# STATISTICAL ESTIMATION OF OPTICAL WAVE ATTENUATION IN PROROGATION THROUGH A CLOUDY ATMOSPHERE

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*Based on different cloud models considered, paper gives a statistical estimate of optical wave attenuation. Using the results obtained, a model of a cloudy atmosphere is introduced and a method is proposed to calculate the probability that a given attenuation is not exceeded at any location within the USSR in any month. It is shown that this probability is considerably increased when diversity reception is employed.*

When optical waves propagate along vertical and slant paths the greatest attenuation is due to clouds.[1] To assess the efficiency of various optical systems when their line of sight passes through clouds, a method is required to calculate different attenuation probabilities based on statistical information about cloud fine structure, geometry, size, and probability of occurance. As regards optical channel serviceability, the probability that a given attenuation $A_0$ due to clouds is not exceeded, i.e., $P(A_0) = P(A \leq A_0)$, is to be estimated. This paper presents a method for the calculation this probability for some types of clouds, cloud formations, and for an entire cloudy atmosphere. In view of the importance of this quantity within the framework of this paper it requires a special name, so it is .called here "a communication probability" because for communication systems, it is the probability of communication provided that the feasibility of of communications is determined solely by optical wave attenuation.

Approximate calculations of probability distributions of attenuation for optical radiation in clouds were presented earlier[1] for four climatic zones of the USSR and for different seasons. These calculations were based on the probability distribution of liquid water content (LWC) in clouds without any classification by cloud type and without taking into consideration the local statistics of clouds. Obviously, such an assessment should be considered to be very approximate and absolutely unacceptable for a determination of the probability characteristics of radiation losses due to clouds at a specific location and for a given time of a year.

The complex refractive index of water does not depend on temperature in the optical range. In practice the specific attenuations for water and ice spheres differ by no more than a factor of 1.5 for typical distributions of particle sizes. Therefore in our calculations all clouds are considered to consist of liquid droplets at a temperature of +20°C[3,4,5].

We first find the communication probability for six types of clouds: St, S As, Ci, Ns, Cu which are chosen for study due to their high recurrence and large attenuation (except Ci). We consider Cirrus clouds, in spite of their relatively small attenuation, because in the absence of low clouds the presence of even weakly absorbing media (and small attenuations are of the greatest importance in practice) will be the major factor in a communication probability limitation.

For the purpose of calculations _ it is necessary to know the LWC profiles $q(h)$ for different types of clouds, and using these profiles to compute the vertical water capacity, we have

$$W(H) = \int_0^H q(h)dh.$$

The typical average LWC profiles vary in clouds of different types. In large cumulus clouds the LWC variation versus the altitude (from the low cloud edge) has a maximum in the upper part of the cloud. The approximation of the experimental data in Ref. 1 gives the following relationship between the averaged LWC and cloud thickness $H$:

$$\bar{q}(H) = 0,71H^{4/3} + 0.1.$$

Therefore the water capacity of a cloud depends on its thickness as

$$W(H) = 0.17H^{7/3} + 0.1H$$

In stratus clouds such as St, Sc, the averaged LWC increases up to 0.6–0.7 km from the low cloud edge and then it diminishes[2]. Such behavior can be approximated by a log-normal distribution,

$$q(H) = \frac{C}{h} \exp\left[ \frac{\ln^2(h/h_0)}{2\sigma^2} \right].$$

The parameters have the following values for different seasons[2]:

winter $C = 0.377$; $h_0 = 6.320$; $\sigma = 1.583$
spring autumn $C = 0.501$; $h_0 = 6.280$; $\sigma = 1.596$
summer $C = 1.305$; $h_0 = 23.221$; $\sigma = 1.900$,

where $h$ is measured in km. The water capacity can be calculated from the formula

$$W(G) = C\sqrt{\frac{\pi}{2}}\,\sigma\left[1 + \mathrm{erf}\left\{\frac{\ln(h/h_0)}{\sigma\sqrt{2}}\right\}\right]$$

where $\mathrm{erf}(z) = \frac{2}{\sqrt{\pi}}\int_0^z e^{-t^2}dt$ is the error function.

