Basic limitations of the phase conjugation algorithm and realization of amplitude–phase control in a two-mirror adaptive system

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Laser radiation propagation through the turbulent atmosphere is analyzed in numerical experiments, and efficiencies of phase and amplitude–phase beam control methods are compared. Since complete correction of turbulent distortions is possible only at wave front reversal, realization of this algorithm is considered in an adaptive system having two mirrors separated by a gap, where a beam propagates under free diffraction conditions. This system is shown to be more efficient as compared to the phase conjugation system.

1. Scheme of numerical experiment

We report investigation results obtained by means of a numerical experiment. The schematic arrangement of the modeled optical system is shown in Fig. 1. Propagating from the aperture of the source *1* to the observation plane *3*, the laser beam passes through a thin layer of a turbulent medium *2* simulated by one phase screen (Fig. 1) or through a distributed turbulent lens filling the space between the aperture and the observation plane. In the case of a single screen, its position on the path, that is, distance Z_S from the screen to the laser, varies. The intensity of atmospheric turbulence is characterized by the Fried radius[1] r_0:

$$r_0 = \left(0.423k^2 \int_0^L C_n^2(l)\mathrm{d}l \right)^{-3/5}, \tag{1}$$

where L is the thickness of the turbulent layer; C_n^2 is the structural constant of the atmosphere; k is a constant. Residual distortions and compensation efficiency are characterized using the focusing criterion[2]:

$$J(t) = \frac{1}{P_0} \iint \rho(x,y) I(x,y,z_0,t)\mathrm{d}x\mathrm{d}y \tag{2}$$

having the meaning of the relative fraction of the beam power falling within the aperture of radius a_0. Here P_0 is the total beam power; $\rho(x, y) = \exp[-(x^2 + y^2]/a_0^2$ is the aperture function.

As known, the laser beam propagating through a randomly inhomogeneous medium distorts, its energy radius increases, and its centroid displaces randomly.[3] This leads to decrease of the field concentration in the observation plane, that is, to decrease of the criterion J. In this case, the smaller the Fried radius, the more distortions are introduced into the beam. This dependence is illustrated in Fig. 2.

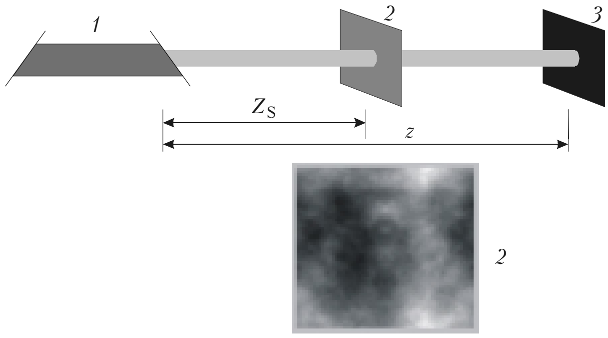

Fig. 1. Scenario of numerical experiment on measurement of beam distortions due to a thin turbulent layer.

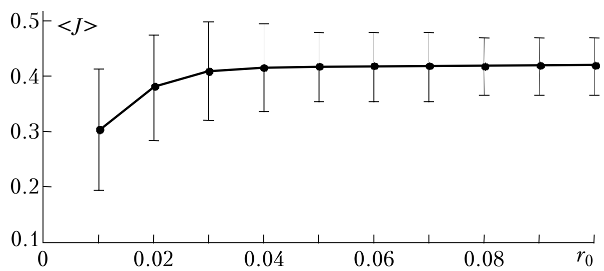

Fig. 2. Field concentration in the observation plane (averaged over 50 realizations) as a function of r_0; the path length $z = 0.5$, the length of the distributed lens is equal to the path length.

To determine the effect of individual atmospheric layers (more precisely, their position) on the intensity of distortions, a single phase screen was set on the propagation path. The values of the criterion obtained for different r_0 and at different distances from the screen to the source aperture Z_S are shown in Fig. 3.

