# Optical model for investigating the characteristics of light transmitted through an oriented plate

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The optical model of a particle is developed for investigating the energy and polarization characteristics of light transmitted through a semitransparent oriented plate. The equations for scattering cross sections proportional to the corresponding parameters of the Stokes vector are obtained for any point of the forward hemisphere. These equations relate the physical parameters of the particle (dimensions, refractive index) and the parameters of the incident radiation (wavelength, state of polarization) to the characteristics of radiation transmitted through the crystal at different positions of the source, receiver, and the crystal.

## Introduction

Clouds play determining role in the thermal budget of the earth–atmosphere system. To date, the crystal clouds of complicated structure are poorly studied. For numerical studies of the transformations of light passing through the atmospheric formations it is necessary, first, to develop a model of an individual particle, which would allow one to adequately reveal the dependence of the light scattering characteristics on the parameters of the incident radiation and the parameters of the scatterer. The problem of extinction by small and large crystals with chaotic orientation is widely presented in the literature.[1,2] The extinction matrices are obtained for the particles of spherical and non-spherical shape. These models allowed the estimation to be done of both the intensity and polarization of the transformed radiation in the direction of propagation of the electromagnetic wave.

A preferred orientation of crystals, if any, leads to anisotropy of scattering. The difficulties of describing the transformation of light passed through a set of oriented semitransparent crystals make the problem of light scattering by ice atmospheric formations open. In determining the extinction of light by oriented crystals with plane-parallel sides, one should take into account the commensurability of the diffraction field and the field of the refracted beams that pass through the particle.[3,4] The method of physical optics allows one to coherently sum the aforementioned fields and to take into account the wave nature of the electromagnetic radiation in determining the total scattered field in the far zone.

The light scattering characteristics are very sensitive to variations of microphysical and optical properties of oriented crystals, especially near the forward scattering direction. Such characteristics as cross section or the extinction efficiency factor (for individual particles)[5,6] and the extinction coefficient (for the system of particles)[7] are mainly obtained for analysis of light extinction by oriented crystals. Numerical investigations have shown that the extinction efficiency factor of ice plates can vary within the limits from 0 up to 4. For larger plane-parallel sides of a particle this characteristic varies within a narrower range.[8] Besides, destruction of parallelism of the sides also leads to the tendency in the extinction efficiency factor to asymptotically approach the value of 2. Thus, ice plates are peculiar objects among all types of crystals due to their extinction properties. The model for calculating the energy and polarization characteristics of light scattered into the forward hemisphere is being developed in this paper for this type of crystals.

## Role of the positions of a radiation source, receiver, and scatterer in determination of the light scattering characteristics into the forward hemisphere

As applied to determination of the light scattering characteristics at any point of the forward hemisphere, let us introduce necessary angular characteristics relating the positions of the source, receiver, and the particle (see Fig. 1). Let the source of radiation be at the point $O_1$, the receiver at the point $O_2$, and the particle at the point $O_3$. Let also the $Oxyz$ be the absolute coordinate system, relative to which three more coordinate systems, $\hat{I}_1\tilde{o}_1\acute{o}_1z_1$, $\hat{I}_2\tilde{o}_2\acute{o}_2z_2$, and $\hat{I}_3\tilde{o}_3\acute{o}_3z_3$ are introduced, that are related, respectively, to the source, the receiver, and the scatterer. The coordinate plane $Oxy$ is parallel to the ground surface, and the normal to it is parallel to the $Oz$ axis.

The incident radiation propagates along the positive direction of $\hat{I}_1z_1$ axis. The Pointing wave vector **k** indicates the direction of the incidence of light on the plate base. It is obvious that $\mathbf{k} \parallel \hat{I}_1z_1$.