In altostratus and nimbostratus, the LWC scarcely changes with altitude up to $h = 1$ km, and then slowly decreases[2]. The LWC value depends on the season[1]. The water capacity can be given by the formula

$$W(H) = \begin{cases} q_0H - 0.01H^2 + 0.01 & \text{for } H > 1\text{km} \\ q_0H & \text{for } H \le 1\text{km}. \end{cases}$$

The values for $q_0$ are as follows:
for Ns $q_0 = 0.15$ (winter),
$q_0 = 0.35$ (summer),
$q_0 = 0.22$ (spring, autumn),

for As $q_0 = 0.14$ (winter),
$q_0 = 0.18$ (summer),
$q_0 = 0.19$ (spring, autumn)

For high clouds, the LWC is assumed to be a constant equal to $q(h) = 0.01$ g*m$^{-3}$, since the experimental data are not available and the thickness of these clouds is relatively small, so that[3]

$$W(H) = 0.01H.$$

Thickness repeatability for different cloud types has been observed[2] at 33 locations of the USSR and these data have been used for calculations of the cumulative thickness probability. For high clouds the recurrence is taken from the Ref. 5.

The total attenuation of direct radiation may be written as[1]

$$A(H) = 4.34\frac{K_{\alpha N}}{K_{qN}}\int_0^H q(h)dh = 4.34\frac{K_{\alpha N}}{K_{qN}}W(H),$$

where $K_{qN}$ is the average particle mass and $K_{\alpha N} = \pi\int_0^\infty r^2Q(r\lambda)f(r)dr$ is a relation between the attenuation and the number density of particles[1]. Here, $Q(r, \lambda)$ is an efficiency factor for radiation attenuation by a single particle of radius $r$, $\lambda$ is the wavelength, $f(r)$ is the two-parameter gamma distribution (its parameters are taken from Ref. 7 for the cirrus and from Ref. 6 for the other cloud types). Hence, the attenuation probability distribution can be expressed in terms of the cloud thickness probability distribution $p_H(H)$ as

$$p_A(A) = \frac{p_H[H(A)], K_{qN}}{4.34\,K_{\alpha N}q[H(A)]}$$

where $H(A)$ is a the inverse of the previous expression and, consequently, the communication probability for the corresponding cloud type will be

$$P_{COM}(A_0) = \int_0^{A_0} P_A(A)dA. \quad (1)$$

Thus the communication probability for the specified cloud types can be calculated as a function of the wavelength and the season. The computations were carried out for 33 locations inside the USSR. For specific types of clouds such an approach seems to be justified because their structure and probability of occurence are much more stable than for the overall cloud population. The communication probabilities for the propagation of optical radiation through a cloudy atmosphere will be now calculated as a function of the threshold attenuation value, wavelength, and month of the yean for a given location for which enough statistical data are available. It will be shown below that the meteorological network of the USSR provide a suitable set of points because statistical data of long standing[8] enable one to perform the required calculations. The meteorological network is quite dense (about 3000 locations), and though they are not very uniformly spaced, their number is such that at any point in our country there are sufficient data on cloud cover.

Consider a cloudy atmosphere with cloud types such as St, S As, Ci, Ns, Cu. In examining the radiation propagation of radiation through a cloudy atmosphere, horizontal nonuniformity of the cloud cover must be taken into consideration. This nonuniformity is expressed as a cloud-cover index (one index unit implies 1/10th of the sky fully covered by clouds).

