

Temporal statistical structure of meteorological fields in the atmospheric boundary layer

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We discuss the results of statistical analysis of temporal structure of temperature, specific humidity, and zonal and meridional wind fields using data of observations at Warsaw and Novosibirsk sites for atmospheric boundary layer. In addition to analysis of temporal correlation functions of temperature, specific humidity, and zonal and meridional wind, we examine their approximations for time lags $\tau = 1, 2, \dots, 5$ days using analytical expressions derived.

Introduction

Recently, there has been shown a growing interest in the study of temporal statistical structure of meteorological fields in the atmospheric boundary layer (ABL), primarily produced by mesoscale atmospheric processes, with a special emphasis on analysis of temporal correlation functions and their analytical approximation.

This is because the information on temporal statistical structure of meteorological fields in the atmospheric boundary layer (and primarily, on correlation functions) is required not only for study of time behavior of meteorological quantities, but also for some practical applications. The latter, in particular, include:

- objective four-dimensional analysis of mesoscale temperature and wind fields in the atmospheric boundary layer, based on the interconnecting procedure of the modified method of clustering of arguments (MMCA) with the methods of optimal interpolation and optimal extrapolation of the random process,¹ with MMCA results usable to solve problems of numerical prediction of pollution in a limited air basin (such as over a big city or industrial zone); and

- construction of the dynamical models describing the temporal variability of fields of meteorological quantities in the atmospheric boundary layer and used in the algorithm of their space-time extrapolation 250–300 km as deep into the data-sparse area, using Kalman filtration technique.²

Also the temporal statistical structure of meteorological fields (unlike the well-studied spatial structure^{3–5}) is still poorly studied, especially for the atmospheric boundary layer.

All this calls for the efforts urgently needed to perform special research in the framework of statistical analysis of time structure of meteorological fields in atmospheric boundary layer to obtain reliable data on the structure variations over limited time periods when mesoscale atmospheric processes play an important role, and to answer the question on what analytical

expressions can best fit the time correlation functions of a meteorological quantity.

We discuss in this paper the results of similar studies performed for the atmospheric boundary layer in terms of the temperature, humidity, and zonal and meridional wind fields.

1. Some methodological aspects and characterization of the initial material

As is well known, any random function of time, usually dealt with in practice, is often examined for the possibility to be treated as a stationary one. However, for atmospheric random processes, the assumption of stationarity can be valid only within short time intervals and it rapidly deteriorates as these time intervals increase. In addition, the presence of diurnal and annual variations of meteorological quantities, as well as some other systematic factors, leads to inconstancy of mathematical expectation, i.e., to its variations in time t .

At the same time, the stationarity, in the sense of independence of correlation function from the choice of origin point, remains valid, at least within a practically acceptable approximation, if not exactly.

In these cases, to determine the mathematical expectation $\bar{\xi}(t)$, it is necessary to remove the periodic contributions. In practice, this is usually done through single or multiple application of moving smoothing of the studied process over some fixed averaging interval n . Mathematically, the procedure of such a smoothing is usually presented by the integral operator of the form⁶

$$\bar{\xi}_n(t) = \frac{1}{n} \int_{t-n/2}^{t+n/2} \xi(t) dt, \quad (1)$$

where $\bar{\xi}_n(t)$ is the moving average; $\xi(t)$ is some random process; n is the length of moving averaging period; $t - n/2$ and $t + n/2$ are the current limits of integration of the process $\xi(t)$ over this period.

To estimate the correlation function, it is necessary to use not the random process itself, but a centered random process, i.e., its deviation from the moving mean $\xi(t) = \xi(t) - \bar{\xi}_n(t)$, determined sequentially for each interval of averaging. In this case, the correlation function of the centered random process $R_\xi(\tau_k)$ can be calculated by use of the following expression:

$$R_\xi(\tau_k) = \frac{1}{N-k} \sum_{i=1}^{N-k} [\xi(t_i) - \bar{\xi}_n(t_i)] \times [\xi(t_i + \tau_k) - \bar{\xi}_n(t_i + \tau_k)], \quad (2)$$

where $\tau_k = k \Delta t$ ($k = 0, 1, 2, \dots, m$) is the time lag; $\xi(t_i)$ and $\xi(t_i + \tau_k)$ are terms of the series of observations; $t_i = i\Delta t$ and $t_i + \tau = (i + k)\Delta t$ (here Δt is discretization time of observations); N is the number of terms in the time series.

