Self-similar solutions of kinetic equations describing the evolution of discharge plasma in pulse gas discharge lasers

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Processes leading to self-similarity of plasma dynamic properties in pulse gas discharge lasers (PGL) in a wide range of variation of initial conditions of the discharge are described. The first integrals of the equations and self-similar solutions of kinetic equations describing the evolution of plasma for a space self-maintained longitudinal high-current pulse-periodic discharge in a mixture of neon with metal vapors (copper, barium) are presented. The characteristics of discharge plasma in similar schemes of excitation are the same and can be described by the self-similar solutions. Possible mechanisms, restricting the plasma self-similarity in a pulse-periodic PGL discharge during the time of a current pulse are discussed.

Introduction

Results of investigations of low-temperature non-isothermal pulse nanosecond gas discharges are reported in many publications. Such discharges are widely applied in pulse gas-discharge lasers (PGL). Despite the available authoritative monographs,[1–3] ambiguous opinions about simulation methods of plasma dynamics and discharge parameters in different moments of evolution of such discharges can be found. The similarity relations[4,5] for pulse gas discharge lasers at self-restricted transitions have been established, but there are still no publications about self-similar solutions of equations for PGL.

Note that the self-similar solutions reveal symmetry in dynamics of plasma formation and demonstrate that changes in scales of independent variables can be compensated by the similarity transformation of other dynamic PGL variables. The self-similar solutions are usually found by a self-similar substitution, whose form is defined by scale transformations of a system of differential equations.

It is known that volt-ampere characteristics for discharges of PGL with different active media are similar in time under similar pumping schemes. The presence of this group attribute indicates a possibility of using methods of group analysis in the search for invariant (scaling) transformations of discharges' plasma and in determination of plasma parameters as typical functions of time for certain phases of discharge evolution. At present, group analysis of differential equations is the most powerful and universal method in searching for wide classes of exact solutions for differential equations of an arbitrary form.

The system of equations describing the evolution of pulse discharge plasma

Let us consider the plasma of pulse-periodic longitudinal low-temperature gas discharge of high-current stage, having a constant chemical composition under conditions of a high pre-pulse electron concentration and a pulse length of several hundreds of nanoseconds. It is known[3] that the plasma evolution of such discharge is caused by the space ionization of the standard and excited states of gas atoms through electrons, accelerated by the external pulse of an electric field. The processes of plasma decomposition due to ambipolar diffusion and shock-radiating recombination proceed much slower, and so they can be neglected at current pulses, shorter than several hundreds of nanoseconds.

Let's write the dimensionless form of the system of differential equations describing plasma kinetics of the positive column of a pulse discharge in a gas:

$$\begin{cases} \dfrac{\mathrm{d}n_\mathrm{e}^*}{\mathrm{d}\tau} = n_\mathrm{e}^* \displaystyle\sum_{k=0}^{m} \nu_{k\mathrm{i}}^* - n_\mathrm{e}^* \nu_\mathrm{a}^*, & (1) \\ \dfrac{\mathrm{d}n_m^*}{\mathrm{d}\tau} = n_\mathrm{e}^* \left(\displaystyle\sum_{k=0}^{m-1} \nu_{km}^* - \nu_{m\mathrm{i}}^* \right), & (2) \\ \dfrac{\mathrm{d}(\varepsilon^* n_\mathrm{e}^*)}{\mathrm{d}\tau} = \dfrac{e^2 n_\mathrm{e}^*}{m_\mathrm{e}\varepsilon_\mathrm{i}\left(\nu_\mathrm{i}^0\right)^2} \dfrac{E^2(\tau)}{\nu_\mathrm{el}^*} - n_\mathrm{e}^* \displaystyle\sum_{k=0}^{i} (\varepsilon^* - \varepsilon_{k\mathrm{i}}^*)\nu_{k\mathrm{i}}^* - \\ \quad - n_\mathrm{e}^* \displaystyle\sum_{k=0}^{i-1} (\varepsilon^* - \varepsilon_{km}^*)\nu_{km}^* + \varepsilon^* n_\mathrm{e}^* \nu_\mathrm{a}^* \delta(\varepsilon), & (3) \end{cases}$$

where $\tau = t\nu_\mathrm{i}^0$ is the reduced time of the discharge evolution; ν_i^0 is the full ionization frequency of an easily ionized admixture at the beginning of the high-current stage of the discharge; $\nu_{k\mathrm{i}}^* = \nu_{k\mathrm{i}} / \nu_\mathrm{i}^0$ is the reduced frequency of single ionization of the admixture atoms by electrons from the state k; $\nu_\mathrm{el} = \left(\sum_{k=0}^{i-1} \nu_{k\mathrm{i}} + \nu_l \right)$ is the total frequency of inelastic collisions of electrons with admixture atoms and elastic ones with atoms of the buffer gas,