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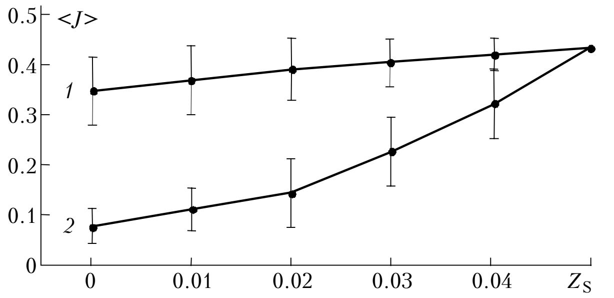


Fig. 3. Intensity of distortions as a function on the position of the turbulent layer (turbulent screen) on the path. Averaging over 50 realizations. Parameters: $r_0 = 0.1$ (curve *1*) and 0.01 (curve *2*). Path length $z = 0.5$.

We can see that the strongest distortions are caused by turbulent layers located near the source aperture. As Z_S increases, distortions decrease. It is quite natural that the screen located in the registration plane does not affect amplitude variations and J.

2. Phase and amplitude–phase control over the beam

To compensate distortions caused by individual phase screens, we used the phase conjugation (PC) algorithm, operating by the following scheme. A laser beam propagates from the source aperture plane to the registration plane in the direction opposite to the reference beam. In the source aperture plane, the beam phase $\varphi(x, y)$ is set equal to the phase $\psi(x, y)$ of the reference beam and of opposite sign

$$\varphi(x, y) = -\psi(x, y), \tag{3}$$

and the beam amplitude is Gaussian.

The results of PC control are shown in Fig. 4 (ideal adaptive system, limitation introduced by individual elements are ignored). We can see that the control provides complete compensation for the screens located in the source aperture plane at any intensity of distortions (Fig. 4*a*, curve *2*), while the layers located at some finite distance from the laser cannot be compensated even in the ideal system (Fig. 4*c*, curve *2*). This is explained by violation of the principle of optical reversibility at PC in the second case. This violation can be illustrated by introducing one more parameter, i.e. the square error ε determined as

$$\varepsilon = \frac{\iint \sqrt{(A(x,y) - A_{\text{ref}}(x,y))^2}\,\mathrm{d}x\mathrm{d}y}{\iint A(x,y)\,\mathrm{d}x\mathrm{d}y} \tag{4}$$

and characterizing how widely the Gaussian amplitude distribution of the beam $A(x, y)$ differs from the amplitude distribution $A_{\text{ref}}(x, y)$ of the reference beam (or how widely the reference beam differs from the Gaussian one). Values of ε obtained at compensation for the screens located at different distances from the source aperture are depicted in Figs. 4*b* and *d*. As can be seen, the ε values are small for the screen located in the source aperture plane. This indicates that the principle of optical reversibility holds with high accuracy. In this case, distortions are compensated almost completely. As the distance from the aperture to the screen increases, the values of ε grow, because the reference beam is no longer Gaussian. Thus, we can conclude that complete PC compensation for screens (turbulent layers) located at a finite (nonzero) distance from the source aperture is conceptually impossible.

