

# Correlation between the IR radiation angular structure and optical characteristics of the atmospheric ground layer

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The thermal radiation transfer equation is considered for the case of observing the horizon under condition of the axial symmetry of the sky radiation. The equation is solved using model representations of the brightness field and scattering phase functions. It is shown that formation of the sky brightness field on the horizon mainly depends on the magnitude of the upward and downward radiation (34% variations), while the asymmetry of the scattering phase function is of minor importance (less than 3% variations). The algorithm is proposed for determining the single scattering albedo based on the measurements of the following parameters: angular distribution (from zenith to nadir) of the incident radiation, sky brightness on the horizon and the blackbody brightness at the temperature of the near-ground atmosphere. The atmospheric conditions preferable for realization of the method, as well as the reasons for the difference between radiative temperatures of the sky on the horizon and near-ground atmosphere are considered.

## Introduction

Obtaining more precise and reliable data on the single scattering albedo in different wavelength ranges is one of important problems of the radiation transfer in the atmosphere. This problem for the IR range is in fact treated as an ambiguity in separation of the radiation total extinction into scattering and absorption components. All the aforesaid stimulates a search for new methods for determination of the sought characteristics. One of such methods using the brightness of the observed horizon in the IR wavelength range is considered in this paper.

The optical horizon region is quite widely used in the spatial orientation of different devices, as well as in solving inverse problems connected with determination of physical characteristics of the atmosphere. This can be exemplified by numerous variants of photometric measurements of the daytime and twilight horizon from the space.<sup>1-3,etc.</sup> Methods of observation of the horizon from the Earth surface are not too various. In particular, in practical implementation are the instrumental-visual techniques for determination of the atmospheric transparency (meteorological range) from the relationship between the brightness of remote objects and the daylight sky on the horizon.<sup>4-6,etc.</sup> Theoretically, the method is based on the Bouguer-Beer law and the Koschmieder light-air equation, which describes variations of the brightness of the atmospheric haze between the observer and the object. Note that the sky background on the horizon is used in this case only as a referential, relative level of the brightness, but its dependence on the characteristics of the atmosphere and conditions of illumination is not given in an explicit form.

Preliminary consideration of the problem has shown that the brightness of the horizon can be a

source of additional information, in particular, on the single scattering albedo. To complete the formulation of the method, one more equation is required, relating optical characteristics of the atmosphere to the radiation coming from the horizon region. In spite of a great quantity of papers devoted to the horizon brightness,<sup>1,7-10,etc.</sup> the near-horizon region ( $-0-5^\circ$ ) remains to be insufficiently studied, i.e., the available results are not yet reduced to simple model representations suitable for solving direct and inverse problems. Therefore, as a first step, we consider the simplest case, i.e. the axial symmetry of the sky radiation, using a number of model representations of the brightness field and scattering phase functions.

## 1. Initial equation and geometry of the problem

It is well known that the relationship between optical characteristics of the atmosphere and the radiation coming to the point of observation (or the brightness  $B$ ) is described by the radiation transfer equation

$$\frac{1}{\varepsilon} \frac{dB(l, \omega)}{dl} + B(l, \omega) = S(l, \omega), \quad (1)$$

where  $l$  is the path length along the direction of observation  $\omega$ ;  $\varepsilon = \kappa + \sigma$  are the extinction, absorption, and scattering coefficients, and  $S$  is the source function.

Let us firstly consider the plane-parallel atmosphere with uniform conditions of illumination in the horizontal direction (Fig. 1a). We pass to new variables: the optical thickness  $\tau = \varepsilon l$  and spherical coordinate system, in which  $\varphi$  is the azimuth angle,  $\mu = \cos\theta$  is the cosine of the zenith angle. In this case, the formal solution for the horizontal direction of observation ( $\mu = 0$ ) can be written in the form<sup>10-12</sup>:

$$B_{hd} = S(\tau, 0, \varphi) = \frac{\kappa}{\varepsilon} B^0(T_{atm}) + \frac{\sigma}{\varepsilon 4\pi} \int_{\varphi'=0}^{2\pi} \int_{\mu'=-1}^1 p(\mu, \mu') B(\tau, \mu', \varphi') d\mu' d\varphi', \quad (2)$$

where  $B_{hd}$  is the brightness in the horizontal direction;  $p(\mu, \mu')$  is the light scattering phase function, and  $B^0(T_{atm})$  is the Plank function (brightness of the blackbody at the atmospheric temperature  $T_{atm}$ ).

