Theory of multiple scattering and its application to the problem of laser sensing of aerosol

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The equation of laser sensing is formulated taking into account multiple scattering ($N > 2$) and polarization effects. A method of calculation of lidar returns from an inhomogeneous cloud is proposed, the accuracy of which is tested by comparison with calculations by the Monte Carlo method. Peculiarities of the lidar signal structure at the boundary between the cloud and the clear atmosphere are considered.

Introduction

A correct account of the effect of multiple scattering (MS) on the return signal is necessary to increase the information capacity of methods of laser sensing of the atmosphere. Undoubtedly, the best way to estimate the MS background is to solve the nonstationary transfer equation by means of the Monte Carlo method (a complete review of methods for solving sensing problems is presented in Ref. 1).

One drawback of such an approach is that it is poorly suited for conversion to automated operation. This is especially important when processing a great quantity of lidar measurements. In recent years this fact has stimulated the development of the method known as the semianalytical approximation of the MS background (see, for example, Refs. 2–4).

One of the factors limiting the range of applicability of such methods is the multiplicative representation of the MS contribution to the total signal. Representation of the signal in the form of a product of the single ($P^{(1)}(z)$) and multiple ($P^{(m)}(z)$) scattered components (as, for example, in the small-angle approximation) removes from consideration regions of sensing beyond the cloud boundary where $P^{(1)}(z) = 0$. This limitation is important for one-layer cloudiness when describing polarization effects. Due to the delay of the cross-polarized component of the lidar signal relative to the parallel one, maximum values of the degree of depolarization can be observed not in the cloud, but beyond its boundary.[5] Taking multiple scattering into account when sensing multi-layer cloudiness is possible only by using Monte Carlo methods, and the construction of an adequate approximation of the lidar equation based on its decomposition into scattering orders is the aim of the present paper.

The theory of multiple scattering ($N > 2$) for a homogeneous scattering layer was discussed in Refs. 6 and 7. The principles of the theory developed by us were stated in Refs. 8 and 9, where the equation of laser sensing of a medium with an arbitrary scattering phase function was formulated for the case of double scattering. An equation relating the power of a return signal of arbitrary scattering order to the parameters of the lidar and the medium is derived in the first part of the present paper.

1. Laser sensing equation taking into account the *N*th order of scattering

1.1. Geometry of sensing

Let a radiation source situated at the point O (Fig. 1) emit a pulse with power P_0 and duration Δt into the solid angle $\pi\theta_0^2$ at the time t_0 in the direction OZ. The return signal due to single scattering in a volume element of the medium centered at the point Z with coordinate $z = [c(t - t_0)]/2$ arrives at the receiving system with field-of-view angle $2\varphi_0$ at some time $t > t_0$. The value of this signal is described by the location equation in the single scattering approximation as

$$P^{(1)}(z) = P_0\, kS(z)\, \Delta z\, T^2(0, z)\, \gamma_\pi(z)\, \sigma(z), \qquad (1)$$

where k is the transmission coefficient of the receiving-transmission system optics, $S(z)$ is a function that depends on the geometric parameters of the locator; in the far zone of a monostatic lidar $S(z) = Az^{-2}$, A is the area of the receiving antenna of the lidar, $\Delta z = c\Delta t/2$; $\gamma_\pi(z)$ is the lidar ratio (scattering phase function in the backward direction), $T(0, z) = \exp\left\{-\int\limits_0^z \sigma(z')\, dz'\right\}$ is the atmospheric transmission of the altitude interval $[0, z]$, and $\sigma(z)$ is the scattering coefficient.

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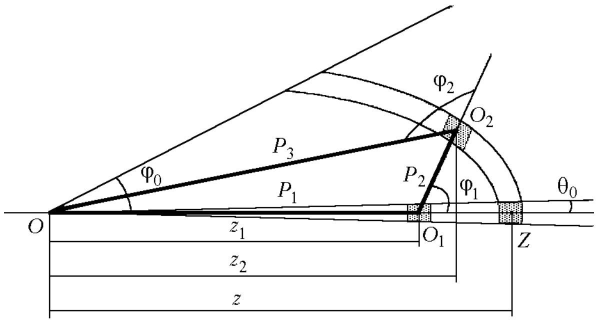

Fig. 1. Diagram of trajectories of elementary beams of doubly scattered radiation.