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respectively; $n^* = n_0^0 / n_e^0$ is the concentration of atoms of an easily ionized admixture reduced to concentration of electrons at the beginning of the discharge high-current stage;

$$n^* = n_e^* + \sum_{k \geq 0}^{m} n_k^*, \quad n_e^* = n_e / n_e^0, \quad n_m^* = n_m / n_e^0$$

are reduced concentrations of gas atoms, plasma electrons, atoms that are excited to the level m, respectively; $\varepsilon^* = \varepsilon / \varepsilon_{0i}$ is the reduced mean energy of plasma electrons; $\nu_a^* = 6D_a / r_p^2$ is the frequency of ambipolar diffusion;

$$\delta(\varepsilon) = \left[\ln \frac{r_p T_g^{0.5}}{2.4 D_a M_i^{0.5}} + \ln \frac{M_i \varepsilon^*}{m_e \varepsilon_a^*} \right]$$

is the part of energy that is taken by electrons in the near-wall layer of plasma; ε_{ki} is the ionization energy of gas atoms from the level k; $E = E_0 \varphi_0(\tau)$, E_0 is intensity of the electric field in the discharge plasma at the beginning of the high-current (arc) stage of the discharge; $\varphi_0(\tau)$ is the function describing the change of intensity of the electric field during the current pulse.

Group analysis of the system of differential equations describing kinetics of gas discharge plasma

The kinetic equations (1)–(3) of the system are linear differential equations of first order with coefficients depending on electron energy, i.e., implicitly depending on time. Let's denote the expression $\sum_{k=0}^{m} \nu_{ki}^*$ (the reduced complete frequency of gas ionization) by ν_i^*, as well as $\sum_{k=0}^{m-1} \nu_{km}^* - \nu_{mi}^*$ (the reduced frequency of excitation of the m-level) by ν_m^*. Introduce new generalized variables into equations (1)–(3) and transform equation (3) via equation (1). The system takes the following form:

$$\begin{cases} \dfrac{dn_e^*}{d\tau} = n_e^* \nu_i^* - n_e^* \nu_a^*, \\ \dfrac{dn_m^*}{d\tau} = n_e^* (\nu_m - \nu_{mi}^*), \\ \dfrac{d\varepsilon^*}{d\tau} + \varepsilon^* \nu_i^* \left(1 - \dfrac{\nu_a^*}{\nu_i^*} [1 + \delta(\varepsilon)] + \dfrac{\nu_{ki}^*}{\nu_i^*} + \dfrac{\nu_m^*}{\nu_i^*} \right) = \\ = \dfrac{e^2}{m_e \varepsilon_i (\nu_i^0)^2} \dfrac{E^2(\tau)}{\nu_{el}^*} + \varepsilon_{mi}^* \nu_i^* + \varepsilon_{km}^* \nu_m^*. \end{cases} \tag{4}$$

It is seen that the equation for ε does not contain n_e^* explicitly. It depends on the initial conditions, time function of the electric field, and frequencies of inelastic collisions.

Let us define the invariant transformations, which are admissible by system of differential equations (4). Using the algorithm of the search for group transformations for the system of differential equations,[6] we come to situation, when the system of kinetic equations admits the transformation group

$$\tau' = \tau e^a; \quad \nu_i' = \nu_i^* e^{-a}; \quad \nu_m' = \nu_m^* e^{-a}; \quad \nu_a' = \nu_a^* e^{-a};$$

$$\nu_{el}' = \nu_{el}^* e^{-a}; \quad (E)' = E e^{-a},$$

where $k = 0, 1, \ldots, i-1$; $m = 1, \ldots, i-1$; with the operator

$$\hat{X} = \tau \frac{\partial}{\partial \tau} + n_e^* \frac{\partial}{\partial n_e^*} - \nu_i^* \frac{\partial}{\partial \nu_i^*} - \nu_k^* \frac{\partial}{\partial \nu_k^*} - \nu_a^* \frac{\partial}{\partial \nu_a^*} - \nu_{el}^* \frac{\partial}{\nu_{el}^*} - E \frac{\partial}{\partial E}.$$

The range of existence of the obtained scale transformation of equation (3) is bounded by the condition $\nu_m^*/\nu_i^*; \nu_{mi}^*/\nu_i^*; \nu_a^*/\nu_i^* \ll 1$, which implies that the losses of electron energy for the working substance ionization considerably exceed other energy losses.