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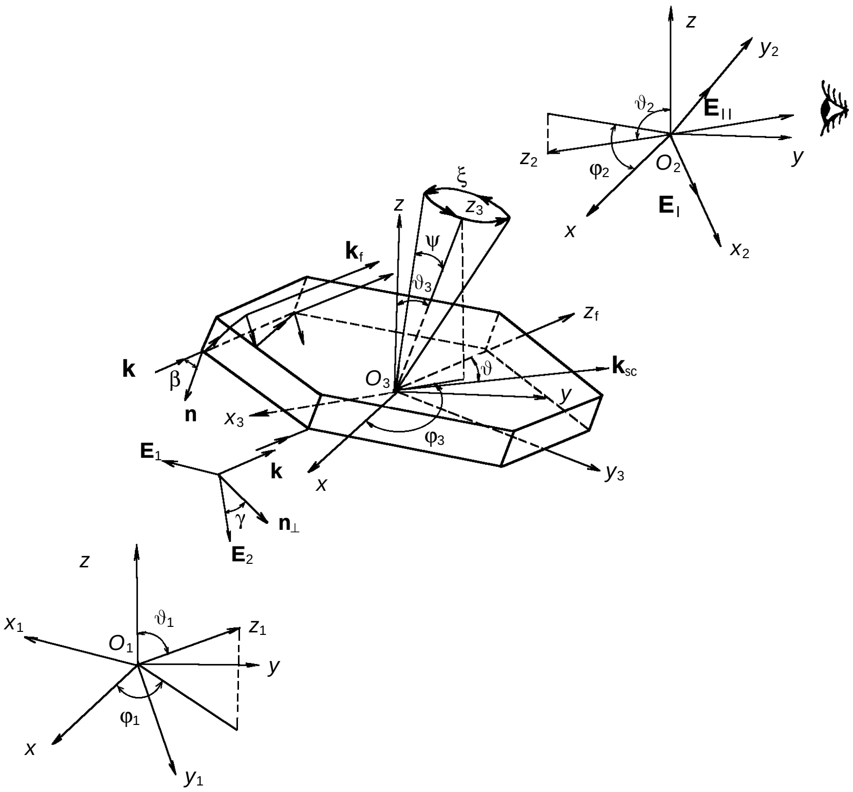


**Fig. 1.** The set of the coordinate systems for description of the light scattering into the forward hemisphere.

Electric components of the incident wave of the elliptic polarization ($\mathbf{Å}_1$, $\mathbf{Å}_2$) are directed, respectively, along the axes Î$_1$õ$_1$ and Î$_1$ó$_1$; $\mathbf{n}_{\perp}$ is the normal to the incidence plane. An oriented plate is introduced as the scattering particle. The plane passing through the crystal base is the coordinate plane Î$_3$õ$_3$ó$_3$ of the coordinate system Î$_3$õ$_3$ó$_3$$z_3$, β is the angle between the direction of the radiation incidence **k** (or the axis Î$_1$$z_1$) and the normal **n** to the plate base (or the axis Î$_3$$z_3$). Electromagnetic field represented by the beams outgoing after a number of internal reflections is formed in the direction Î$_3$$z_3$. The vector $\mathbf{k}_f$ indicates the direction of the refracted beams outgoing from the plate. Obviously, $\mathbf{k} || \mathbf{k}_f ||$ Î$_3$$z_f$. The elevation angle ψ and azimuth angle ξ set the possible positions of the plate caused by its oscillation about the axis Î$_3$$z_3$. At a fixed ψ value and continuous change of ξ from 0 up to 2π the normal direction to the plate base circumscribes the cone with the axis Î$_3$$z_3$. The scattered radiation is received from the direction $\mathbf{k}_{sc}$ (or axis Î$_2$$z_2$), and the axis Î$_2$$y_2$ is parallel to the horizontal plane (or the ground surface). The vectors $\mathbf{E}_{\perp}$ and $\mathbf{E}_{||}$ are the components of the field detected at the receiver. They are directed along the Î$_2$$x_2$ and Î$_2$$y_2$ axes, respectively. Let us denote the deviation of the receipt direction from the line the outgoing beams escape from the plate (i.e. the angle between the directions Î$_3$$z_f$ and $\mathbf{k}_{sc}$) as ϑ.