In addition, expression (2) can be used to determine the values of normalized correlation function

$$\mu_\xi(\tau_k) = R_\xi(\tau_k) / R_\xi(0). \quad (3)$$

We used expressions (1)–(3) to determine the moving means and normalized correlation functions of temperature, humidity, and orthogonal wind velocity components (in what follows, the word “normalized” will be omitted for brevity). The length of the interval for moving average, performed for a single standard time (12 GMT) was taken to be 9 days, during which a gradual decrease of the time correlation is observed.⁷

Here, it is important to note the following. The nonstationary behavior in time (annual variations) of the meteorological quantities (most notably, temperature) with respect to time average is observed primarily in transition (spring and fall) months,⁸ and so it should be taken into account in calculating the correlation functions. At the same time, when these functions are calculated for winter and summer, the stationarity with respect to mathematical expectation is usually assumed.

Despite this, to check the validity of this assumption and estimate its effect on the time correlation, we compared the correlation functions for temperature (this meteorological quantity has most

strong annual variations), calculated for the cases of stationary and nonstationary variations of this meteorological quantity about its average value. To calculate the correlation functions for the case of stationary variations about the average, i.e., $\bar{\xi}(t) = \bar{\xi}$, we used the following expressions:

$$R_\xi(\tau_k) = \frac{1}{N-k} \sum_{i=1}^{N-k} [\xi(t_i) - \bar{\xi}] [\xi(t_i + \tau_k) - \bar{\xi}]; \quad (4)$$

$$\mu_\xi(\tau_k) = R_\xi(\tau_k) / \sigma_\xi^2, \quad (5)$$

where σ^2 is the variance of the meteorological quantity, calculated over the entire data set.⁸

The calculations using many year (1970–1975) temperature data obtained at Novosibirsk site (geographic location given below) have shown (see Table 1) that the correlation functions of this meteorological quantity, calculated for January and July with account of stationary and nonstationary behavior of the mathematical expectation, do differ little and are within standard error of their calculation. Therefore, the mathematical expectation $\bar{\xi}$ of a stationary process was used subsequently throughout the computation of time correlation functions of all meteorological quantities considered (temperature, humidity, and wind velocity components) in the atmospheric boundary layer.

We tried to choose the best fits and approximated (by the method to be described below) the obtained correlation functions by⁹

$$\mu_\xi(\tau) = \exp(-\alpha\tau), \quad \alpha > 0, \quad (6)$$

$$\mu_\xi(\tau) = \exp(-\alpha\tau^2), \quad \alpha > 0, \quad (7)$$

$$\mu_\xi(\tau) = \{\exp(-\alpha\tau)\} \cos \beta\tau, \quad \alpha > 0 \quad (8)$$

$$\mu_\xi(\tau) = \{\exp(-\alpha\tau^2)\} \cos \beta\tau, \quad \alpha > 0 \quad (9)$$

$$\mu_\xi(\tau) = \begin{cases} 1 - \tau/\tau_0 & \text{for } |\tau| \leq \tau_0 \\ 0 & \text{for } |\tau| > \tau_0 \end{cases}, \quad (10)$$

where $\tau = |t_2 - t_1|$ is time lag; α and β are empirical coefficients; and τ_0 is the correlation time scale.

Table 1. Time correlation functions of temperature $\mu_T(\tau) \cdot 10^2$ for stationary (1st column) and nonstationary (2nd column) atmospheric processes

Time lag, h	Height, m									
	100		400		800		1200		1600	
	1	2	1	2	1	2	1	2	1	2
Winter										
24	68	67	70	70	71	72	72	75	72	75
48	50	48	54	56	54	56	53	56	52	55
72	44	42	48	48	49	49	48	47	47	47
96	39	36	44	41	45	43	45	43	45	42
120	33	32	40	38	41	39	42	39	42	40
Summer										
24	62	62	64	64	67	67	71	69	73	70
48	42	42	44	42	46	44	47	43	48	43
72	24	24	26	26	27	26	29	24	30	23
96	15	15	16	14	17	14	18	13	18	14
120	07	07	06	06	06	07	06	06	07	05

In conclusion, a brief account of characteristics of the initial material will be provided.

For statistical estimation of empirical time correlation functions of temperature, specific humidity, and zonal and meridional wind velocity components, we used data from 15 summer coincident (12 GMT) radiosonde observations at two aerological stations: Warsaw (52°11'N, 20°58'E) and Novosibirsk (55°02'N, 82°54'E), located in different physical and geographical regions.

All data used for analysis pertain to 2 central months (January and July) of winter and summer seasons and to atmospheric layer 0–1.6 km. The use of data for just central months, and not for the whole winter and summer seasons, to calculate time correlation functions is quite justified because, according to Ref. 7, these functions are identical throughout each of the seasons studied.

