Numerical retrieval of temperature and wind fields in the meso-β-scale area on the base of the dynamic-stochastic approach

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We consider the methodological basis and investigation results of an algorithm for numerical retrieval of temperature and wind fields in the meso-β-scale area. The algorithm is realized by data of a single aerologic station and a local net of meteorological stations with the use of the Kalman filtering and two-dimension dynamic-stochastic model.

A new methodological approach to the problem of numerical retrieval of vertical profiles of temperature and wind in the boundary layer of the atmosphere (BLA) in the absence of data of height observations was proposed in Refs. 1 and 2. The approach is based on the algorithm of the Kalman filter and two-dimensional dynamic-stochastic model. It is realized for the space points of aerologic measurements. In practice, however, there arises another problem, which is connected with availability of information about physical state of BLA not only at the point of aerologic observations, but also at meteorological stations located at small (less than 100–200 km) distances from a single station of height sounding. In this case, the numerical retrieval of the 3-dimensional structure of meteorological fields in the meso-β-scale area (i.e., in the area whose characteristic dimensions are 20–200 km [Ref. 3]) is necessary. The retrieval is realized on the base of data from a single aerologic station (or from a system of lidar or acoustic sounding) and a few meteorological stations.

The growing interest to the soonest solutions of such problems is caused by the fact that the results of numerical retrieval of meteorological fields (and, in particular, temperature and wind fields) in BLA make it possible

– to extend functional potentials of lidar and acoustic systems of remote sensing due to numerical estimation (by the measurement data) of the physical state of the atmospheric boundary layer not only at the place, where such a system is located, but also within the limits of the whole surrounding meso-β-scale area;

– to forecast spatial propagation of pollutants to small distances (up to 50–100 km) from the source;

– to improve the efficiency of meteorological support for solving special problems of military geophysics, etc.

Some time ago (see, for instance, Refs. 4 and 5) the specialists of the Institute of Atmospheric Optics, Siberian Branch of the Russian Academy of Sciences, proposed several methodological approaches to the problem of numerical retrieval of meteorological fields in the mesoscale area, based on application of the Kalman filtering technique and different types of dynamic-stochastic models. But the problem cannot be solved through the use of the dynamic-stochastic models, proposed in Refs. 4 and 5, because their realization requires meteorological data from three or more stations of height sounding.

Therefore, in this paper we propose another methodological approach, which makes it possible to retrieve the three-dimensional structure of temperature and wind fields in the meso-β-scale area, using observation data from a local network of meteorological stations and data from a single station (or a single remote system) of height sounding.

Formulate the problem and briefly consider the features of the proposed method and algorithm of the solution of the problem.

Let a given meso-β-scale area have two points with coordinates (x_0, y_0) and (x_1, y_1). At the first point, only land-based meteorological observations are performed, while at the second – only aerologic measurements. In such a case, the problem of numerical retrieval of any height meteorological field ξ, which is a centered, homogeneous, and stochastic one, at the point (x_0, y_0) is in the numerical estimation (with the use of a given mathematical model) of its values at the height h; as well as at the instant t_0 by land-based data at the same point and data of aerologic measurements at the point (x_1, y_1)

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that are taken at the instant t_0 and previous instants $t_0 - j$ (j = 1, 2, ..., K). Here it should be noted that the meso-β-scale area is characterized by a sufficiently homogeneity and isotropy of meteorological fields.[6]

To solve this problem, the following two-dimensional dynamic-stochastic model is used:

$$\xi_h(k) = \sum_{m=h}^{h+i}\sum_{j=0}^{K} d_{j,m}\xi_m(k-j) + \varepsilon_h(k), \qquad (1)$$

where $\xi_h(k)$ is the magnitude of some meteorological field at a fixed height h (within the limits of the boundary layer of the atmosphere) at an instant k; m is the index of the height level, for which the field ξ is retrieved, m varies from h to $h+i$ (here i = 1, 2, ..., n is the maximal number of height levels, which are taken into account in retrieving the field at the height h); j = 0, 1, ..., K is the discrete time, determining the size of the Kalman filter algorithm predictor; $d_{j,m}$ are unknown model parameters, which are to be estimated and defining the relations between the values of the field $\xi_h(k)$ and its values at previous instants of time $k - j$ at a given height and at higher ith levels (in this paper, j = 3 and i = 2); $\varepsilon_h(k)$ is the model's discrepancy, which is defined by stochasticity of atmospheric processes.