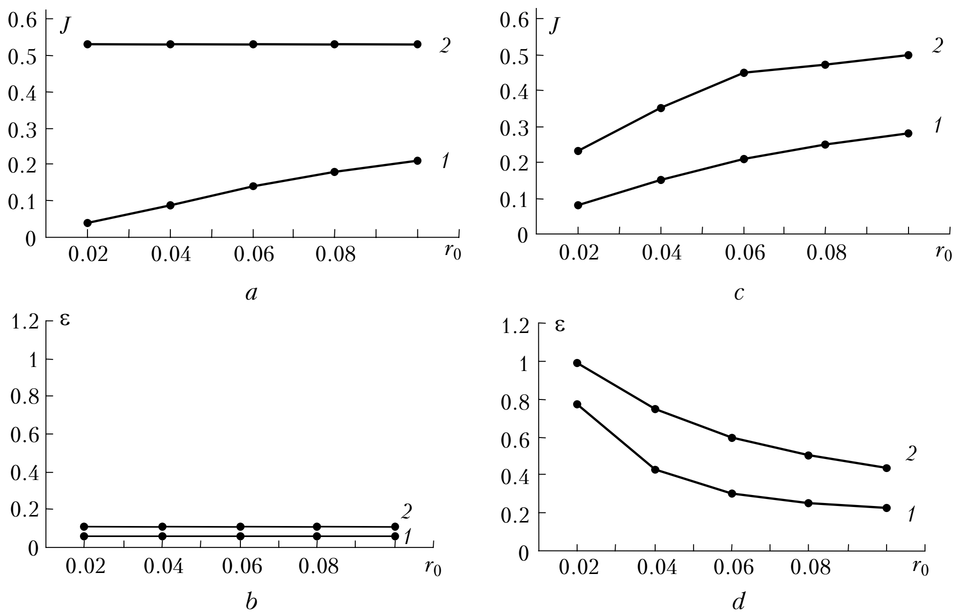


Fig. 4. Focusing criterion J (upper panels; result of control (curve *2*); no control (curve *1*)) and square error ε (lower panels; source aperture plane (curve *2*); registration plane (curve *1*)) as functions of the intensity of turbulent distortions. Turbulence is simulated by one screen located at different distances from the laser aperture. The distance from the aperture to the distorting screen $Z_S = 0$ (left panels) and $Z_S = 0.25$ (right panels). The total path length $z = 0.5$.

The complete correction of distortions can be achieved providing the wave front reversal (WFR) operation is performed[2]:

$$\mathrm{Im}(E) = -\,\mathrm{Im}(E_{\mathrm{ref}}), \quad (5)$$

where $\mathrm{Im}(E)$ is the imaginary part of the complex amplitude of the beam field, and $\mathrm{Im}(E_{\mathrm{ref}})$ is the same for the reference beam. Physically, this operation means that the beam phase is taken inverse with respect to the reference beam phase, and the amplitude is taken equal to the reference beam amplitude.

3. Realization of the amplitude–phase control in a two-mirror adaptive system

Wave front reversal can be realized in an adaptive system including two mirrors (Fig. 5) separated by a gap, in which the beam propagation is free of distortions.[4,5]

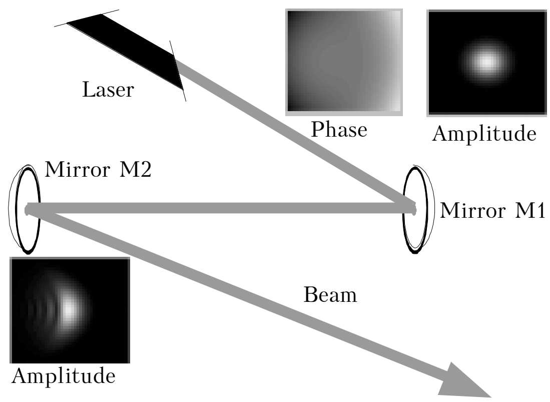


Fig. 5. Formation of the preset amplitude distribution of a beam in a two-mirror adaptive system.

The first mirror (M1) specifies the beam phase. As the beam propagates in the free space, phase variations lead to amplitude variations. Thus, the required intensity distribution of the light field is achieved in the plane of mirror M2 (at the entrance into the medium). Mirror M2 compensates the introduced phase variations and performs the operation of conjugation. As a result, the beam with the preset distribution of the amplitude and phase profiles is formed at the entrance into the medium. The main difficulty in realization of this operation is specifying the phase profile providing for the needed amplitude distribution.

The problem can be easily solved in the presence of a single distorting screen located at the path center. Therewith, the following property of optical radiation should be used: beams having equal initial amplitude and phase profiles and propagating along equal paths acquire equal amplitude and phase distributions. The possibility of realization of the amplitude–phase control based on this property is illustrated in Figs. 6 and 7.