Let us divide the cloudy atmosphere into layers. Each layer may contain one or more cloud types. Assuming that radiation propagating through the atmosphere is attenuated in layers $i_j$, and hence falls through the breaks in the clouds of layers $k_l$. It is obvious that the probability that this will occur is

$$P_m = \prod_{j=1}^{N}\prod_{l=1}^{M}\sum_{n_{i_j}}\sum_{n_{k_l}} P_S(\{n\})n_{i_j}(10-n_{k_l})/100$$

where $m$ is the serial order of occurrence, $n$ is the cloud-cover index in the layer, $N$ is the number of absorbing layers, $M$ is the number of layers with breaks, and $P_S(\{n\})$ is the probability of simultaneous appearance of cloudy layers with the corresponding cloud-clever index. If the cloud layers are assumed to be independent of one another, then this expression takes the form

$$P_m = \prod_{j=1}^{N}\prod_{l=1}^{M}\langle n_{i_j}\rangle(10-\langle n_{k_l}\rangle)/100, \quad (2)$$

where $\langle n_{i_j}\rangle$ is the mean of the corresponding cloud-cover index. Now let each layer $i_j$ have the communication probability $P_{COM}(A)$, where $A$ is the attenuation. Then the communication probability at the $m$-th occurrence is obviously

$$P_{COMm}(A) = \int \cdots \int p_s(\xi_{i_1} \cdots \xi_{1_N}) d\xi_{i_1} \cdots d\xi_{1_N},$$

where $p_s(\xi_{i_1} \ldots \xi_{1_N})$ is the joint distribution of attenuation values $\xi_{i_1} \ldots \xi_{1_N}$ in absorbing layers $i_j$ or, assuming again the independence of the cloud layers,

$$P_{COMm}(A) = \int \cdots \int p_{1_N}(\xi_{1_1}) \cdot \cdots p_{1_N}(\xi_{1_N}) d\xi_1 \cdot \cdots d\xi_{1_N} \quad (3)$$

where $p_{i_j}(\xi)$ is a joint distribution for attenuation $\xi_{i_1} \ldots \xi_{1_N}$ by cloudy layer $i_j$. In both cases, the integrals are evaluated over the volume which is bounded by the hyperplanes $\xi_{i_j}$ and $\sum_{j=1}^{N} \xi_{i_j} A$. The communication probability for the entire cloudy atmosphere will be

$$P_{COM}(A) = \sum_{m=1}^{2^{(N+M)}} P_m P_{COMm}(A). \quad (4)$$

To make use of this formula it is necessary to have expressions for $p_{i_j}(\xi)$, which can be derived from

$$p_{i_j}(\xi) = \frac{d}{d\xi}\left(P_{COMi_j}(\xi)\right), \quad (5)$$

where $P_{COMi_j}(\xi)$ is the communication probability for the corresponding layer.

To calculate these probabilities let us again use the proposed technique. Consider the layer $i_j$ consisting of specific cloud types. Using the same j reasoning as above, expressions similar to (2), (3), (4), (5) are obtained (where cloud types will be used rather than layers). Equation (1) gives в the expressions required to calculate of communication probability for individual cloud types.

The most detailed information on cloudiness over the USSR can be found in Ref. 8, where different characteristics of clouds are presented separately for two categories: low-level and overall cloudiness. Clouds with an altitude limit of 2000 m are classified as low cloudiness. All clouds observed at a given time fall under the heading of overall cloudiness. In accordance with this definition, the atmosphere is divided into 2 layers: low and high cloudiness:

In Ref. 8, no probability characteristics for, high cloudiness are given, but they may be derived f from the tables therein. How to do so will be shown below.

To smooth observational inaccuracies in Ref. 8, the cloud-cover index values are divided into three groups: clear sky (index 0–2), overcast (index 8–10), and partly cloudy (index 3–7). It is then sufficient to calculate the. high cloudiness probability characteristics of these three groups alone. It is easily seen that the clear and overcast probabilities for high cloudiness are described by the following expressions respectively:

$$P_h(0-2) = \frac{P_0(0-2)}{P_1(0-2)}$$

$$P_h(8-10) = \frac{P_0(8-10) \quad P_1(8-10)}{P_1(8-10)}$$

and

$$P_h(3-7) = 1 - P_h(0-2) - P_h(8-10),$$

where $P_0(\xi) = P_{overall}(\xi)$, $P_1(\xi) = P_{low}(\xi)$, $P_h(\xi) = P_{high}(\xi)$.