The problem of numerical retrieval of a height meteorological field ξ in the meso-β-scale area is solved in two steps.

At the first step, parameters of the model $d_{j,m}$ are estimated by data of aerologic observations of the field ξ at the point (x_1, y_1) at the instant t_0 and previous instants $t_0 - j$ for the height level h and ith higher levels and by land-based measurements at the point (x_0, y_0).

At the second step, based on the assumption that the vector of estimated parameters of the model $\boldsymbol{D} = d_{j,m}$ varies weakly within the considered atmospheric layer and throughout the given meso-β-scale testing area, the field ξ is estimated at the given point (x_0, y_0) at the instant t_0 at the height h + 1 (on the base of the retrieval equation and aerologic data at the point (x_1, y_1)).

The retrieval equation can be written in the following form:

$$\hat{\xi}^{(0)}_{h+1}(k) = \sum_{m=h}^{h+1}\sum_{j=0}^{K} \hat{d}_{j,m}\xi^{(1)}_{m+1}(k-j), \qquad (2)$$

where $\hat{\xi}^{(0)}_{h+1}(k)$ is the estimate of the meteorological field ξ at the height $h + 1$ at the instant k (the estimate is obtained for the point (x_0, y_0), where aerologic measurements are not performed); $\hat{d}_{j,m}$ are the estimates of the model (1) parameters for the hth height level from data of aerologic measurements at the point (x_1, y_1); $\xi^{(1)}_{m+1}(k-j)$ are the measured values of the same field ξ at the point (x_1, y_1) at the time from k to $k - j$ and at the height levels from h + 1 to $h + i$ + 1.

Since as the initial data in the expressions (1) and (2) are not the measured values of the field ξ themselves but its centered values, the retrieving procedure is performed by the same scheme as in Ref. 1, when the resulting estimate of the height meteorological field ξ at the points (x_0, y_0) is a sum of the estimate of the regular component of the field $\overline{\xi}$ and the estimate of its fluctuation component ξ', i.e.,

$$\xi = \overline{\xi} + \xi'. \qquad (3)$$

Note that the designation of the prime was omitted in Eqs. (1) and (2). In the sequel this prime will be also omitted.

The regular component of the field $\overline{\xi}$ is computed as a smoothed average over several previous observations taken for the point (x_1, y_1) at a fixed height:

$$\overline{\xi}_h(k) = \frac{1}{p}\sum_{j=1}^{p}\xi_h(k-j). \qquad (4)$$

Here p is the depth of the time window, which is used to estimate the regular component of the field ξ (in our case, p = 3).

Now we briefly dwell on the algorithm of numerical retrieval of the vertical structure of the field ξ at a given point (x_0, y_0), located within the limits of the meso-β-scale and having only land-based meteorological data. First, the unknown parameters $d_{j,m}$ in the model (1) are estimated by data of aerologic observations at the point (x_1, y_1), which is in the same meso-β-scale area. For this purpose, we use

1) the system of difference equations of state

$$\mathbf{X}^t_{k+1} = \mathbf{\Psi}_k \cdot \mathbf{X}^t_k + \boldsymbol{\omega}^t_k, \qquad (5)$$

where $\mathbf{X}^t_k$ is the state vector of the dimension $n = (i+1)K$, which includes all unknown parameters $d_{j,m}$, which are to be estimated for current discrete time k; $\mathbf{\Psi}_k$ is the transition matrix for a discrete system of the dimension $(n \times n) = [(i+1)K(i+1)K]$; $\boldsymbol{\omega}^t_k$ is the vector of random perturbations of the system (the state noise vector) of the dimension $n = (i + 1)K$.

Note that if $\mathbf{X}^t_k$ do not vary on the average at the taken time interval (this may be assumed), then $\mathbf{\Psi}_k$ corresponds to the identity matrix $\mathbf{I}$ of dimensions $(n \times n)$ and the expression (5) takes the form

$$\mathbf{X}^t_{k+1} = \mathbf{X}^t_k + \boldsymbol{\omega}^t_k; \qquad (6)$$

2) the model of measurements, by data of which the state of the system is estimated in the Kalman filtering algorithm. In the general case, the model is described by an additive mixture of a useful signal and a measurement error:

$$\mathbf{Y}_k^0 = \xi_k^0 = \mathbf{H}_k \cdot \mathbf{X}_k^t + \varepsilon_k^0, \qquad (7)$$

where $\mathbf{Y}_k^0$ is the vector of actual measurements, which represents some measurement at the height h at instant k; $\mathbf{H}_k$ is the observation vector of the dimension $n = (i + 1)K$, which defines the functional connection between the true values of state variables and actual measurements. Its components are values, which are obtained by measurements of the field $\xi_h(k)$ at given levels in a fixed atmospheric layer at a given and previous instants to the depth K:

$$\mathbf{H}_k = \| y_0(k-1),\ y_0(k-2), ., \ y_0(k-K), y_1(k-1), \dots, y_{i+1}(k-K) \|;$$