The reference beam propagates from the registration plane. At its incidence on the screen, the difference of its amplitude from the initial Gaussian one and the phase from the plane one is caused only by diffraction. Immediately behind the screen the amplitude remains Gaussian; the passage through the screen leads only to the change in the phase. As the reference beam reaches mirror M2, amplitude distortions already show themselves. If mirror M2 performs conjugation of the phase of the reference beam, keeping the amplitude unchanged, then in plane M1 we obtain the beam absolutely identical to that immediately behind the distorting screen, that is, having the Gaussian amplitude and some phase different from the plane one.

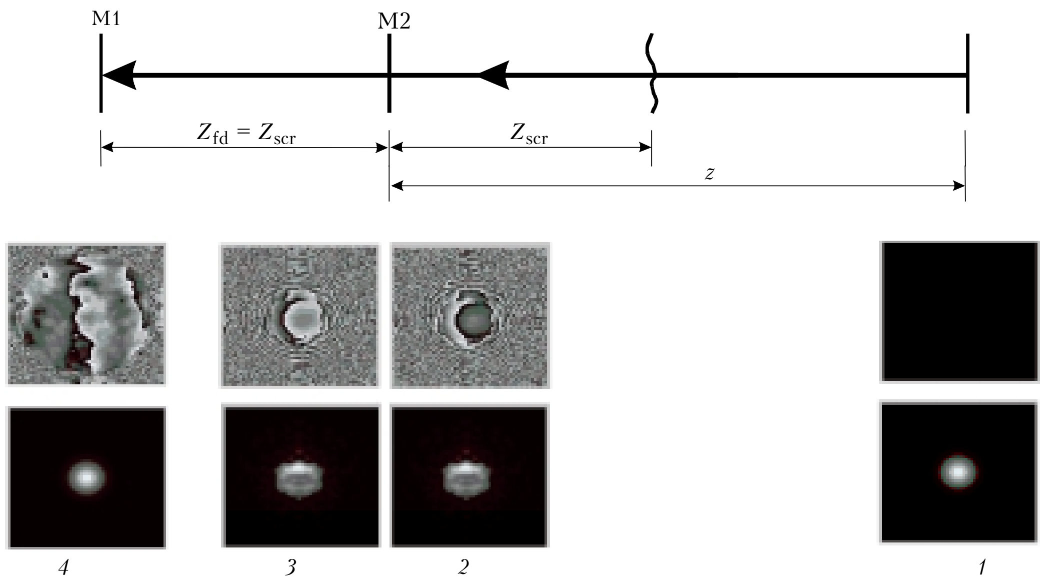


Fig. 6. Propagation of the reference beam in a two-mirror adaptive system. Distortions are simulated by one screen: phase and amplitude of the reference beam in the registration plane (*1*); phase and amplitude of the beam in the plane of mirror M2 before conjugation (*2*); phase and amplitude of the beam in the plane of mirror M2 after conjugation (*3*); phase and amplitude of the beam in the plane of mirror M1 (*4*).