The possible situations for optical beam propagation are presented in Table 1.

*TABLE 1.*

| m – situation | | | | |
|---|---|---|---|---|
| high cloudines | + | + | – | – |
| low cloudiness | + | – | + | – |

Here signs "+" and "–" mean that optical radiation is absorbed by or propagates freely in the layer under consideration.

Corresponding values of probability $P_m$ for these situations, using Eq. (2) and communication probabilities $P_{COMm}(A)$ in accordance with (3), will be

$$P_1 = \langle n_1 \rangle \langle n_h \rangle / 100, \quad P_{COM1}(A) = \iint p_1(\xi_1) p_h(\xi_h) d\xi_1 d\xi_h,$$

$$P_2 = \langle n_1 \rangle (10 - \langle n_h \rangle)/100, \quad P_{COM2}(A) = P_{COMh}(A)$$

$$P_3 = \langle n_h \rangle (10 - \langle n_1 \rangle)/100, \quad P_{COM3}(A) = P_{COM1}(A)$$

$$P_4 = (10 - \langle n_1 \rangle)(10 - \langle n_h \rangle)/100, \quad P_{COM4}(A) = 1 \quad (6)$$

where $\langle n_l \rangle$ and $\langle n_h \rangle$ are the average cloud-cover indices for of the corresponding cloudiness at a given location and in a specific season. The integration is over a portion of the plane bounded by the straight lines $\xi_l = 0$, $\xi_h = 0$, $\xi_l + \xi_h = A$. Here, as in (5),

$$p_1(\xi) \ \frac{dP_{COM1}(\xi)}{d\xi} \quad \text{and} \quad p_h(\xi) \ \frac{dP_{COMh}(\xi)}{d\xi}$$

Thus, the functions $P_{COMl}(\xi)$ and $P_{COMh}(\xi)$ are critical. To establish their values, the above method may be used again. But in that case it is necessary to obtain information on local $p_{i_j}(\xi)$ (here $p_{i_j}(\xi)$ is a probability distribution of attenuation by clouds of the $i_j$ types) as well as on the cloud-cover repeatability for the corresponding cloud types. However this information is not usually available, and it can be obtained only for a specific region and season[2]. As mentioned above, these characteristics are usually quite stable, so the regional and seasonal characteristics of specific cloud types can be related to a given location and time.

In our case, cloud types St, S As, Ci belong to the high layer and Ns, Ci- to the low layer. Therefore the table of situations for the low layer will be similar to that presented above. As in the above case, the probabilities of situations and corresponding communication probabilities will be defined in a similar manner, but the parameters in (6) should be changed:

$$\langle n_1\rangle \longrightarrow \langle n_{Ns}\rangle,\quad p_1(\xi) \longrightarrow p_{Ns}(\xi),\quad P_{COM1}(\xi) \longrightarrow P_{COMNs}(\xi),\quad \langle n_h\rangle \longrightarrow \langle n_{Cu}\rangle,\quad p_h(\xi) \longrightarrow p_{Cu}(\xi),\quad P_{COMh}(\xi) \longrightarrow P_{COMCu}(\xi).$$

For the high layer the number of possible situations is 16. The corresponding $P_m$ values are equal to the product of averaged absorbing cloud-cover indices with $10^{-4}$, and with the result multiplied by the products $(10 - \langle n\rangle)$ for clouds with breaks (in accordance with (2)).

This method has been used to develop a computer program for communication probability calculations. As an example, the communication probability calculations have been carried out for three locations: Moscow, Gillicule (38°N, 72°E) and Tompo (64°N, 136°E) (the locations in the USSR territory which have maximum communication probability are Gillicule in summer and Tompo in winter). The calculations have shown that the dependence between $P_{COM}$ and the wavelength in the 1–11 μm range is low. The results of the calculations are presented in Figs. 1–5.