ε_k^0 is the vector of measurement errors (noises) at an instant k.

On the base of expressions (6) and (7), the problem of estimating the state vector is solved by the linear Kalman filter.[7]

Then, by the use of the obtained estimates for the state vector $\mathbf{X}_k^a$, the value of the height meteorological field $\xi_{h+1}^{(0)}(k)$ is calculated for the point (x_0, y_0), where the aerologic measurements are absent. For this purpose, we use the expression (2) written in the matrix form

$$\hat{\xi}_{h+1}^{(0)}(k) = \mathbf{Y}_k^0 = \mathbf{H}_k^* \cdot \mathbf{X}_k^0, \qquad (8)$$

where $\mathbf{H}_k^*$ is the transition vector of the dimension $n = (i + 1)K$ applied to calculations of a given meteorological field at a height $h + 1$ at an instant k; $\mathbf{X}_k^0$ is the estimate of the state vector at an instant k, which was obtained at the first step.

The efficiency of the above-described algorithm of numerical retrieval of meteorological fields in the meso--β-scale area was considered for the case when it is applied to the problem of three-dimensional (for the horizontal and height) extrapolation of temperature and wind fields within the boundary atmospheric layer.

The quality of the extrapolation can be estimated only from data of at least two closely located aerologic stations, one of which should be taken as a reference station. So, we used the following data from three European stations as initial information: Stuttgart (48°50′ N, 09°12′ E), Kümmersbruck (49°26′ N, 11°54′ E), and Munich (48°15′ N, 11°33′ E), located at minimal distances from each other. All data of radiosonde observations (two times per day, at 00 and 12 h GMT) obtained in January and July, 2007 were reduced (using linear interpolation with allowance for data of special points and principal isobaric surfaces: 1000, 925, 850, and 700 hPa) to the system of geometric heights: 0 (the ground level), 100, 200, 300, 400, 600, 800, 1000, 1200, and 1600 m.

To estimate the accuracy and efficiency of the numerical retrieval algorithm, the Kümmersbruck station was taken as a basic aerologic station (parameters of the mathematical model $d_{j,m}$ were determined by its data), and Munich and Stuttgart stations were taken as control points (for these stations, extrapolation with regard for the height was realized). These stations are located respectively at distances of 175 and 195 km from Kümmersbruck.

At the same time, the same estimation was performed by the use of the root-mean-square errors δ_ξ of such retrieval with the numerical retrieval of mean values in the layer of the temperature and zonal and meridional wind velocity components as the examples (these parameters were required for forecasting of the pollutant cloud propagation). The values of δ_ξ were compared with the admissible error Δ_ξ in determination of the means over the layer of some meteorological parameter values. According to Ref. 8, such an error is 1.0–1.2°C for the temperature and 1.0–1.2 m/sec for the orthogonal components of wind velocity. The mean values of the temperature and orthogonal components of the wind velocity in the layer $h - h_0$ (here h_0 is the ground level and h is the height of the upper boundary of the considered layer) were computed as[8]:

$$\langle \xi \rangle_{h_0,h} = \frac{1}{h - h_0} \int_{h_0}^{h} \xi(z) \mathrm{d}z, \qquad (9)$$

where ξ is a meteorological parameter; z is the height, and $\langle \bullet \rangle$ denotes the procedure of data vertical averaging.

The figure presents root-mean-square errors δ_ξ of spatial three-dimensional extrapolation (for the horizontal and height) of the parameters $\langle T \rangle_{h_0,h}$, $\langle U \rangle_{h_0,h}$, and $\langle V \rangle_{h_0,h}$ as functions of distance. They were obtained for winter, summer, and typical atmospheric layers.

For comparison, the admissible errors (Δ_ξ) in determination of mean values in the layer of temperature and orthogonal components of wind velocity are presented as straight lines.