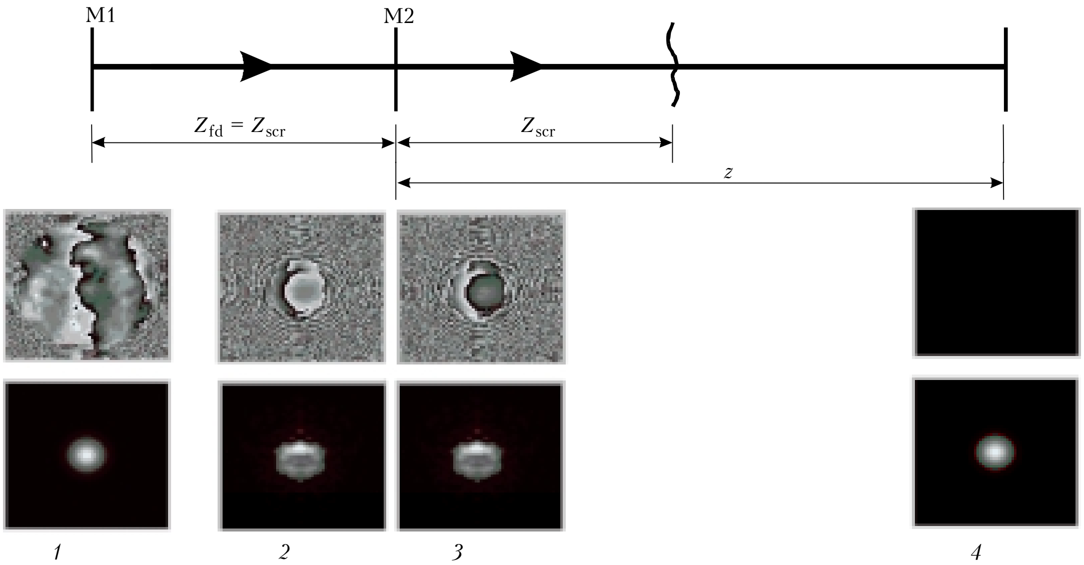


Fig. 7. Propagation of the beam in a two-mirror adaptive system. Distortions are simulated by one screen: phase and amplitude of the beam in the plane of mirror M1 after correction (*1*); phase and amplitude of the beam in the plane of mirror M2 before conjugation (*2*); phase and amplitude of the beam in the plane of mirror M2 after conjugation (*3*); phase and amplitude of the beam in the registration plane (*4*).

In plane M1, the reference beam is replaced by the "direct" beam, whose phase is conjugate with respect to the reference one and the amplitude is Gaussian. Now all the requirements of the principle of optical reversibility are fulfilled. In plane M2, the amplitude distribution is the same as in the reference beam, and the phase is specified conjugate with respect to the reference beam. Propagation of the "direct" beam is shown in Fig. 7. The control leads to the absolute compensation of distortions. Emphasize that this result cannot be obtained by using the PC algorithm.

The next problem considered in this paper is realization of correction of a distributed turbulent lens. The problem is solved based on the assumption that distortions of beams passed through a single phase screen located at the path center are roughly equal to distortions of beams passed through a set of identical phase screens. This assumption is confirmed by the data presented for these two cases (Tables 1 and 2). Tables 1 and 2 give the values of the criterion J, beam energy radius σ, and displacements of the beam energy center X_c and Y_c along the axes normal to the propagation direction:

$$\sigma(t) = \left[\frac{1}{P_0 a_0^2} \iint (\mathbf{r}_\perp - \mathbf{r}_c)^2 I(x,y,z,t) \mathrm{d}x\mathrm{d}y \right]^{1/2}. \quad (6)$$

Here $\mathbf{r}_\perp = \{x, y\}$ is the coordinate vector of a point on the plane; $\mathbf{r}_c$ is the coordinate vector of the beam energy centroid; $I(x, y, z, t)$ is the radiation intensity;

$$X_c = \frac{1}{P_0 a_0^2} \iint x I(x, y, z, t) \mathrm{d}x\mathrm{d}y, \quad (7)$$

Y_c is determined by the similar equation. The results are obtained at different intensities of turbulent distortions. We can see that the values of all parameters introduced to characterize radiation are actually close.