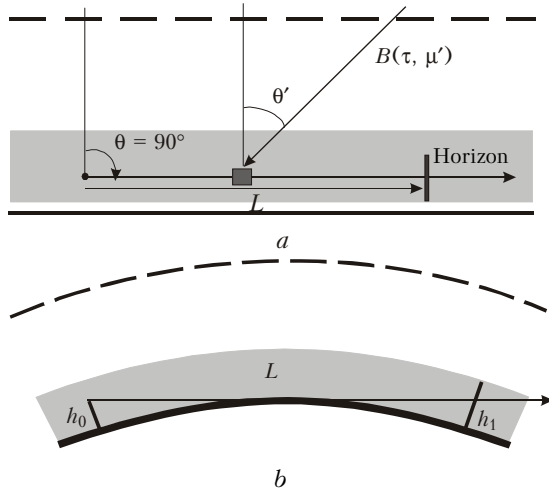


Fig. 1. Geometry of the problem in the plane-parallel (a) and spherical (b) atmospheres.

The solution (2) is called formal, because function  $B(\tau, \mu', \varphi')$  is not defined and needs solving the radiation transfer equation as well. However, this is not an obstacle in the case, because the algorithm for solving the task (see below) allows for setting the value of  $B(\mu')$  as measured or model. As for the physical meaning of equation (2), note, that its first term represents the contribution of the inherent radiation of the atmospheric ground layer into the source function  $S$ , and its second term – the contribution of the radiation  $B(\tau, \mu', \varphi')$  re-scattered from all directions of the space, taking into account the scattering phase function  $p(\mu, \mu')$ .

It can be shown that the observed brightness of the atmospheric haze  $B_{hd}$  relatively fast reaches the state of “brightness saturation” under real atmospheric conditions and becomes equal to the source function  $S$ . To prove this, we set an artificial screen of small size with low inherent brightness  $B_{sc} \rightarrow 0$  along the path of observation, which shuts over the field of view of the receiver, but does not disturb the considered field of brightness. In this case, the recorded brightness at variation of distance  $L$  to the screen can be written as:

$$B_{hd}(L) = B_{sc} \exp(-\varepsilon L) + S(0) [1 - \exp(-\varepsilon L)] \approx S(0) [1 - \exp(-\varepsilon L)], \quad (3)$$

$$B_{hd}(L) \rightarrow S(0) \text{ at } L \rightarrow \infty,$$

where  $S(0)$  is the source function for the horizontal direction ( $\mu = 0$ ) in the near-ground layer.

Estimates for a number of typical values of the extinction coefficient  $\varepsilon$  (Fig. 2) show that even in conditions of high transparency ( $\varepsilon = 0.08 \text{ km}^{-1}$ ) the brightness  $B_{hd}$  becomes practically equal to the source function at the distances of 50–60 km (the difference is about 1%).

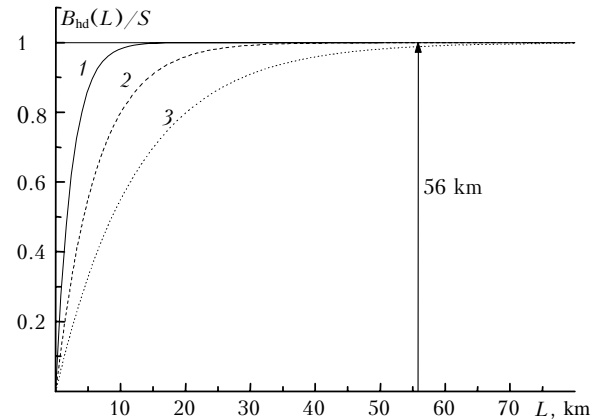


Fig. 2. Illustration of the “brightness saturation” ( $B_{hd}(L) \rightarrow S$ ) at increase of the optical thickness of the atmosphere on the horizontal path:  $\varepsilon = 0.4$  (curve 1); 0.16 (2), 0.08  $\text{km}^{-1}$  (3).

In other words, at the optical thickness of the atmosphere  $\tau = \varepsilon l > 4$  the brightness in the horizontal direction reaches the level of  $S(0)$ , and the possible inhomogeneities become insignificant at long distances. As applied to the geometry of observation in the spherical atmosphere (Fig. 1b), the obtained estimates mean the following. If the optical characteristics of the atmosphere along the horizontal path within the near-ground layer (for example,  $h_1 = 100 \text{ m}$ ) remain constant, then the brightness  $B_{hd}$  at the optical distance  $\tau > 4$  also reaches the “saturation” state and becomes equal to the source function. For example, when observing the horizon from the height  $h_0 = 20 \text{ m}$ , the path length  $L$  is  $\sqrt{2h_1 R} + \sqrt{2h_0 R} \approx 56 \text{ km}$  ( $R$  is the Earth radius taking into account the mean refraction), and the optical thickness  $\tau$  is not less than  $0.08 \cdot 56 = 4.48$ . The corresponding value of  $B_{hd}/S$  is marked by the arrow in Fig. 2.