The signals due to greater scattering orders arrive at the receiving system of the lidar simultaneous with the single scattering signal. The doubly scattered radiation is due to scattering from a pair of elementary volumes of the medium: the volume element $\mathrm{d}V_1$ centered at the point O_1 along the sensing path and the volume element $\mathrm{d}V_2$ centered at the point O_2 situated on the ellipse, whose foci are the points O and O_1, where the symmetry axis coincides with the axis of sensing. The parameters of this ellipse have the form[8]

$$2a = \rho_2 + \rho_3 = 2z - \rho_1;\ 2c = |OO_1| = \rho_1;\ b = \sqrt{a^2 - c^2},$$

where a and b are the semi-major and semi-minor axis, respectively, of the ellipse. Integrating over all possible pairs of volume elements situated within the field of view of the receiver, we obtain the signal due to double scattering.

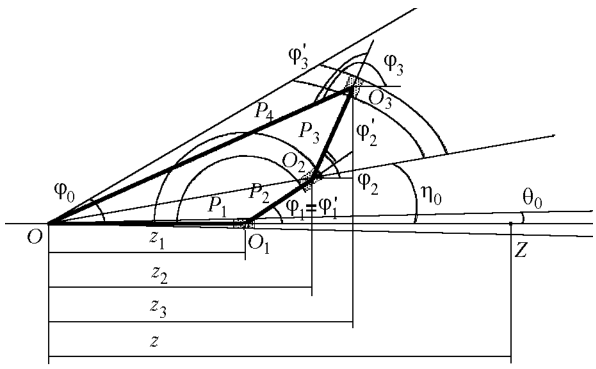

Fig. 2. Diagram of trajectories of elementary beams of triply scattered radiation.

Using this idea, we can continue. The triply scattered radiation comes from interaction of three volumes with the centers at the points O_1, O_2 and O_3 (Fig. 2), O_1 lies on the path of sensing, O_2 lies on the circle, and it is the second focus of the ellipse, on which, in turn, the third point O_3 lies (the first focus, as before, coincides with the point O). The parameters of the ellipse in this case have the form

$$2a = \rho_3 + \rho_4 = 2z - \rho_1 - \rho_2;$$
$$2c = |OO_2| = \rho_1 + \rho_2 \cos(\varphi_1).$$

Integrating with respect to all possible triples of the volumes, we obtain the signal cause by the triple scattering on condition that O_1, O_2 and O_3 are situated in the field of view of the receiver.

One can continue this reasoning. Radiation belonging to the Nth scattering order (generated by N scattering events) arises from the joint action of N elementary volumes of the medium centered, respectively, at the points O_1, ..., O_N, where O_1 lies on the sensing path, O_2, ..., O_{N-1} lie on the circles (the coordinates of these points $z_i = \rho_1 + \sum_{j=1}^{i} \rho_j \cos(\varphi_{j-1})$; $y_i = \sum_{j=2}^{i} \rho_j \sin(\varphi_{j-1})$ are determined, to a certain degree, randomly); and the point O_N lies on an ellipse whose coordinates and angle of emission are fixed and related to the coordinates of the previous points. The parameters of the ellipse in this case have the form

$$2a = \rho_N + \rho_{N+1} = 2z - \sum_{j=1}^{N-1} \rho_j;$$
$$2c = |OO_{N-1}| = \rho_1 + \sum_{j=2}^{N-1} \rho_j \cos(\varphi_{j-1}). \qquad (2)$$

1.2. Principal relations

Let us derive a formula for the elementary radiation flux $\mathrm{d}P$ arriving at the receiving system after N scattering acts. We can write the primary scattering act taking place in the volume element $\mathrm{d}V_1 = \pi\theta_0^2\rho_1^2\mathrm{d}\rho_1$ lying on the sensing beam axis as follows[8]:

$$\mathrm{d}P^{(1)} = \frac{P_0}{\rho_1^2}\,\sigma(\rho_1, \varphi_1)\,T(0, \rho_1)\,\mathrm{d}V_1,$$