In addition, the initial data, given for standard isobaric surfaces and levels of special points, were rearranged and referenced to the system of geometric heights including such levels as: 0, 100, 200, 400, 800, 1200, and 1600 m. This referencing of the initial data to the chosen system of geometric heights was made

using linear interpolation of individual values of the meteorological quantities considered from isobaric surfaces and levels of special points to all these levels mentioned above.

2. Specific features of temporal statistical structure of temperature, humidity, and wind fields in the atmospheric boundary layer

In this section, we will focus on the results of statistical analysis of temporal structure of temperature, specific humidity, and zonal and meridional wind velocity components in the atmospheric boundary layer.

As an example, Tables 2 and 3 present time correlation functions of temperature, specific humidity, and zonal and meridional wind velocity components, calculated using data of measurements at Novosibirsk and Warsaw sites for altitudes 0, 100, 200, 400, 800, 1200, and 1600 m, all lying within the atmospheric boundary layer, and for different time lags $\tau = 1, 2, \dots, 5$ days.

Table 2. Time correlation functions ($\mu_{\xi}(\tau) \times 10^2$) of temperature, humidity, and zonal and meridional wind components, derived from measurements at Warsaw and Novosibirsk sites for different time lags τ in January

Height, m	Warsaw					Novosibirsk				
	τ , day									
	1	2	3	4	5	1	2	3	4	5
	Temperature									
0	62	47	39	31	25	67	47	42	37	31
100	66	49	40	35	31	68	50	44	39	33
200	68	49	40	35	30	69	52	45	40	35
400	69	49	38	35	30	70	54	48	44	40
800	71	50	39	34	29	71	54	49	45	41
1200	72	53	41	34	30	72	53	48	45	42
1600	74	56	43	35	31	72	52	47	45	42
	Specific humidity									
0	62	46	36	26	22	60	40	37	33	30
100	68	49	37	28	25	61	42	37	34	31
200	69	51	37	28	26	62	44	38	34	32
400	70	52	38	29	26	63	47	40	35	33
800	65	44	33	23	20	61	45	39	34	31
1200	58	37	27	18	14	56	43	37	32	28
1600	52	33	23	15	10	50	38	32	28	25
	Zonal wind									
0	36	28	23	18	15	42	25	20	14	09
100	46	35	27	19	16	47	27	23	18	14
200	49	33	25	20	16	52	32	24	20	15
400	51	35	27	21	17	56	35	27	21	16
800	56	39	30	24	19	62	39	27	21	15
1200	61	44	33	27	22	64	40	27	21	14
1600	66	45	35	28	24	65	43	30	22	14
	Meridional wind									
0	29	16	10	04	02	46	24	22	15	09
100	32	18	11	04	02	47	24	21	17	12
200	36	19	12	05	03	48	24	21	13	11
400	37	20	14	06	04	49	24	20	12	06
800	45	23	19	09	06	50	25	21	16	10
1200	55	34	23	16	14	51	28	23	19	17
1600	61	44	31	24	20	56	34	30	26	23

Table 3. Time correlation functions ($\mu_{\xi}(\tau) \times 10^2$) of temperature, humidity, and zonal and meridional wind, derived from measurements at Warsaw and Novosibirsk sites for different time lags τ in July

Height, m	Warsaw					Novosibirsk				
	τ , day									
	1	2	3	4	5	1	2	3	4	5
	Temperature									
0	61	41	31	29	27	61	35	21	14	07
100	65	44	34	31	28	62	42	24	15	07
200	67	47	37	33	29	63	43	25	16	07
400	68	48	37	33	29	64	44	26	16	06
800	69	47	36	33	28	67	46	27	17	06
1200	69	46	33	30	26	71	47	29	18	06
1600	70	45	33	29	26	73	48	30	18	07
	Specific humidity									
0	52	32	27	23	21	55	28	12	06	02
100	53	34	29	25	21	58	32	12	07	03
200	54	36	31	27	24	59	33	12	07	03
400	60	44	37	34	32	61	34	13	08	04
800	58	42	35	33	27	60	34	15	09	05
1200	56	39	34	32	27	54	31	13	07	05
1600	55	37	32	30	27	50	28	10	06	02
	Zonal wind									
0	36	21	19	17	12	35	16	10	06	04
100	42	24	21	18	13	37	17	11	05	01
200	46	29	25	21	17	39	19	13	05	01
400	48	31	24	21	15	43	22	16	05	01
800	54	36	25	21	13	50	30	17	05	01
1200	64	41	26	20	13	52	33	18	06	03
1600	65	43	30	19	11	57	31	20	07	05
	Meridional wind									
0	31	16	10	07	05	40	25	21	14	07
100	35	17	12	08	07	41	26	22	14	05
200	39	23	14	09	06	42	27	23	15	05
400	43	27	17	10	05	45	30	25	16	06
800	48	30	20	10	07	52	37	28	18	09
1200	54	32	21	11	07	57	43	29	20	12
1600	59	34	22	12	09	62	48	32	23	14