Thus, in the majority of events the sky brightness just above the horizon is equal to the source function and is described by formula (2). The problem of the “brightness saturation” of the atmospheric haze in the visible range was discussed many times in the framework of the theory of horizontal visibility of objects.<sup>4,5,13,14</sup>

We will treat the problem for the case of axial symmetry of radiation  $B(\tau, \mu', \varphi') = B(\tau, \mu')$ . This condition is strongly fulfilled in the case of cloudless night and can be expanded to other situations. At the axial symmetry of radiation the cosine of the scattering angle is  $\cos\theta_0 = \mu_0 = \sqrt{1 - \mu'^2}$ , and formula (2) becomes

$$\begin{aligned}
 B_{hd} &= S(0) = \frac{\kappa}{\varepsilon} B^0(T_{atm}) + \\
 &+ \frac{\sigma}{\varepsilon} \left[ \frac{1}{4\pi} \int_{\mu'=-1}^1 B(\tau, \mu') \int_{\varphi'=0}^{2\pi} p(\mu_0) d\varphi' d\mu' \right] = \\
 &= (1 - \Lambda) B^0 + \Lambda B_S, \quad (4) \\
 B_S &= \left[ \frac{1}{4\pi} \int_{\mu'=-1}^1 B(\tau, \mu') \int_{\varphi'=0}^{2\pi} p(\mu_0) d\varphi' d\mu' \right], \quad (4a)
 \end{aligned}$$

where  $B_S$  is the “scattered” component of function  $S$ ; and  $\Lambda = \sigma/\varepsilon$  is the single scattering albedo. According to the data of Ref. 15,  $\Lambda$  in the thermal range is estimated as 0.25–0.5.

The brightness  $B^0(T_{atm})$  and  $S(0) = B_{hd}$  can be easily measured. For determination of the albedo  $\Lambda$  the quantity  $B_S$  remains as yet unknown. So, we will use the model representations.

### 2. Model of the brightness $B(\tau, \mu')$ and scattering phase functions

Angular distribution of radiation coming from the upper hemisphere (the sky brightness),  $B^\downarrow(\tau, \mu')$ , is satisfactorily described by the empirical Linke formula.<sup>9</sup> For convenience of the model representation we rewrite the formula in the form

$$B^\downarrow(\tau, \mu') = B_{hd} [1 - (1 - m) \mu'^n]. \quad (5)$$

According to the data of Ref. 9, parameters  $m$  and  $n$  can take the following values under different atmospheric conditions:  $m = B_Z(\mu' = 1)/B_{hd} = 0.09\text{--}0.44$ ;  $n = 0.1 - 1$  at their mean values of approximately 0.3. Temperature stratification and transparency of the atmosphere mainly affect the formation and variability of the brightness field. In their turn, the transparency variations (as well as  $m$  and  $n$  variations) are determined by the water vapor and aerosol content. To take into account different situations, we consider three variants:  $a$  – transparent,  $b$  – intermediate, and  $c$  – turbid atmosphere. The  $m$  and  $n$  values corresponding to these cases are shown in Table 1. Graphically, the model of  $B^\downarrow(\tau, \mu')$  is shown in Fig. 3.

**Table 1. Parameters of model distributions of brightness  $B(\tau, \mu')$**

Model	$B^\downarrow(\tau, \mu')$			$B^\uparrow(\mu')$		
	$a$	$b$	$c$	$d$	$e$	$f$
Parameters						
$m, m^*$	0.1	0.3	0.5	1	0.9	0.8
$n, n^*$	0.2	0.3	0.8	–	0.15	0.15

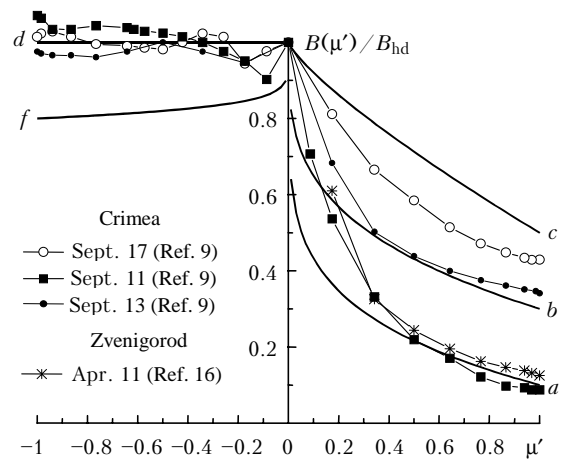
Note: different variants of upwelling radiation  $B^\uparrow(\mu')$  are symbolized in Table 1 by the letters  $d, e,$  and  $f$ .