where $\sigma(\rho, \varphi) = \sigma(\rho)\,\gamma(\rho, \varphi)$ is the directed scattering coefficient and $\gamma(\rho, \varphi)$ is the normalized scattering phase function. The scattering volumes for $i \in [2, N-1]$ are formed by the conical surface with opening angle $2\varphi_0$ and two concentric spheres centered at the point O_i and having radius-vectors ρ_i and $\rho'_i = \rho_i + \mathrm{d}\rho_i$ (see Fig. 2):

$$\mathrm{d}V_i = \sin(\varphi_{i-1})\,\rho_i^2\,\mathrm{d}\rho_i\,\mathrm{d}\varphi_{i-1}\,\mathrm{d}\theta_{i-1}. \qquad (3)$$

Here φ_i is the polar angle associated with the scattering angle φ'_i by the relation $\varphi'_i = \varphi_i - \varphi_{i-1}$ ($\varphi'_1 \equiv \varphi_1$), and θ_i is the angle of rotation about the $|OO_i|$ axis.

$$\mathrm{d}P^{(i)} = \frac{\mathrm{d}P^{(i-1)}}{\rho_i^2}\,\sigma(\rho_i, \varphi'_i)\,T(\rho_{i-1}, \rho_i)\,\mathrm{d}V_i. \qquad (4)$$

The volume defined by the Nth scattering event is formed by the conical surface with opening angle $2\varphi_0$ and two ellipsoids of rotation with symmetry axis forming the angle η_0 with the axis OO_1 and radius-vectors ρ_N and $\rho_N' = \rho_N + d\rho_N$, where $\rho_N = \dfrac{a^2 - c^2}{a + c\cos(\varphi_{N-1})}$; $d\rho_N = \dfrac{\partial \rho_N}{\partial z}\Delta z$; the parameters a and c are defined by formula (2). Since $\eta_0 \ll 1$ (see Fig. 2), the slope of the axis of the ellipse can be neglected. The relations for dV_N and $dP^{(N)}$ are analogous to Eqs. (3) and (4).

Finally, the expression for the elementary radiation flux arriving at the receiving systems after N scattering acts has the form

$$dP = \frac{P_0\, kA}{\rho_{N+1}^2} T(0, \rho_1, \dots, \rho_{N+1}) \prod_{j=1}^{N} \frac{\sigma(\rho_j, \varphi_j')\, dV_j}{\rho_j^2}. \quad (5)$$

Since $\varphi_0 \ll 1$, it may be assumed that σ depends only on the coordinate z, $\varphi_N' = \pi - \varphi_{N-1}'$. Thus $T(\rho_1, \dots, \rho_{N+1}) = \exp\left\{-\left[\sum_{i=1}^{N+1} \int_0^{\rho_i} \sigma(\rho')\, d\rho'\right]\right\} \cong T^2(0, z)$ is the transmission of the atmosphere on the path $OO_1 + O_1O_2 + \dots + O_NO = 2z$. Substituting the expression for dV_i into Eq. (5) and integrating over θ_i and ρ_i from 0 to 2π and from ρ_i to ρ_i', respectively, we obtain

$$P^{(N)}(z) = (2\pi)^{N-1} P_0\, kA\, \Delta z\, T^2(0, z) \int_{\rho_1} \sigma(z_1)\, d\rho_1 \dots \times$$
$$\times \int_{\rho_i}\int_{\varphi_{i-1}} \sin(\varphi_{i-1})\, \gamma(\varphi_{i-1}')\, \sigma(z_i)\, d\rho_i\, d\varphi_{i-1} \dots \times$$
$$\times \int_{\varphi_{N-1}} \frac{\sin(\varphi_{N-1})\, \gamma(\varphi_{N-1}')\, \gamma(\pi - \varphi_{N-1})\, \sigma(z_N)}{R^{(N)}}\, d\varphi_{N-1}, \quad (6)$$

where

$$\frac{1}{R^{(N)}} = \frac{d\rho_N}{\rho_{N+1}^2} = \frac{1}{a^2 + 2ac\cos(\varphi_{N-1}) + c^2}.$$