Table 1. Beam parameters obtained at simulation of turbulence by a single screen located at the path center (z = 0.5)

r_0	0.1	0.08	0.06	0.04	0.02
J	0.28	0.25	0.21	0.15	0.08
σ	1.40	1.48	1.60	1.80	2.59
X_c	−0.22	−0.27	−0.34	−0.47	−0.83
Y_c	0.10	0.12	0.16	0.22	0.38

Table 2. Beam parameters obtained at simulation of turbulence by a distributed lens consisting of identical screens distributed uniformly over the path (z = 0.5)

r_0	0.1	0.08	0.06	0.04	0.02
J	0.30	0.27	0.23	0.17	0.08
σ	1.35	1.40	1.49	1.64	2.10
X_c	−0.25	−0.30	−0.37	−0.51	−0.84
Y_c	0.12	0.14	0.18	0.25	0.43

Since correction of a single screen is absolute, the results obtained at compensation for the distributed lens simulated by a set of identical random screens (Table 3) are good enough as well.

In this case, we do not have absolute compensation, but the values of the criterion obtained from correction at small Fried radii are much higher than in the case of phase conjugation (compare Tables 3 and 4).

Further inaccuracies show themselves at compensation for the distributed lens consisting of different screens generated using random numbers. However, the results obtained in this case with the use of the two-mirror system are rather high as well (Table 5).

Table 3. Correction of a distributed turbulent lens simulated by a set of identical screens in a two-mirror adaptive system. Path length z = 0.5; J are values of the criterion without control; J_{cor} are values obtained as a result of control

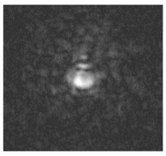
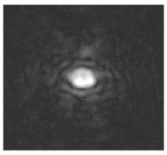
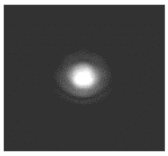
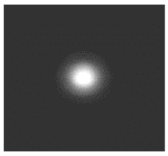
Light field distribution obtained as a result of correction				
r_0	0.005	0.01	0.05	0.1
J	0.002	0.039	0.192	0.292
J_{cor}	0.341	0.462	0.531	0.532
ε (z = 0.5)	0.66	0.46	0.13	0.07

Table 4. Correction of a distributed turbulent lens simulated by a set of identical screens based on the phase conjugation algorithm

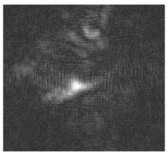
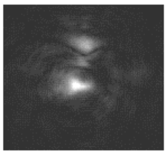
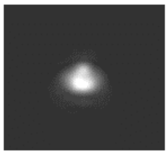
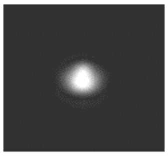
Light field distribution obtained as a result of correction				
r_0	0.005	0.01	0.05	0.1
J	0.002	0.039	0.192	0.292
J_{cor}	0.105	0.163	0.452	0.501
ε (z = 0.5)	1.06	0.92	0.32	0.21

Table 5. Beam parameters obtained at compensation for distributed turbulence represented by several screens (z = 0.5)

r_0	0.005	0.01	0.02	0.03
J	0.049	0.143	0.262	0.310
J_{cor}	0.41	0.47	0.50	0.50
ε (z = 0.5)	0.45	0.31	0.17	0.12

From the above data, we can conclude that the proposed method of amplitude–phase control does not provide for absolute compensation for the turbulent lens, but the obtained values of the focusing criterion are higher than in the case of purely phase control.

References

1. V.P. Lukin and B.V. Fortes, *Adaptive Beaming and Imaging in the Atmosphere* (SB RAS Publishers, Novosibirsk, 1999), 211 pp.
2. M.A. Vorontsov and V.I. Shmal'gauzen, *Principles of Adaptive Optics* (Nauka, Moscow, 1985), 335 pp.
3. V.A. Vysloukh, V.P. Kandidov, S.S. Chesnokov, and S.A. Shlenov, Izv. Vyssh. Uchebn. Zaved., Ser. Fiz., No. 11, 30–41 (1985).
4. F.Yu. Kanev and V.P. Lukin, Atm. Opt. **4**, No. 12, 856–863 (1991).
5. N.V. Vysotina, N.N. Rozanov, V.E. Semenov, and V.A. Smirnov, Izv. Vyssh. Uchebn. Zaved., Ser. Fiz., No. 11, 42–51 (1985).