Angular anisotropy of radiation from the lower hemisphere  $B^\uparrow(\mu')$  is of less significance,<sup>7–9,15</sup> and the brightness of the underlying surface is close to  $B^0(T_{atm})$

and to the horizon brightness  $B_{hd}$ . The formula analogous to Eq. (5) was used by us for uniformity of the model representation:

$$B^\uparrow(\mu') = B_{hd} [1 - (1 - m^*) (-\mu')^{n^*}], \quad (6)$$

where  $m^* = B_N(\mu' = -1)/B_{hd}$  may exceed 1, if the brightness of the underlying surface in the nadir direction  $B_N$  exceeds  $B^0(T_{atm})$ .



**Fig. 3.** Model dependence of the angular structure of radiation  $B(\tau, \mu')$  in comparison with real data.

It follows from the comparison with the experimental data<sup>9,16</sup> (see Fig. 3) that the chosen model dependences are in good agreement with the realistic brightness distribution  $B(\mu')$ . Note that the model  $B^\uparrow(\mu')$  covers a wide range of situations from radiation temperature of the surface  $T_N \approx T_{atm}$  to somewhat exotic case ( $T_{atm} - T_N$ )  $\approx 10$  K (see variants  $d$  and  $f$ ).

When setting the scattering phase function, we used the isotropic, Rayleigh, and Henyey–Greenstein<sup>10,11</sup> scattering phase functions with different asymmetries depending on parameter  $g$ :

$$p(\theta_0) = (1 - g^2) (1 + 2g^2 - 2g \cos\theta_0)^{-1.5}. \quad (7)$$

The asymmetry coefficient of the scattering phase function was estimated by the formula

$$\Gamma = \frac{\int_0^{\pi/2} p(\theta_0) \sin\theta_0 d\theta_0}{\int_{\pi/2}^{\pi} p(\theta_0) \sin\theta_0 d\theta_0}. \quad (8)$$

### 3. Discussion

#### 3.1. Results of modeling and properties of $B_S$

The solution for the isotropic scattering phase function  $p(\mu_0) = 1$  can be most readily obtained. Equation (4a) is easily integrated and gives the following results for the brightness distribution  $B(\mu')$  in cases  $e$  and  $f$ :

$$B_S = \int_{-1}^1 \frac{B(\mu')}{2} d\mu' = \frac{B_{hd}}{2} \left\{ \int_0^1 [1 - (1-m)\mu'^n] d\mu' + \int_{-1}^0 [1 - (1-m^*)\mu'^{n^*}] d\mu' \right\} = \frac{B_{hd}}{2} \left( 2 - \frac{1-m}{1+n} - \frac{1-m^*}{1+n^*} \right). \quad (9)$$

It is also easy to obtain the solution for  $B_S$  in the case of the Rayleigh scattering phase function<sup>11</sup>  $p(\mu_0) = 0.375(3 - \mu^2)$  for upwelling radiation  $B^\uparrow(\mu')$  described by variant  $d$ :

$$B_S = \frac{3B_{hd}}{16} \left\{ \int_0^1 [1 - (1-m)\mu'^n] (3 - \mu'^2) d\mu' + \int_{-1}^0 (3 - \mu'^2) d\mu' \right\} = \frac{3B_{hd}}{16} \left( \frac{16}{3} - \frac{3(1-m)}{n+1} + \frac{1-m}{n+3} \right). \quad (10)$$

When calculating  $B_S$  with the model Henyey-Greenstein scattering phase functions, the standard computer program Mathcad was used. The results of calculation of the normalized component of the

scattering radiation ( $B_S/B_{hd}$ ) are presented in Tables 2 and 3 and in Figs. 4 and 5.