For introducing the normalized light scattering characteristics, it is sufficient to determine the angular position of the unit vector setting the components of the scattered field. It this relation, let us superpose the centers of all four coordinate systems at the point *O* and define the angular dependences of the unit vectors (**x**, **y**, **z**) of the absolute coordinate system with the unit vectors ($\mathbf{õ}_i$, $\mathbf{ó}_i$, $\mathbf{z}_i$) by the following relationship:

$$\begin{pmatrix} \mathbf{x} \\ \mathbf{y} \\ \mathbf{z} \end{pmatrix} = S_i \begin{pmatrix} \mathbf{x}_i \\ \mathbf{y}_i \\ \mathbf{z}_i \end{pmatrix}, \quad i=1, 2, 3, \tag{1}$$

where

$$S_i = \begin{pmatrix} \cos\varphi_i \cos\vartheta_i & -\sin\varphi_i & \cos\varphi_i \sin\vartheta_i \\ \sin\varphi_i \cos\vartheta_i & \cos\varphi_i & \sin\varphi_i \sin\vartheta_i \\ -\sin\vartheta_i & 0 & \cos\vartheta_i \end{pmatrix}.$$

Obviously, the angles $\vartheta_i$ and $\varphi_i$ determine the position of the unit vectors $\mathbf{õ}_i$, $\mathbf{ó}_i$, $\mathbf{z}_i$ ($i$ = 1, 2, 3) of each of the three coordinate systems $Ox_i$ó$_i$$z_i$ ($i$ = 1, 2, 3) relative to the absolute system *Oxyz*.

As conventionally accepted the rotations of a body in space relative to the Cartesian coordinate system are described by the Euler matrix.[2] For this reason, let us determine the position of the components of the incident field ($\mathbf{Å}_1$, $\mathbf{Å}_2$) in the coordinate system related to the scatterer. The unit vectors of the coordinate systems Îõ$_1$ó$_1$$z_1$ and Îõ$_3$ó$_3$$z_3$ can be represented by means of the linear transformation

$$\begin{pmatrix} \mathbf{x}_3 \\ \mathbf{y}_3 \\ \mathbf{z}_3 \end{pmatrix} = A \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{y}_1 \\ \mathbf{z}_1 \end{pmatrix}, \tag{2}$$

where

$$A = \begin{pmatrix} \cos\alpha\cos\beta\cos\gamma - \sin\alpha\sin\gamma & -\cos\alpha\cos\beta\sin\gamma - \sin\alpha\cos\gamma & \cos\alpha\sin\beta \\ \sin\alpha\cos\beta\cos\gamma + \cos\alpha\sin\gamma & -\sin\alpha\cos\beta\sin\gamma + \cos\alpha\cos\gamma & \sin\alpha\sin\beta \\ -\sin\beta\cos\gamma & \sin\beta\sin\gamma & \cos\beta \end{pmatrix}.$$

The Euler angles $\alpha$, $\beta$, $\gamma$ are defined as some combinations of the angles $\vartheta_1$, $\varphi_1$ and $\vartheta_3$, $\varphi_3$.[9]

The elements of the matrix $A$ determine the positions of the vectors $\mathbf{k}$, **Å**$_1$, and **Å**$_2$ in the coordinate system Î õ$_3$ó$_3$$z_3$. One should note that the component $\mathbf{E}_1$ or $\mathbf{E}_2$ at arbitrary orientation angle of the polarization plane $\gamma$ does not lie in the plane of the wave incidence. So, for further calculations of the light scattering characteristics and application of the Fresnel formulas it is necessary to transform the components $\mathbf{E}_1$ and $\mathbf{E}_2$ so that one of them is perpendicular to the plane of incidence and the other one lies in it. To do this, let us use the following linear transformation:

$$\begin{pmatrix} \mathbf{E}_\parallel \\ \mathbf{E}_\perp \\ \mathbf{k} \end{pmatrix} = F \begin{pmatrix} \mathbf{E}_1 \\ \mathbf{E}_2 \\ \mathbf{k} \end{pmatrix}, \qquad (3)$$

where

$$F = \begin{pmatrix} -\cos\gamma & \sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