Analysis of data presented in Tables 2 and 3 shows that:

– the time correlations of temperature, specific humidity, and zonal and meridional wind components markedly decrease at all altitudes with the increasing time lag τ , however remaining positive even for $\tau = 5$ days;

– independent of season and altitude, the time correlations of temperature decrease most slowly so that for time lag $\tau = 1$ day (most frequently used to construct dynamic models for dynamic-stochastic prediction of meteorological fields²) the temperature correlation coefficients lie in the range 0.62–0.74 in winter and 0.61–0.73 in summer; whereas zonal and meridional wind correlations decrease most strongly (so that, for this same time lag, they range from 0.29 to 0.66 in winter and from 0.31 to 0.65 in summer), and even correlation coefficient of 0.29 is much higher than its threshold value of 0.08 taken for the probability $P = 0.95$;

– the closeness of time correlations of temperature and zonal and meridional wind increases with height. Indeed, for time lag $\tau = 1$ day, the time correlation

coefficients are 0.61–0.67 (for temperature) and 0.29–0.36 (for zonal and meridional wind) at the surface ($h = 0$); whereas toward 1600 m height, they increase, respectively, to 0.70–0.74 and 0.56–0.65;

– the time correlation coefficients of specific humidity, for the same time lag $\tau = 1$ day, behave with altitude in a different way as those of temperature and wind velocity components: they first increase with height, reach a maximum near 400 m of 0.60–0.70, and then decrease to a minimum (of about 0.50–0.55) toward 1600 m height.

This behavior of time correlations of specific humidity with height is associated with the fact that, in the 800–1600 m layer, cloud formation processes play an important role; exerting a disturbing effect on the behavior of water vapor concentration with time and, thereby, leading to a decrease in correlation for this meteorological quantity.

These are some specific features in the behavior of time correlation functions of temperature, specific humidity, and zonal and meridional wind, revealed by analyzing the temporal statistical structure of the corresponding meteorological fields.

3. Some results of analytical approximation of time correlation functions of temperature, humidity, and zonal and meridional wind

As is well known, in practical applications, such as in construction of dynamic models, used to construct the algorithms of dynamic-stochastic prediction, it is more convenient to use different analytical expressions approximating these functions instead of empirical time correlation functions $\mu_{\xi}(\tau)$, (see discussion above for some examples). Therefore, we tried to find the best analytical expressions that could describe the time correlation functions of temperature, specific humidity, and zonal and meridional wind, obtained for atmospheric boundary layer, with minimum error.

For this, we have considered all the above-mentioned approximate expressions of the form (6)–(10). Their careful comparison with empirical correlation functions $\mu_{\xi}(\tau)$ has shown that none can approximate these functions with acceptable for practice accuracy. This made us to try the approximation of the empirical correlation functions of temperature, specific humidity, and zonal and meridional wind by a unified analytical formula, namely:

$$\mu_{\xi}(\tau) = (1 - \alpha\tau) e^{-\beta\tau}, \tag{11}$$

whose parameters depend on altitude and can be determined from the following expressions:

$$\alpha(h) = a + bh; \tag{12}$$

$$\beta(h) = c + dh, \tag{13}$$

where $\tau = 1, 2, \dots, 5$ is the time lag (in days), and h is height in meters.

The coefficients $a, b, c,$ and d in expressions (12) and (13) have (for all seasons and sites) the following values:

for temperature

$$a = 0.017; \quad b = -1.706 \cdot 10^{-3};$$

$$c = 0.008; \quad d = -5.125 \cdot 10^{-3};$$

for the specific humidity (for $0 \leq h < 400$ m)

$$a = 0.0019; \quad b = -9.277 \cdot 10^{-3};$$

$$c = 0.009; \quad d = -1.889 \cdot 10^{-3};$$

for the specific humidity (for $h = 400\text{--}1600$ m)

$$a = 0.016; \quad b = 3.985 \cdot 10^{-3};$$

$$c = 0.008; \quad d = 1.184 \cdot 10^{-3};$$

and for zonal and meridional wind

$$a = 0.028; \quad b = -6.804 \cdot 10^{-3};$$

$$c = 0.012; \quad d = -2.327 \cdot 10^{-3}.$$

The accuracy of this approximation can be judged based on data given in Table 4, which gives, as an example, the absolute deviations of the analytical function of the form (11) from the corresponding empirical correlation functions at heights 0, 800 and 1600 m.