Analysis of the data obtained makes it possible to see the following peculiarities. Variability of  $B$  is mainly caused by illumination conditions (see variations in the limits of each row of Tables 2 and 3). The total range of the ( $B_S/B_{hd}$ ) variation reaches 46% for the considered examples covering the majority of real conditions. Naturally, maximum effect on the variations of ( $B_S/B_{hd}$ ), 34%, is exerted by variations of the downwelling radiation (see  $a$ ,  $b$ , and  $c$  variants of illumination). The contribution of the underlying surface brightness  $B^\uparrow(\mu')$  although being no less than a half, is characterized by a great stability. Relative variation of ( $B_S/B_{hd}$ ) at different  $B^\uparrow(\mu')$  does not exceed 12% (see variants  $d$ ,  $e$ , and  $f$ ).

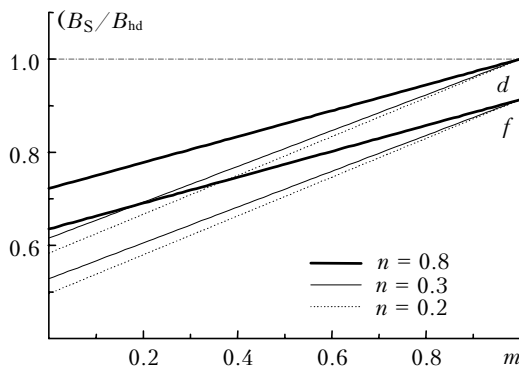
In contrast to the brightness filed, the type of the scattering phase function weaker affects the variation of  $B_S$ . The value of  $B_S/B_{hd}$  varies within small limits  $\pm 2.5\%$  in the wide range of the considered scattering phase functions ( $\Gamma = 1-10.88$ ) (Fig. 5). The component  $B_S$  magnifies as the scattering phase function asymmetry increases, due to the raise of contribution of the higher brightness from the near-horizon region (see Fig. 3).

**Table 2. Relative brightness ( $B_S/B_{hd}$ ) for isotropic and Rayleigh scattering phase functions**

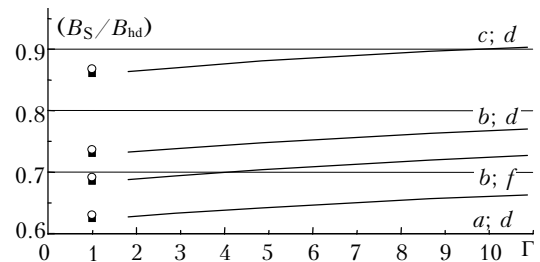
Scattering phase function	Model $B(\mu')$								
	$a$			$b$			$c$		
	$d$	$e$	$f$	$d$	$e$	$f$	$d$	$e$	$f$
Isotropic	0.625	0.582	0.538	0.731	0.687	0.644	0.861	0.818	0.774
Rayleigh	0.631	0.588	0.545	0.737	0.694	0.651	0.868	0.826	0.783

**Table 3. Relative brightness  $B_S/B_{hd}$  for the Henyey-Greenstein scattering phase function**

Scattering phase function (7)		Model $B(\mu')$			
$g$	$\Gamma(p_\mu)$	$a+d$	$b+d$	$b+f$	$c+d$
0.2	1.83	0.627	0.733	0.688	0.864
0.346	2.89	0.633	0.738	0.694	0.870
0.5	4.86	0.642	0.748	0.704	0.881
0.65	8.68	0.657	0.763	0.720	0.897
0.7	10.88	0.663	0.770	0.727	0.903



**Fig. 4.** The influence of illumination conditions (parameters  $m$  and  $n$ ) on formation of brightness  $B_S$  at isotropic and Rayleigh scattering.



**Fig. 5.** The influence of asymmetry of the scattering phase function (the asymmetry coefficient  $\Gamma$ ) on brightness  $B_S$ : isotropic ( $\bullet$ ); Rayleigh ( $\circ$ ).

### 3.2. The algorithm for determining $\Lambda$

As it was mentioned above, Eq. (4) offers the possibilities for determination of the single scattering albedo:

$$\Lambda = \sigma/\varepsilon = (B^0 - B_{hd})/(B^0 - B_S). \quad (11)$$

To obtain the unknown value of  $B_S$ , we will use the results of modeling (see part 3.1). Practically linear dependence  $B_S(\Gamma)$  and small deviation from the results of calculation for the isotropic scattering allow one to write the simple relationship

$$B_S \approx \bar{B}(\bar{\mu}') + 0.004 B_{hd}(\Gamma - 1), \quad (12)$$

where  $\bar{B}(\bar{\mu}') = 0.5 \int_{-1}^1 B(\mu') d\mu'$  is the brightness  $B_S$  at

the isotropic scattering, which is equal to the mean value of  $B(\mu')$  in the interval  $\mu'[-1, 1]$  and can be easily determined experimentally.