Analysis of the figure has shown that the proposed algorithm can be successfully applied in practice, because it yields quite satisfactory results, especially for the lower 600 m atmospheric layer. The best retrieval was obtained for the temperature field at the largest distance from the aerologic station.

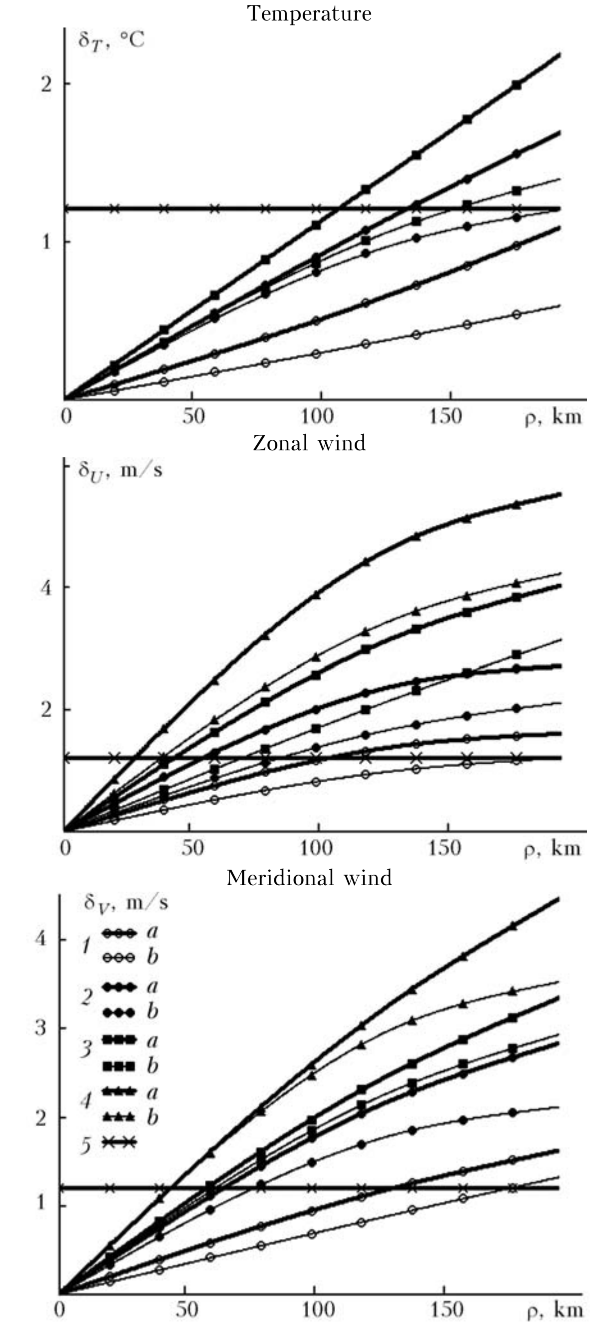

Fig. The root-mean-square error δ_ξ of spatial extrapolation for mean values of temperature, zonal and meridional components of wind velocity in the layer of 0–200 (*1*), 0–400 (*2*), 0–800 (*3*) and 0–1600 m (*4*) as a function of the distance ρ and admissible errors Δ_ξ (*5*), winter *(a)* and summer *(b)*. The mean values were extrapolated on the base of the dynamic-stochastic algorithm.

Actually, the conditions, under which the root-mean-square error of spatial extrapolation of the mean values in the layer of temperature does not exceed the admissible error of 1.2°C do not change up to a distance of 100–120 km. At the same time, for wind, whose spatial variability is higher than for temperature, the conditions, under which the root-mean-square errors $\langle U\rangle_{h_0,h}$ and $\langle V\rangle_{h_0,h}$ are less than the admissible error of 1.2 m/sec, are stable only to a distance of 60 km, and only in the lower 600 m layer. Above level of 600 m, the retrieving quality for the same parameters is stable up to 30–50 km and decreases with height.

Thus, mean values were retrieved in the layer of temperature and orthogonal components of wind velocity in the meso-β-scale on the base of the Kalman filtering algorithm with two-dimensional dynamic-stochastic model and data from a single station of height sounding (with available data from existing meteorological stations). Experimental studies of quality of the retrieval demonstrate that this algorithm is efficient and can be used in different applied problems, where the physical state of the boundary atmospheric layer in a given mesoscale area should be taken into account.

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