The vectors **Å**$_\perp$ and $\mathbf{E}_\parallel$ in the coordinate system related to the plate are determined by the elements of the first and second columns of the matrix $A_F = AF$. Let us determine the angles setting the position $\mathbf{z}_2$ relative to the vectors **Å**$_\perp$, $\mathbf{E}_\parallel$, $\mathbf{k}_\mathrm{f}$. If defining the matrix B as $S_3^{-1}S_2$,[9] the sought projections of the vector $\mathbf{z}_2$ are written as

$$\cos\psi_x = A_{F_{11}}B_{13} + A_{F_{21}}B_{23} + A_{F_{31}}B_{33},$$

$$\cos\psi_y = A_{F_{12}}B_{13} + A_{F_{22}}B_{23} + A_{F_{32}}B_{33}, \qquad (4)$$

$$\cos\psi_z = A_{F_{13}}B_{13} + A_{F_{23}}B_{23} + A_{F_{33}}B_{33}.$$

Taking into account that

$$\cos\psi_x = \sin\vartheta\cos\varphi,$$

$$\cos\psi_y = \sin\vartheta\sin\varphi,$$

$$\cos\psi_z = \cos\vartheta,$$

it is easy to determine $\vartheta$ and $\varphi$. The angles $\vartheta$ and $\varphi$ are counted from the direction $\mathbf{k}_\mathrm{f}$. Let us introduce a new coordinate system Î õ$_s$ó$_s$$z_s$ related to Î õ$_3$ó$_3$$z_3$:

$$A_T = A_F S P,$$

where

$$S_i = \begin{pmatrix} \cos\varphi\cos\vartheta & -\sin\varphi & \cos\varphi\sin\vartheta \\ \sin\varphi\cos\vartheta & \cos\varphi & \sin\varphi\sin\vartheta \\ -\sin\vartheta & 0 & \cos\vartheta \end{pmatrix}$$

$$\text{and } P = \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

The electric components of the scattered field **Å**$_{\mathrm{s}_1}$, **Å**$_{\mathrm{s}_2}$ are determined by the elements of the first and the second columns of the matrix $A_T$. These vectors lie in the plane perpendicular to the direction of radiation receipt Î $z_2$. Let $\mathbf{E}_\mathrm{I}$ and $\mathbf{E}_\mathrm{II}$ be the components of the field detected at the receiver. Then the ratios between **Å**$_{\mathrm{s}_1}$ **Å**$_{\mathrm{s}_2}$ and $\mathbf{E}_\mathrm{I}$, $\mathbf{E}_\mathrm{II}$ are determined by the linear relationships:

$$\mathbf{E}_\mathrm{I} = -\cos u\mathbf{E}_{\mathrm{s}_1} + \sin u\mathbf{E}_{\mathrm{s}_2},$$

$$\mathbf{E}_\mathrm{II} = \sin u\mathbf{E}_{\mathrm{s}_1} + \cos u\mathbf{E}_{\mathrm{s}_2}, \qquad (5)$$

where

$$\cos u = A_{T_{12}}B_{12} + A_{T_{22}}B_{22} + A_{T_{32}}B_{32};$$

$$\sin u = A_{T_{11}}B_{12} + A_{T_{21}}B_{22} + A_{T_{31}}B_{32}.$$

## Statement of the problem

Let us consider a hexagonal plate with the base side of size $a$ and the thickness $d$ as a scatterer. According to the law of crystallographic growth, the following dependence is true between the diameter and the thickness of the plate[1]: $d = 2.020(2a)^{0.449}$. The semitransparent ice plate has the refractive index $\tilde{n} = n + i\chi$. As was mentioned above, the position of the hexagonal plate is set in the coordinate system $Ox_3y_3z_3$, and the incident wave is set in the coordinate system $Ox_1y_1z_1$. These two coordinate systems are related to each other by the Euler angles $\alpha$, $\beta$, $\gamma$. The angle $\alpha$ determines the turn of the plate around the axis $Oz_3$, the angle $\beta$ coincides with the angle of incidence of the beam on the plate base, and the angle $\gamma$ sets the orientation of the polarization plane relative to the incidence plane, or, in other words, it determines the possible positions of mutually orthogonal vectors $\mathbf{E}_1$ and $\mathbf{E}_2$ at their turn around the axis $Oz_1$. After incidence of the field on the plane, a part of it is reflected along the direction $(\pi-\beta)$, and another part passes into the particle where it undergoes refraction and absorption. After a number of internal reflections, the refracted beams outgoing from the plate are formed mainly along the direction $\beta$. To determine the total scattering, one should take into account the diffraction field caused by the appearance of geometric shadow in the wave front after passing through the medium containing the scatterer, in addition to the refracted beams.