Formula (12) allows us to estimate the brightness  $B_S$  calculating  $\bar{B}(\bar{\mu}')$  and taking into account the scattering phase function asymmetry  $\Gamma$ . In the absence of *a priori* data on the asymmetry coefficient, the brightness  $B_S$  can be estimated for some mean scattering phase function. For example, at  $\Gamma_{mean} = 6$  we obtain the approximate formula from Eq. (12):

$$B_S \approx \bar{B}(\bar{\mu}') + 0.02 B_{hd} \approx \bar{B}(\bar{\mu}') + 0.02 B^0. \quad (13)$$

Thus, the procedure of solving the problem can be as follows.

1. The angular (through zenith angle) distribution of the brightness of the sky and underlying surface  $B(\mu')$  is measured, including the sky brightness on the horizon  $B(\mu' = 0) = S(0) = B_{hd}$ .

2. The brightness of the blackbody is measured (or calculated from the temperature of the atmosphere)  $B^0(T_{atm})$ .

3.  $\bar{B}(\bar{\mu}')$  is calculated from the data obtained for  $B(\mu')$  by numerical integration of values from zenith to nadir.

4. The real value of  $B_S$  is estimated by formulae (12) and (13), and then  $\Lambda$  is determined through Eq. (11).

### 3.3. The effect of “cool horizon”

Prerequisites to the difference between temperatures of the horizon and surrounding region  $T_{atm}$  were discussed earlier.<sup>7,12,17</sup> Since under conditions of the cloudless atmosphere relationships  $B^\downarrow(\mu') < B_{hd}$  and  $B^\uparrow(\mu') \approx B_{atm}^0$  (or  $B_{hd}$ ) usually hold, then  $B_S$  is always less than  $B^0 \approx B_{hd}$  (see Table 3). Thus, it follows from Eq. (4) that the horizon brightness is equal or less than  $B_{atm}^0$ . The fact is confirmed by the results of field experiments. According to Ref. 7, the temperature difference ( $T_{atm} - T_{hd}$ ) under cloudless conditions reaches 2.5 K above the sea and 1 K above the ground. According to the data of Ref. 17, the mean value of the ratio ( $B_{hd}/B_{atm}^0$ ) interpreted as the atmospheric emittance near the horizon, is 0.95. (the difference  $B_{hd} \neq B_{atm}^0$  can be explained in this case by the fact that the sphericity of the atmosphere was not taken into account).

Now we estimate the effect of different factors on the horizon brightness, using the isotropic scattering as an example (the asymmetry of the scattering phase function is of less concern). Taking into account Eq. (9), one can rewrite formula (4) in the form

$$b_{hd}^0 = \frac{B_{hd}}{B_{atm}^0} = 1 - \Lambda (1 - B_S/B_{atm}^0) = \frac{1 - \Lambda}{1 - \Lambda\{1 - 0.5[(1-m)(1+n)^{-1} + (1-m^*)(1+n^*)^{-1}]\}}. \quad (14)$$

The calculations have shown (Fig. 6) that the maximum “cooling of the horizon” (decrease of  $b_{hd}^0$ ) can be expected at great  $\Lambda$ , small brightness  $B_S$  ( $m \rightarrow 0$ ), and small scattering phase function asymmetry. The magnitude of the scattering component  $B_S$  is of principal importance for manifestation of the effect.

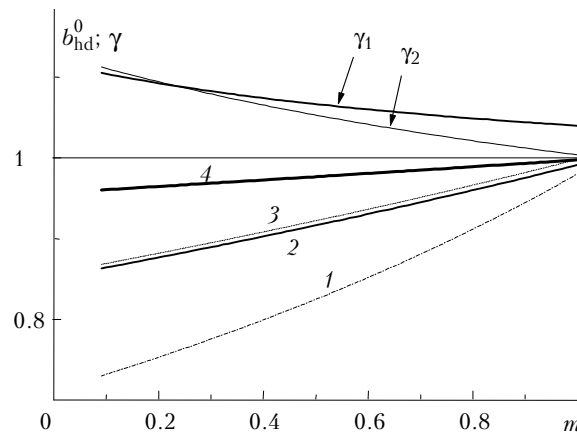


Fig. 6. Dependences of the normalized values of the horizon brightness and the total flux  $\gamma = (\Sigma/2\pi B_{hd})$  on the magnitude of the downwelling radiation determined by the parameter  $m$ :  $\Lambda = 0.5$ ,  $\Gamma = 0.6$  (1);  $\Lambda = 0.3$ ,  $\Gamma = 6$  (2);  $\Lambda = 0.3$ ,  $\Gamma = 2$  (3);  $\Lambda = 0.1$ ,  $\Gamma = 6$  (4).