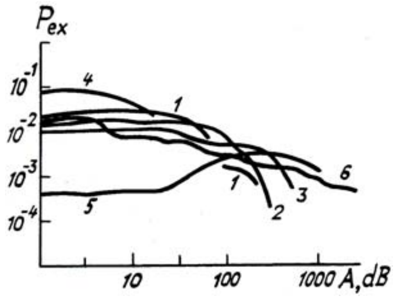


*FIG. 1. Cloud attenuation probability distributions for Moscow. Summer. Numbers of curves correspond to 1 – St, 2 – Sc, 3 – As, 4 – Ci, 5 – Ns, 6 – Cu.*

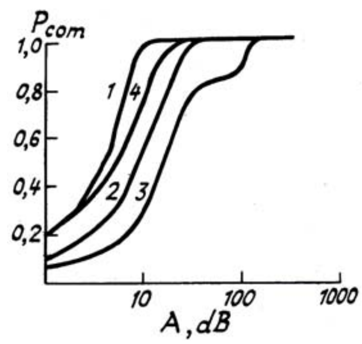


*FIG. 2. Communication threshold attenuation values 1 – St, 2 – Sc, 3 – As, 4 – Ci.*

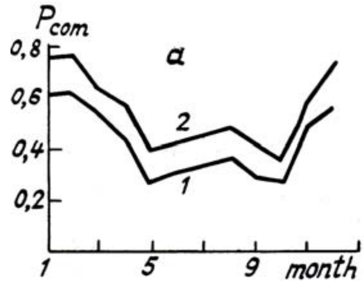


*FIG. 3. Annual behaviour of communication probability in Tompo. 1 – 5 dB attenuation, 2 – 50 dB attenuation.*

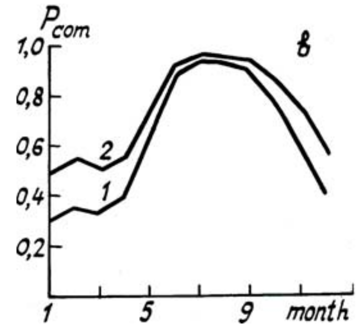


*FIG. 4. Annual behaviour of communication probability in Gillicule. 1 – 5 dB attenuation, 2 – 50 dB attenuation.*

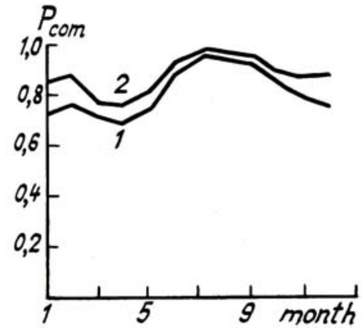


*FIG. 5. Annual behaviour of communication probability with diversity reception (Tompo and Gillicuie). 1 – 5 dB attenuation, 2 – 50 dB attenuation.*

It can be seen from these curves that a high communication probability at a single location cannot be provided during the whole year. The use of diversity reception (the separation distance between two locations must exceed the correlation length for cloud occurrence) enables the communication probability to be drastically increased during the whole year. Figure 5 shows the annual communication probability behaviour with diversity reception (the distance between Gillicule and Tompo is about 6000 km).

It should be mentioned that this method does not explicitly employ the wavelength as a parameter. In principle it may be extended to ranges other than optical.

## CONCLUSION

1. For the most critical cloud types (from the standpoint of energy loss) expressions for the vertical LWC profiles are given. Based on these profiles, the radiation attenuation versus cloud thickness in these clouds is obtained.

2. To calculate the communication probability in the chosen cloud types for four seasons and 33 locations within the USSR, a special method is proposed.

3. This method, which is applicable for any month and the whole of the USSR, is based on cloudiness statistical data of long standing, and takes into consideration the three-dimensional structure of the cloudy atmosphere.

4. The proposed method may be used over a rather wide range of wavelengths for which information on attenuation in specific types of clouds is available.

5. This method enables a suitable location to be evaluated as regards the operation of devices which use optical wave propagation to and from the zenith (for example, in an optical communication link).

6. A drastic improvement of communication probabilities for the whole year when using diversity reception is demonstrated.

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