Let us determine the scattered field in the forward hemisphere. To do this, the following procedure for solving the problem is proposed. First, let us obtain the total scattered field as coherent sum of the refracted and diffracted fields. Then, to determine the transformed field in the far zone, let us use the method of physical optics, which takes into account all necessary maxima in calculation of the Fraunhofer integral. Then let us obtain the relationships for the scattering cross sections reduced to the corresponding parameters of the Stokes vector for the characteristics of light scattered into the forward hemisphere.

## Field of light scattered into the forward hemisphere

One can represent the electromagnetic field of the plane incident wave in the coordinate system $\hat{I}\,\tilde{o}_1\acute{o}_1 z_1$ in the form of the electric component $\mathbf{E} = \mathbf{x}_1 E_1 + \mathbf{y}_2 E_2$, magnetic component $\mathbf{H} = -\mathbf{x}_1 H_2 + \mathbf{y}_1 H_1$, and the wave vector $\mathbf{k}$. Let us note that the amplitudes of the electric and magnetic components are related to each other. So, we shall not present the formulas for the magnetic components in this paper.

Let us determine the electric component of the total electromagnetic field scattered into the forward hemisphere. Let us consider the total field to mean the field formed in the far zone resulting from coherent sum of the fields diffracted at and refracted through the scatterer. Let us represent the scattered field by the known relationship[10]:

$$\mathbf{E}_s = \mathbf{A}\frac{e^{ikr}}{ikr}. \tag{6}$$

Let us set the vector $\mathbf{A}$ in the form of the sum of two mutually perpendicular vectors $\mathbf{A}_1$ and $\mathbf{A}_2$: $\mathbf{A} = \mathbf{A}_1 + \mathbf{A}_2$. The complex amplitudes of the scattered ($\mathbf{A}_1$, $\mathbf{A}_2$) and the incident ($\mathbf{E}_1$, $\mathbf{E}_2$) fields are related by the relationship[10]:

$$\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \begin{pmatrix} E_1 \\ E_2 \end{pmatrix}, \tag{7}$$

where $b_{ij}$ ($i$ = 1, 2; $j$ = 1, 2) are the elements of the amplitude matrix.

To determine the scattered field, let us use the method of physical optics,[3] which allows one to coherently sum the diffraction field and the field of the refracted beams taking into account the phase shift:

$$\begin{aligned} A_1 &= (A_{\mathrm{D}_1} - A_{\mathrm{R}_1})(\boldsymbol{\vartheta}_0\cos\varphi - \boldsymbol{\varphi}_0\sin\varphi), \\ A_2 &= (A_{\mathrm{D}_2} - A_{\mathrm{R}_2})(\boldsymbol{\vartheta}_0\sin\varphi + \boldsymbol{\varphi}_0\cos\varphi). \end{aligned} \tag{8}$$

Let us note that the unit vectors of the spherical coordinate system $\boldsymbol{\vartheta}_0$, $\boldsymbol{\varphi}_0$, $\mathbf{r}_0$ are represented by the elements of the matrix $A_T$, and the angles $\vartheta$ and $\varphi$ are defined in the coordinate system $Ox_1y_1z_1$ related to the incident wave. According to the Babine principle, the amplitudes of the diffraction field $\grave{A}_{\mathrm{D}_1}$ and $\grave{A}_{\mathrm{D}_2}$ are defined as

$$\begin{aligned} A_{\mathrm{D}_1} &= \frac{k^2}{4\pi}(1+\cos\vartheta)E_1 F(\vartheta,\varphi)e^{i\psi}, \\ A_{\mathrm{D}_2} &= \frac{k^2}{4\pi}(1+\cos\vartheta)E_2 F(\vartheta,\varphi)e^{i\psi}, \end{aligned} \tag{9}$$

where $k = 2\pi/\lambda$ is the wavenumber, $\lambda$ is the wavelength. The angular function $F(\vartheta, \varphi)$ is the Fraunhofer integral

$$F(\vartheta,\varphi) = \iint_S \exp(-ikx\cos\varphi\sin\vartheta - iky\sin\varphi\sin\vartheta)\,\mathrm{d}x\mathrm{d}y, \tag{10}$$