It is more convenient to use the following relation for numerical integration:

$$P^{(N)}(z) = (2\pi)^{N-1} P_0\, kA\, \Delta z\, T^2(0, z) \int_{z_0}^{z} \sigma(z_1)\, dz_1 \dots \times$$
$$\times \int_{z_0}^{z} \sigma(z_i) \int_0^{\varphi_{i-1}^*} \tan(\varphi_{i-1})\, \gamma(\varphi_{i-1}')\, dz_i\, d\varphi_{i-1} \dots \times$$
$$\times \int_0^{\varphi_{N-1}^*} \frac{\sin(\varphi_{N-1})\, \gamma(\varphi_{N-1}')\, \gamma(\pi - \varphi_{N-1})\, \sigma(z_N)}{R^{(N)}}\, d\varphi_{N-1}. \quad (7)$$

The assumption of horizontal homogeneity of the atmosphere and the relationship $d\rho_i = dz_i/\cos(\varphi_{i-1})$ were used to derive Eq. (7). The limits of integration are determined, first of all, by the position of the cloud layer, and second, by the field of view of the receiving system. The angles φ_{i-1}^* are determined by the intersection of the straight lines $z = z_i$ and $|OF|$ (Fig. 3).

$$\varphi_{i-1}^* = \arctan \frac{z_i \tan(\varphi_0) - y_{i-1}}{|z_i - z_{i-1}|}. \quad (8)$$

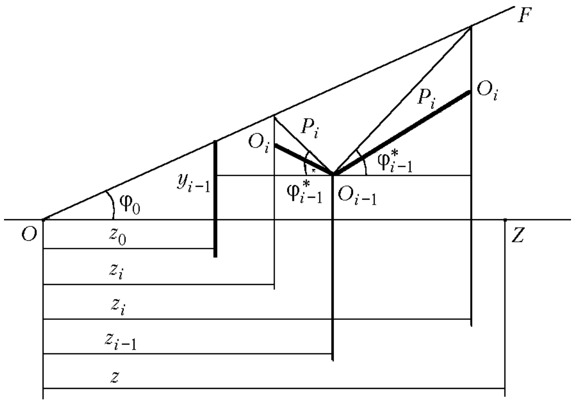

Fig. 3. Determination of the integration limits.

Analogously to Eq. (8), the angle $\varphi_{N-1}^* \in [0, \min(\varphi_{N-1}^1, \varphi_{N-1}^2)]$, where the angle φ_{N-1}^1 is determined by the intersection of the ellipse with the straight line $|OF|$:

$$\varphi_{N-1}^1 = \arccos \frac{2ac - (a^2 + c^2)\cos(\varphi_0)}{2ac\cos(\varphi_0) - (a^2 + c^2)}, \quad (9)$$

the angle φ_{N-1}^2 is determined by the intersection of the ellipse with the straight line $z = z_0$:

$$\varphi_{N-1}^2 = \arccos \frac{a(z_0 - z_{N-1})}{(a^2 - c^2) - c(z_0 - z_{N-1})}.$$

according to Eq. (2), the parameters a and c have the form

$$2a = 2z - z_1 - \sum_{j=1}^{N-2} \frac{z_{j+1} - z_j}{\cos(\varphi_j)}, \quad 2c = z_{N-1}.$$

1.3 Results of calculation

The results of calculation of $P^{(i)}(z)$ $(i = 2 \dots 4)$ using the Monte Carlo method[4] are shown in Figs. 4 and 5. Profiles of the scattering coefficient are shown in Fig. 4*a*. The calculations were carried out for spaceborne (see Fig. 4, the wavelength is $\lambda = 532$ nm, the height above the Earth is $H = 270$ km, $\theta_0 = 0.6$ mrad, $\varphi_0 = 1.6$ mrad) and airborne (see Fig. 5, the wavelength is $\lambda = 532$ nm, the height above the Earth is $H = 270$ km, $\theta_0 = 0.6$ mrad, $\varphi_0 = 1.6$ mrad) lidars.