Even at strong effect of aerosol ( $\Lambda \rightarrow 1$ ) the increase of the sky brightness ( $m \rightarrow 1$ ) due to low cloudiness and strong turbidity of the atmosphere, or enhanced temperature of the underlying surface ( $m^* > 1$ ) lead to the fact that the horizon brightness becomes close to  $B_{atm}^0$ . This peculiarity limits the area of application for the above-described method (see part 3.2).

It is of interest to know whether the situation of “hot horizon” ( $B_{hd} > B_{atm}^0$ ) is possible. Such facts are unknown because of lack of experimental investigations of the horizon brightness, therefore, the problem can be treated only hypothetically on the basis of model estimates. It follows from Eq. (14) that, according to the illumination conditions, the excess of brightness  $B_{hd}$  over  $B_{atm}^0$  is possible, if the magnitude of the expression in braces is greater than 1. From the standpoint of atmospheric conditions, such a situation can be expected during dust storms in arid regions, i.e., at the set of conditions: very heated underlying surface ( $m^* > 1$ ) and high content of aerosol at low humidity ( $\Lambda \rightarrow 1$ ,  $m \rightarrow 1$ ).

It follows from the left part of Eq. (14) taking into account Eq. (13), that the key condition of difference between  $B_{hd}$  and  $B_{atm}^0$  is the magnitude of the ratio of the mean brightness  $\bar{B}(\bar{\mu}')$  to  $B_{atm}^0$ :

$$\frac{B_S}{B_{\text{atm}}^0} \approx \left( \frac{\bar{B}(\bar{\mu}')}{B_{\text{atm}}^0} \right) = \begin{cases} > 1 - \text{“hot horizon”}, \\ < 1 - \text{“cold horizon”}. \end{cases} \quad (15)$$

Thus, the total range of possible values of the sky brightness near the horizon in the limits from  $\bar{B}(\bar{\mu}')$  to  $B_{\text{atm}}^0$  is set by conditions of illumination of the horizontal path  $B(\mu')$ , and the particular value of  $B_{\text{hd}}$  inside this range is determined by scattering properties of the atmospheric near-ground layer, i.e., by the magnitude of  $\Lambda$ .

### 3.4. Relation with components of the radiation budget

Formally, the aforementioned formulae for the scattered component of the source function  $B_S$  are close to sum of fluxes  $\Sigma$  of the downwelling  $Q$  and upwelling  $R$  thermal radiation, as well as to the total flux  $G$  (Ref. 11):

$$G = 2\pi \int_{-1}^1 B(\mu') \mu' d\mu' = 2\pi \left[ \int_0^1 B^\downarrow(\mu') \mu' d\mu' + \int_{-1}^0 B^\uparrow(\mu') \mu' d\mu' \right] = Q - R; \quad (16)$$

$$\Sigma = Q + R = 2\pi \left[ \int_0^1 B^\downarrow(\mu') \mu' d\mu' + \left| \int_{-1}^0 B^\uparrow(\mu') \mu' d\mu' \right| \right] = 2\pi B_{\text{hd}} \left\{ 1 - \frac{1-m}{1+n} - \frac{1-m^*}{1+n^*} \right\}; \quad (17)$$

$$B_S = \frac{1}{2} \left[ \int_0^1 B^\downarrow(\mu') \mu' d\mu' + \int_{-1}^0 B^\uparrow(\mu') \mu' d\mu' \right] + \delta_B(\Gamma) \approx B_{\text{hd}} \left\{ 1 - \frac{1-m}{2(1+n)} - \frac{1-m^*}{2(1+n^*)} + 0.004(\Gamma - 1) \right\}; \quad (18)$$

$$B_{\text{hd}} = B_{\text{atm}}^0 - \Lambda (B_{\text{atm}}^0 - B_S). \quad (19)$$

All presented characteristics are determined by the integrals of the upwelling and downwelling radiation  $B(\mu')$ . The parameters  $\Sigma$  and  $G$  characterize the sum and the difference of radiations incident on a horizontal plate from the upper and lower hemispheres. The brightness  $B_S$  is the scattering of this radiation in the horizontal direction. The contribution of the radiation from the directions nearby zenith and nadir into the total flux (16) and (17) is increased relative to the mean brightness  $\bar{B}(\bar{\mu}')$  due to the cosine dependence (factor  $\mu'$ ). The contribution of the radiation from the horizon region, which is close to the brightness of the blackbody at the temperature of the atmospheric near-ground layer, into the component  $B_S$  (18) increases due to asymmetry of the real scattering phase functions. The effect of  $B(\mu')$  through the component  $B_S$  also manifests itself in the sky brightness near the horizon (19), which is intermediate between  $B_S$  and  $B_{\text{atm}}^0$ .