where $S$ is the geometric shadow area. In Eq. (9) $\psi$ is the phase shift of the undisturbed part of the wave front after passing through the volume containing the plate:

$$\psi = 2ka\sin\beta + kd\cos\beta.$$

The amplitudes $\grave{A}_{\mathrm{R}_1}$ and $\grave{A}_{\mathrm{R}_2}$ of the scattered field passing through the plate are determined as the sum of the scattered beams formed at multiple passages of the part of the front of the primary wave through the plate. Using the method presented in Refs. 9 and 11 for calculation of the amplitudes of the scattered field, we determine the values $\grave{A}_{\mathrm{R}_1}$ and $\grave{A}_{\mathrm{R}_2}$ in the form

$$\begin{aligned} A_{\mathrm{R}_1} &= \frac{k^2}{4\pi}(1+\cos\vartheta)(E_1 a_{11} + E_2 a_{12}), \\ A_{\mathrm{R}_2} &= \frac{k^2}{4\pi}(1+\cos\vartheta)(E_1 a_{21} + E_2 a_{22}). \end{aligned} \tag{11}$$

$$\begin{aligned} a_{11} &= \cos u\cos\gamma B_{\parallel} + \sin u\sin\gamma B_{\perp}, \\ a_{12} &= \cos u\cos\gamma B_{\perp} - \cos u\sin\gamma B_{\parallel}, \\ a_{21} &= \cos u\sin\gamma B_{\perp} - \sin u\cos\gamma B_{\parallel}, \\ a_{22} &= \sin u\sin\gamma B_{\parallel} + \cos u\cos\gamma B_{\perp}. \end{aligned} \tag{12}$$

Angular functions $B_{\parallel}$ and $B_{\perp}$ are determined as combinations of the Fresnel coefficients ($\tilde{T}_{\parallel}, \tilde{T}_{\perp}, T_{\parallel}, T_{\perp}, R_{\parallel}, R_{\perp}$) and the angular scattering functions of the $j$th beams $F_j(\vartheta,\varphi)e^{i\psi_j}$ as follows:

$$\begin{aligned} B_{\parallel}(\vartheta,\varphi) &= T_{\parallel}\tilde{T}_{\parallel}\sum_{j=1}^{J} R_{\parallel}^{2(j-1)} F_j(\vartheta,\varphi)e^{i\psi_j}, \\ B_{\perp}(\vartheta,\varphi) &= T_{\perp}\tilde{T}_{\perp}\sum_{j=1}^{J} R_{\perp}^{2(j-1)} F_j(\vartheta,\varphi)e^{i\psi_j}. \end{aligned} \tag{13}$$

The transmission $\tilde{T}_{\parallel}, \tilde{T}_{\perp}, T_{\parallel}, T_{\perp}$ and reflection $R_{\parallel}, R_{\perp}$ coefficients are determined by the known formulas

$$\begin{aligned} T_{\parallel} &= \frac{2\tilde{n}\cos\tilde{\vartheta}}{\cos\tilde{\vartheta} + \tilde{n}\cos\beta}, \quad T_{\perp} = \frac{2\tilde{n}\cos\tilde{\vartheta}}{\tilde{n}\cos\tilde{\vartheta} + \cos\beta}, \\ \tilde{T}_{\parallel} &= \frac{2\cos\beta}{\tilde{n}\cos\beta + \cos\tilde{\vartheta}}, \quad \tilde{T}_{\perp} = \frac{2\cos\beta}{\cos\beta + \tilde{n}\cos\tilde{\vartheta}}, \end{aligned} \tag{14}$$

$$R_{\parallel} = \frac{\cos\tilde{\vartheta} - \tilde{n}\cos\beta}{\cos\tilde{\vartheta} + \tilde{n}\cos\beta}, \quad R_{\perp} = \frac{\tilde{n}\cos\tilde{\vartheta} - \cos\beta}{\tilde{n}\cos\tilde{\vartheta} + \cos\beta}.$$