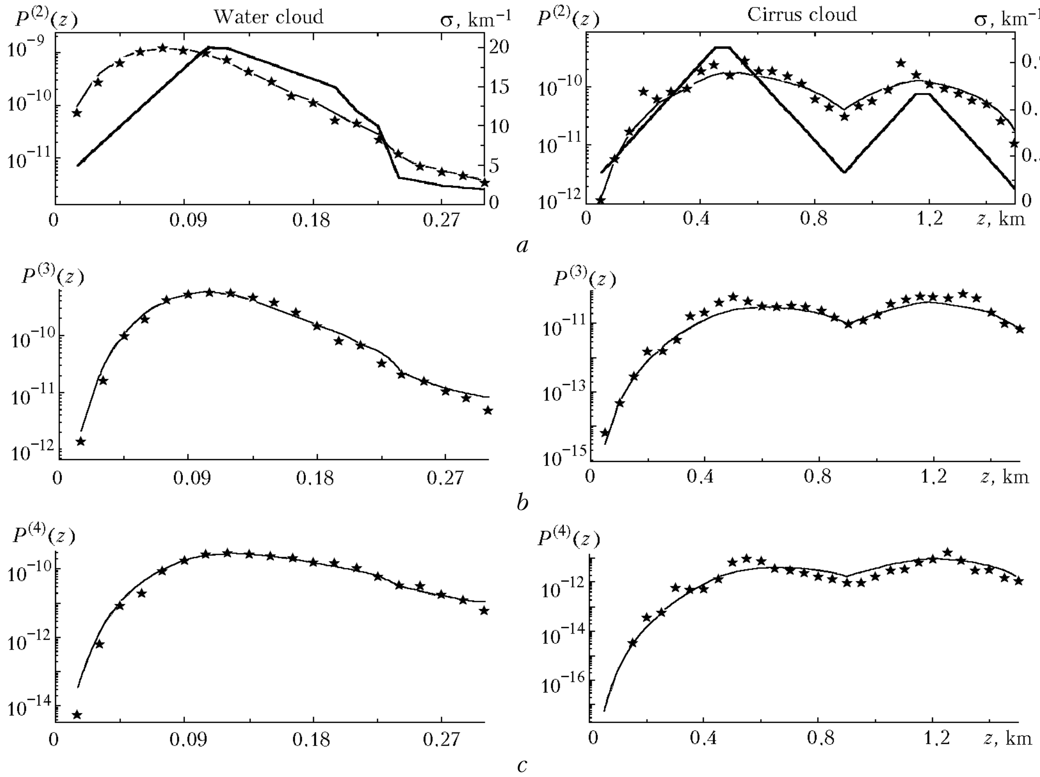

Fig. 4. Comparison of methods for calculation of the lidar return signal from the first four scattering orders for the spaceborne lidar: lidar return signal, second scattering order (a), third scattering order (b), fourth scattering order (c); —— $\sigma(z)$; ι $P(z)$, Monte Carlo method; —— $P(z)$, analytical estimates; $\lambda = 0.53$ μm; $z_0 = 264$ km for a liquid cloud and $z_0 = 260$ km for a crystal cloud; $\varphi_0 = 1.6 \cdot 10^{-3}$ rad.

