angle $\Omega_s^{(1)}$ in the lidar image forming system to be sufficiently small $(D_s \ll D_r)$. This enables one in the processing of the image, e.g., when the operator observes the image formed on the screen, to choose an optimal element of spatial integration $\Sigma_{\rm el}^{\rm opt}$ for which the signal-to-noise ratio δ and the target detection range are maximum. However, to scan a large search zone with a small solid angle, it is necessary to have a source which has a high pulse repetition frequency. Indeed, during the time interval of frame formation one needs to send $m = \Omega_s^{(2)} / \Omega_s^{(1)}$ pulses into the medium. In this respect the television system "2", in which a low value of $\Omega_{\rm rec}^{(2)}$ is realized simply enough, at present retains certain technical advantages. One may further ask, what are the limits for increasing the solid angle $\Omega_s^{(1)}$ in the lidar system, and, respectively, for lowering the pulse repetition frequency without significantly losing in detection range? Clearly, such an increase may go as far as $\Omega_s^{(1)} \approx \Sigma_{\rm el}^{\rm opt} / z_0^2$.

We assumed above that the solid angle $\Omega_s^{(2)} = \Omega_{\rm rec}^{(1)}$ is sufficiently large. While this condition is of fundamental importance for the television system "2", since the value $\Omega_s^{(2)}$ determines the field of view (the frame) of the system, it does not appear to be mandatory for the lidar system "1". Indeed, in this case the frame size is determined by the scanning field, i.e., by the value of $\Omega_s^{(1)}$ and by the pulse repetition frequency. The question however arises of the optimal choice of the solid angle $\Omega_{\rm rec}^{(1)}$. It is easy to understand that the same question can be reformulated as follows: does a system of type 3 ("narrow-narrow")⁶ have any fundamental physical advantages, compared with systems of types 1 and 2, if the same amount of source energy is spent on frame formation?

It is known that system "3" provides a higher apparent image contrast, however remaining inferior to system "1" in its energetics. Therefore at high enough source energies, when the limitations on the viewing distance are determined by the threshold contrasts, system "3" gains the advantage over system "1". In that case the small receiving angle $\Omega_{\rm rec}^{(1)}$ becomes optimal (see the analysis of the optimal receiving angles for single-pulse sounding).

Nevertheless, in practice a different situation is usually encountered, in which the limitation on the range is determined by the shot noise, i.e., by a lack of energy. Then the optimum in $\Omega_{rec}^{(1)}$ exists only under condition (9), which, however, is not satisfied, as a rule. That is why the signal-to-noise ratio in that case is higher, the larger is $\Omega_{rec}^{(1)}$. We may thus conclude that at moderate source energies, when condition (9) is violated, the "narrow-narrow" system remains fundamentally inferior to systems of type 1 as far as the detection range is concerned (and not only because of its technical realization difficulties as stated in Ref. 8).

We therefore arrive at the physically obvious but important conclusion that if the same amount of information is to be retrieved from the search zone, the fundamental capabilities of both the television and lidar systems remain identical.

However in certain cases. e.g., when a-priori information on the target size is available, the detection range may be increased by the choice of an optimal search strategy. Indeed, it is then no longer necessary to scan across the whole frame with a narrow solid angle of the source $\Omega_s^{(1)}$. The simplest situation would then be the one in which the search is restricted to separate points within the frame, located at a distance from each other close to the target size d_{tar} . Then the number of elements in the frame can be reduced to $m_1 = \Omega_{\rm rec}^{(1)} z_0^2 / d_{\rm tar}^2 < \Omega_{\rm rec}^{(1)} / \Omega_s^{(1)} = m$, thus increasing the energy spent on each single element in the image during the recording of the frame. It is easy to understand that the cost of such an increase in the detection range is poorer accuracy in the determination of the target position.



FIG. 1. Detection range for a square target of albedo $\langle A \rangle = 0.1$ with side length 1 m (1) and 30 m (2) using single-pulse lidar (solid curve) and the television system (dashed curve).

What are the possibilities for increasing the detection range in this way? Generally speaking, this question is not so simple. However, the upper limit of the detection range may be easily found. It is equal to the range of location in the given direction when all the energy available for frame formation is used just for that purpose. Figure 1 compares the target detection ranges for single-pulse lidar and television systems. It may be seen that the availability of a-priori information on the target direction makes it possible to increase the detection range of small targets severalfold.

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