The complex refraction angle $\tilde{\vartheta}$ is determined from the Snell's law $\sin\tilde{\vartheta} = \frac{\sin\vartheta}{\tilde{n}}$. The factor $F_j(\vartheta,\varphi)$ in Eq. (13) is the Fraunhofer integral

$$F_j(\vartheta,\varphi) = \iint_{S_j} \exp(-ikx\cos\varphi\sin\vartheta - iky\sin\varphi\sin\vartheta)\mathrm{d}x\mathrm{d}y, \quad (15)$$

where $S_j$ is the cross section of the beam outgoing from the plate after $2j - 1$ passages through it in the direction $\beta$. The phase shifts of the beams of different reflection multiplicity are determined as follows:

$$\psi_j = \frac{kd\tilde{n}}{\cos\vartheta_r}(2j-1) + 2ka\sin\beta - kd(2j-1)\tan\vartheta_r\sin\beta, \quad (16)$$

where $\vartheta_r$ is the real refractive angle related to the complex refractive angle $\tilde{\vartheta}$ as follows:

$$\tan\vartheta_r = \tilde{n}\sin\tilde{\vartheta} / [\mathrm{Re}(\tilde{n}\cos\tilde{\vartheta})].$$

One should note that formulas (9)–(15) can be further simplified. In particular, if considering a round plate as a scatterer, the Fraunhofer integrals are reduced to analytical form. In particular, we obtain the following formula for the relationship (15):

$$F_j(\vartheta,\varphi) = \pi a_{\min} a_{\max}\cos\beta \times \exp\left(-ik\frac{h_j}{2}\cos\beta\sin\vartheta\cos\varphi\right)\frac{2J_1(R)}{R}, \quad (17)$$

$$R = k\sin\vartheta\sqrt{a_{\min}^2\cos^2\beta\cos^2\varphi + a_{\max}^2\sin^2\varphi},$$

where $J_1$ is the Bessel function of the first order,

$$h_j = (2j-1)d\tan\vartheta_r; \quad a_{\min} = a - h_j/2,$$

$$a_{\max} = \sqrt{a^2 + h_j^2/4}.$$

In the case of $\vartheta = 0$ the Eqs. (10) and (13) are significantly simplified, $F(\vartheta,\varphi) = S$ and $F_j(\vartheta,\varphi) = S_j$.

## Light scattering cross section

Let us determine the light scattering characteristics, which represent both energy and polarization properties of the radiation at any point of the forward hemisphere. To do this, let us consider the scattering cross sections proportional to the respective parameters of the Stokes vector:

$$\sigma_{f_j} = \frac{4\pi r^2}{I_1} I_{f_j}, \quad (18)$$

where $I_1$ is the intensity of electromagnetic field of the incident wave. The Stokes vector parameters $I_{f_j}$ are represented by the amplitudes of the transformed field:

$$I_{f_1} = |E_{\mathrm{I}}|^2 + |E_{\mathrm{II}}|^2, \quad I_{f_2} = |E_{\mathrm{I}}|^2 - |E_{\mathrm{II}}|^2,$$
$$I_{f_3} = 2\mathrm{Re}(E_{\mathrm{I}}E_{\mathrm{II}}), \quad I_{f_4} = 2\mathrm{Im}(E_{\mathrm{I}}E_{\mathrm{II}}). \quad (19)$$

Taking into account the relationships (5)–(19) and performing the necessary algebraic operations, we obtain the following formulas for the scattering cross sections:

$$\sigma_{f_j} = WM_{ij}\frac{I_i}{I_1}, \ i = 1, 2, 3, 4; \ j = 1, 2, 3, 4, \quad (20)$$

where

$$W = \frac{k^2}{\pi}\frac{(1+\cos\vartheta)^2}{2};$$

$I_i$ ($i$ = 1, 2, 3, 4) are the Stokes vector parameters of the incident radiation. The elements of the scattering phase matrix $M_{ij}$ are the combinations of the elements $b_{kl}$ ($k$ = 1, 2; $l$ = 1, 2) of the amplitude matrix (7):