Solutions for  $\Sigma$  and  $B_S$  (expressions in braces in Eqs. (17) and (18)) show similar behavior and differ by some constant depending on illumination conditions. The dependence of the flux normalized magnitude  $\gamma = (\Sigma/2\pi B_{\text{hd}})$  on the sky brightness (parameter  $m$ ) is shown in Fig. 6 for two extreme cases: 1)  $\gamma_1$  at  $\Gamma = 11$ ,  $n = 0.1$ ; and  $\gamma_2$  at  $\Gamma = 2$ ,  $n = 0.8$  as an example. As the downwelling radiation increases ( $m \rightarrow 1$ ),  $\Sigma$  and  $B_S$  approach each other, and  $2\pi$  is the asymptote of deviation of  $\Sigma$  from  $B_S$  and  $B_{\text{hd}}$ .

## Conclusion

Analysis of the solution of the radiation transfer equation for the horizontal direction ( $\mu = 0$ ) and axial symmetry has shown that near the horizon the sky brightness, equal to the source function  $S(0)$ , is represented by a sum of two components:  $\bar{B}(\bar{\mu}')$  with accounting for a small correction for the asymmetry of the scattering phase function, and  $B_{\text{atm}}^0$ , the relative contribution of which is determined by the magnitude of the single scattering albedo. Based on the conducted modeling, we propose the algorithm for determining  $\Lambda$ . The most preferable condition for realization of the method is a small value of the averaged brightness  $\bar{B}(\bar{\mu}')$ .

It follows from the model estimates of the dependence of  $B_S$  and  $B_{\text{hd}}$  on illumination conditions that under typical conditions of the cloudless atmosphere the following relation holds:

$$B^0(T_{\text{atm}}) \geq B_{\text{hd}}(B^0; B_S; \Lambda) > B_S(\rho_{\mu 0}; B_{\mu'}) > \bar{B}(\bar{\mu}') > \Sigma/2\pi. \quad (20)$$

Under conditions of strong turbidity of the atmosphere or at low cloudiness these characteristics become close to  $B_{\text{atm}}^0$ . At high transparency of the atmosphere the range of values  $[B_{\text{atm}}^0; \bar{B}(\bar{\mu}')]$  expands, and, depending on  $\Lambda$ , one or another difference between  $B_{\text{hd}}$  and  $B_{\text{atm}}^0$  is possible. The maximum difference or the effect of “cooling the horizon” is expected at high aerosol content and minimum absorption ( $\Lambda \rightarrow 1$ ). This is a qualitative difference from the shortwave range ( $\lambda < 3 \mu\text{m}$ ), where the sky brightness on the horizon increases as  $\Lambda$  increases (the absorption decreases) due to smallness of the inherent radiation  $B_{\text{atm}}^0$ .

To complete the discussion, we shall dwell briefly on the degree of applicability of the condition of axial symmetry of  $B^\downarrow(\tau, \mu')$  to the reality. (In the case of the homogeneous underlying surface the limitation concerns only the sky brightness). Apart from the case of the cloudless night, the condition is fulfilled at the continuous cloudiness, which case is of small interest for applied problems, because all considered types of brightness tend to  $B_{\text{atm}}^0$ , and application of the method to determination of  $\Lambda$  becomes impossible (see formula (11)).

Considering the integral character of  $B_S$  (formulae (2) and (4a)), obviously, it is sufficient to use some weaker requirement, namely, the averaged axial symmetry. Therewith, the azimuth distribution  $B^\downarrow(\tau, \mu', \varphi')$  varies in the vicinity of some mean value, and the latter does not depend on the angle  $\varphi$ . This situation corresponds to homogeneous distribution of small clouds in the sky. Fulfillment of the "averaged axial symmetry" condition is indirectly confirmed by experimental data,<sup>7-9</sup> which indicate that the azimuth dependence of brightness  $B_{hd}$  in the wavelength range 8–12  $\mu\text{m}$  does not practically manifest itself.

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