The operator $\hat{X}$ of the transformation group, which are admitted by the system, corresponds to the expansion group. A set of invariants of such a transformation for equations of discharge plasma kinetics (4) $\tilde{X}\varphi = 0$ is

$$I_1 = \tau \nu_i^*; \; I_2 = \frac{\nu_m^*}{\nu_i^*}; \; I_3 = \frac{n_e^*}{\tau}; \; I_4 = \frac{n_m^*}{\tau},$$
$$I_5 = \frac{\nu_a^*}{\nu_i^*}; \; I_6 = \frac{\nu_{el}^*}{\nu_i^*}; \; I_7 = \frac{e^2 E^2 \tau}{m_e \nu_{el}^* \varepsilon_i}; \; I_8 = \varepsilon^*. \tag{5}$$

Note that the invariants $I_2 - I_4$, I_7 of the transformation of system (4) coincide with those of the transformation of Boltzmann equations for different plasma particles,[7,8] and invariants

$$I_1 = \tau \nu_i^*, \; I_6 = \frac{e^2 E^2 \tau^2}{m_e \varepsilon_i}$$

define the dynamic similarity of the discharge plasma evolution.

Self-similar solutions of equations

Self-similar solutions of equations of system (1)–(3), which admit the extension group with the operator $\hat{X}$ have the form

$$u(\tau^*; \nu_i^*; \nu_m^*; \nu_a^*) = \tau^k \varphi_n \left(\frac{\varepsilon^*}{\tau^{\frac{\beta}{k}}}, \frac{E}{\tau^{\frac{\gamma}{k}}}, I_2, I_5 \right),$$

where φ_n is a new sought function of generalized variables and invariants defining the dynamic similarity of processes in the discharge plasma.

Following the idea of self-similarity, represent functions entering equations of the system (4) in the form

$$n_e^* = \tau\varphi_e;\ n_m^* = \tau\varphi_m;\ \varepsilon^* = \frac{1}{\tau}\varphi_\varepsilon. \qquad (6)$$

Here φ_k are implicit functions of time. Substituting representations (6) into equations of system (4), we obtain the following solutions:

$$\varphi_e = \exp\left(-\int_1^\tau \varepsilon^* \nu_a(\varepsilon_0)\,d\tau\right);$$

$$\varphi_m = \int_1^\tau \varphi_e I_1\left(I_2 - \frac{\nu_{mi}^*}{\nu_i^*}\right)d\tau;$$

$$\varphi_\varepsilon = \int_1^\tau \left(\frac{e^2}{m_e\varepsilon_i(\nu_i^0)^2}\frac{E^2(\tau)\tau^2}{\nu_{el}^*} + \varepsilon_{mi}^* I_1 + \varepsilon_{km}^* I_2\right)d\tau.$$

The self-similar solutions of equations of system (4) have the form

$$n_e^*(\tau) = n_e^0\tau\exp\left(-\nu_a(\varepsilon_0)\int_1^\tau \frac{\varphi_\varepsilon}{\tau}\,d\tau\right);$$

$$n_m^*(\tau) = n_m^0\tau\int_1^\tau \exp\left(-\nu_a(\varepsilon_0)\int_1^\tau \frac{\varphi_\varepsilon}{\tau}\,d\tau\right) I_1\left(I_2 - \frac{\nu_{mi}^*}{\nu_i^*}\right)d\tau;$$

$$\varepsilon^*(\tau) = \frac{1}{\tau}\varphi_\varepsilon.$$

Conclusion

The self-similar solutions of equations for n_e^*, n_m^*, and ε^* of system (4) depend on φ_ε, i.e., on the form of the dependence of the electric field on time. Note that the form of the function φ_ε depends on a particular scheme of the pumping source, which forms the pumping pulse and defines the time dependence of the electron temperature during the pumping pulse. Therefore, if the discharge pumping schemes are similar, the dependences of line intensities and discharge plasma parameters for plasma of such discharges are similar in time.

If the initial conditions (scales of variables) $\nu_i^0, n_e^0, \nu_k^0, \nu_{el}^0, \nu_a^0, E_0, \varepsilon_{0i}$, are changed in the same pumping scheme ($\varphi_e(\tau)$=const), the self-similar solutions undergo scale expansions. This agrees with the experimentally observed similarity of time dependencies of levels' population and intensities of spontaneous plasma radiation.

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