$$M_{11} = \frac{|b_{11}|^2 + |b_{12}|^2 + |b_{21}|^2 + |b_{22}|^2}{2},$$

$$M_{12} = \frac{|b_{11}|^2 - |b_{12}|^2 + |b_{21}|^2 - |b_{22}|^2}{2},$$

$$M_{13} = \mathrm{Re}(b_{11}b_{12}^* + b_{21}b_{22}^*),$$

$$M_{14} = -\mathrm{Im}(b_{11}b_{12}^* + b_{21}b_{22}^*), \quad (21)$$

$$M_{21} = \frac{|b_{11}|^2 + |b_{12}|^2 - |b_{21}|^2 - |b_{22}|^2}{2},$$

$$M_{22} = \frac{|b_{11}|^2 - |b_{12}|^2 - |b_{21}|^2 + |b_{22}|^2}{2},$$

$$M_{23} = \mathrm{Re}(b_{11}b_{12}^* - b_{21}b_{22}^*), \quad M_{24} = -\mathrm{Im}(b_{11}b_{12}^* - b_{21}b_{22}^*),$$

$$M_{31} = \mathrm{Re}(b_{11}b_{21}^* + b_{21}b_{22}^*), \quad M_{32} = \mathrm{Re}(b_{11}b_{21}^* - b_{12}b_{22}^*),$$

$$M_{33} = \mathrm{Re}(b_{11}b_{22}^* + b_{12}b_{21}^*), \quad M_{34} = -\mathrm{Im}(b_{11}b_{12}^* - b_{12}b_{21}^*),$$

$$M_{41} = \mathrm{Im}(b_{11}b_{21}^* + b_{12}b_{22}^*), \quad M_{42} = \mathrm{Im}(b_{11}b_{21}^* - b_{12}b_{22}^*),$$

$$M_{43} = \mathrm{Im}(b_{11}b_{22}^* + b_{12}b_{21}^*), \quad M_{44} = \mathrm{Re}(b_{11}b_{22}^* - b_{12}b_{21}^*).$$

The elements $b_{kl}$, in their turn, are determined as the difference between the corresponding components of the amplitudes of the diffracted and refracted fields (11). Obviously, the extinction matrix derived from $M_{ij}$ when the direction of radiation receipt coincides with the direction of beams outgoing from the plate ($\vartheta$ = 0), has more simple form, because $a_{12} = a_{21}$ (see Eq. (12)). Nevertheless, even in this case the algebraic expression for the matrix element explicitly depending on the geometric size of the particle, its refractive index, parameters of the incident radiation, angular characteristics determining the position of the particle relative to the source (or the receiver) has quite a cumbersome form. For the case of $\vartheta = 0$, using Eqs. (6)–(16) and the optical theorem, we obtain the following formula for the cross section of extinction of polarized radiation by the plate[3,11]:

$$C_{\text{ext}} = 2S - \text{Re}(Q_{\parallel} + Q_{\perp}) - \frac{I_2}{I_1}\text{Re}(Q_{\parallel} - Q_{\perp})\cos 2\gamma + \frac{I_3}{I_1}\text{Re}(Q_{\parallel} - Q_{\perp})\sin 2\gamma,$$

where

$$Q_{\parallel} = T_{\parallel}\tilde{T}_{\parallel}\sum_{j=1}^{J} R_{\parallel}^{2(j-1)} S_j e^{i(\psi_j - \psi)},$$

$$Q_{\perp} = T_{\perp}\tilde{T}_{\perp}\sum_{j=1}^{J} R_{\perp}^{2(j-1)} S_j e^{i(\psi_j - \psi)}.$$

## Conclusion

The numerical model is developed in the frameworks of the method of physical optics for studying the characteristics of electromagnetic radiation passing through the atmospheric ice formations. The semitransparent oriented plate characterized by the greatest interval of the possible values of the extinction factor (0, 4) is considered as the scatterer. The relationships for the scattering cross section are obtained in the frameworks of the method of physical optics in the form of combinations of the elements of the scattering phase matrix. The obtained formulas allow one to calculate both energy and polarization characteristics of light scattering at any point of the forward hemisphere as functions of the particle size and refractive index at different positions of the source, receiver, and the scatterer, as well as at any wavelength from the optical range and at any state of polarization of the incident radiation.

The numerical model presented allows one to study the fine structure of the dependence of extinction on the small-angle displacements caused by either particle oscillations about their stable position (angles $\psi$ and $\xi$, see Fig. 1) or deviation of the radiation receipt line from the "forward" direction, i.e., the direction of propagation of the plane front of the wave.

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