Table 4. Absolute deviations of analytical functions of the form (11) from the empirical correlation functions of temperature, specific humidity, and zonal and meridional wind for different time lags τ

Height, m	τ , day				
	1	2	3	4	5
Winter					
Temperature					
0	0.02	0.04	0.05	0.06	0.07
800	0.02	0.04	0.05	0.07	0.08
1600	0.02	0.04	0.05	0.07	0.08
Specific humidity					
0	0.02	0.03	0.05	0.06	0.08
800	0.03	0.04	0.04	0.04	0.04
1600	0.01	0.02	0.02	0.02	0.02
Zonal wind					
0	0.02	0.02	0.01	0.00	0.00
800	0.02	0.01	0.02	0.03	0.03
1600	0.00	0.00	0.02	0.03	0.05
Meridional wind					
0	0.02	0.02	0.01	0.00	0.00
800	0.00	0.01	0.02	0.03	0.03
1600	0.00	0.00	0.02	0.03	0.05

Table 4 (continued)

Height, m	τ , day				
	1	2	3	4	5
	Summer				
	Temperature				
0	0.02	0.04	0.05	0.06	0.07
800	0.02	0.04	0.06	0.07	0.08
1600	0.02	0.04	0.05	0.07	0.09
	Specific humidity				
0	0.02	0.04	0.05	0.06	0.07
800	0.03	0.04	0.04	0.05	0.04
1600	0.01	0.02	0.02	0.02	0.02
	Zonal wind				
0	0.01	0.02	0.01	0.00	0.00
800	0.00	0.01	0.02	0.03	0.03
1600	0.00	0.00	0.01	0.03	0.05
	Meridional wind				
0	0.01	0.02	0.01	0.00	0.00
800	0.00	0.01	0.02	0.03	0.03
1600	0.00	0.00	0.01	0.03	0.05

From analysis of Table 4 it follows that, for all meteorological quantities, seasons, and altitudes, the discrepancy between analytical and empirical correlation functions is small enough. Therefore, the obtained approximate formula of the form (11) can be successfully used for adequate description of empirical correlation functions up to $\tau = 5$ days.

Thus, we have considered some results of analysis of the temporal statistical structure of temperature, specific humidity, zonal and meridional wind fields in the atmospheric boundary layer, as well as analytical approximation of the corresponding empirical correlation functions.

References

1. V.S. Komarov and A.V. Kreminskii, Atmos. Oceanic Opt. **12**, No. 8, 700–705 (1999).
2. V.S. Komarov, V.I. Akselevich, A.V. Kreminskii, Yu.B. Popov, and K.Ya. Sineva, in: *Proceedings of International Scientific Conference on Ecological and Hydrometeorological Problems of Big Cities and Industrial Zones* (St. Petersburg, 2000), pp. 74–75.
3. L.S. Gandin and R.L. Kagan, *Statistical Methods of Interpretation of Meteorological Data* (Gidrometeoizdat, Leningrad, 1976), 359 pp.
4. L.S. Gandin, V.N. Zakhariyeva, and R. Tselnai, eds., *Statistical Structure of Meteorological Fields* (Budapest, 1976), 365 pp.
5. V.S. Komarov and Yu.B. Popov, Atmos. Oceanic Opt. **11**, No. 8, 686–691 (1998).
6. M.M. Voronchuk and E.V. Kulish, in: *Proceedings of Third All-Union Symposium on Use of Statistical Methods in Meteorology* (Moscow Branch of Gidrometeoizdat, Moscow, 1978), pp. 211–217.
7. O.A. Drozdov, V.A. Vasiliev, N.V. Kobysheva, A.N. Raevskii, L.K. Smekalova, and E.P. Shkol'ny, *Climatology* (Gidrometeoizdat, Leningrad, 1989), 567 pp.
8. N.V. Kobysheva and G.Ya. Narovlyanskii, *Climatological Processing of Meteorological Information* (Gidrometeoizdat, Leningrad, 1978), 295 pp.
9. D.N. Kazakevich, *Fundamentals of Theory of Random Functions and Its Application in Hydrometeorology* (Gidrometeoizdat, Leningrad, 1977), 319 pp.