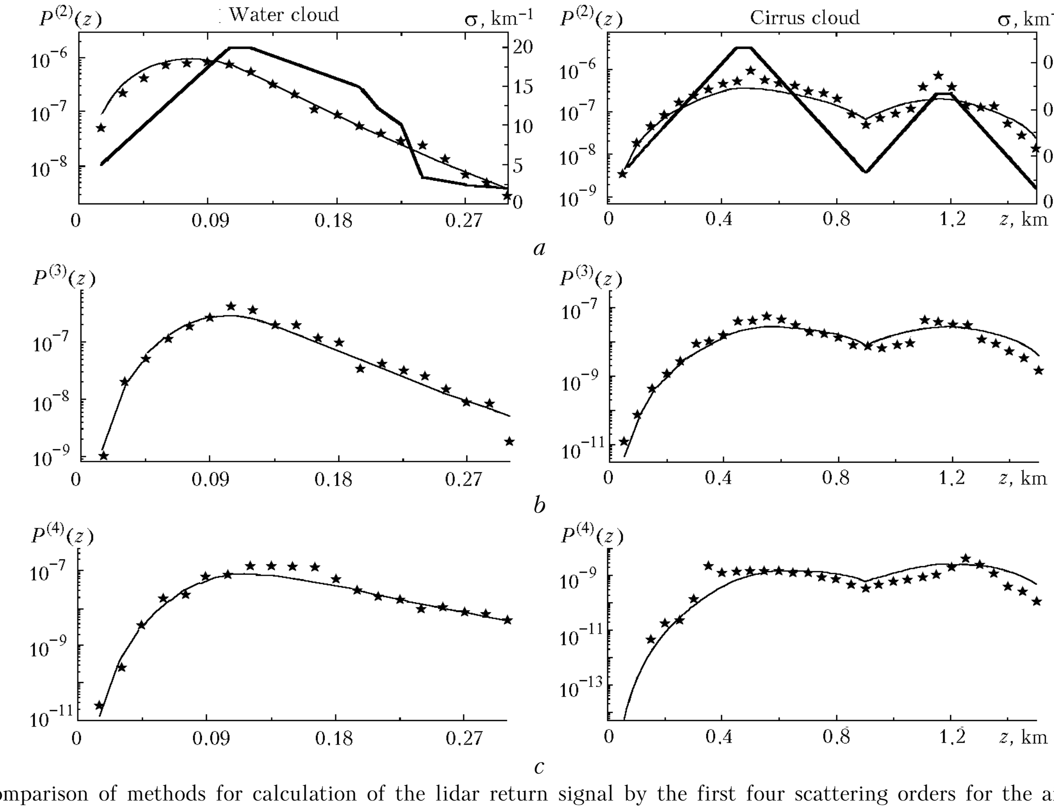

Fig. 5. Comparison of methods for calculation of the lidar return signal by the first four scattering orders for the airborne lidar: lidar return signal, second scattering order (a), third scattering order (b), fourth scattering order (c); ——$\sigma(z)$; ι $P(z)$, Monte Carlo method; —— $P(z)$, analytical estimates; $\lambda = 0.53$ μm; $z_0 = 7.6$ km for a liquid cloud and $z_0 = 3.6$ km for a crystal cloud; $\varphi_0 = 1.1 \cdot 10^{-3}$ rad.

The scattering phase function of the water cloud (left part of the figures) was assigned according to the C_1 model in Deirmendjian's classification[10]; the height of the upper boundary above the Earth was 6 km. When assigning the scattering phase function of cirrus clouds (right part of the figures, the height above the Earth was taken to be 10 km), we used the results of calculations[11] carried out in the geometrical optics approximation for a polydisperse mixture of hexagonal crystals randomly oriented in space. Results of calculation of $P^{(i)}(z)$ (i = 2–4) according to Eqs. (7)–(9) are shown in Fig. 4 and 5, the conditions of the calculations correspond to the ones used above for the Monte Carlo method.

Analysis of the results demonstrates good agreement between our data and the data obtained by the Monte Carlo method for different types of cloudiness and patterns of sensing. This makes it possible to analyze in detail the structure of the lidar return signal with regard to scattering orders and to apply the developed theory to a number of problems of atmospheric optics.

1.4. Sensing of multi-layer cloudiness

When sensing multi-layer cloudiness with large variations in the extinction coefficient, a peculiarity of multiple scattering is manifested which is associated with "smoothing" of the stratification of the medium due to the delay of photons scattered twice or more at large angles. Results of calculation[5] of the signal from a cloud consisting of seven layers with the extinction coefficient of the even layers equal to zero are shown in the upper part of Fig. 6. The total optical thickness of all the layers is equal to 5. Results of our calculation of the lidar return signal formed by the sum of low scattering orders are shown in the lower part of Fig. 6.

Analysis of the results shows that in the region where the scattering coefficient vanishes, the signal does not vanish but instead undergoes a gradual falloff with increasing distance from the scattering layer. With penetration into the cloud, the multiple scattering increases, and, hence, the signal from the non-scattering layers increases, and the signals becomes practically equal in the sixth layer. No existing theory of semianalytical representation of the lidar equation can describe this last peculiarity. Obviously, perspectives should be sought in the analysis of the signal structure using the Monte Carlo method[12] of the theory of multiple scattering. Another approach consists in taking into account the diffuse component of the light field, an equation for which was formulated in Ref. 2.

2. Scattering of arbitrary order taking into account the polarization of the sensing radiation

We write an equation for $P^{(N)}(z)$ in terms of the Stokes vector. We assume that the scattering medium is isotropic, and the scattering plane coincides with the reference plane Q.

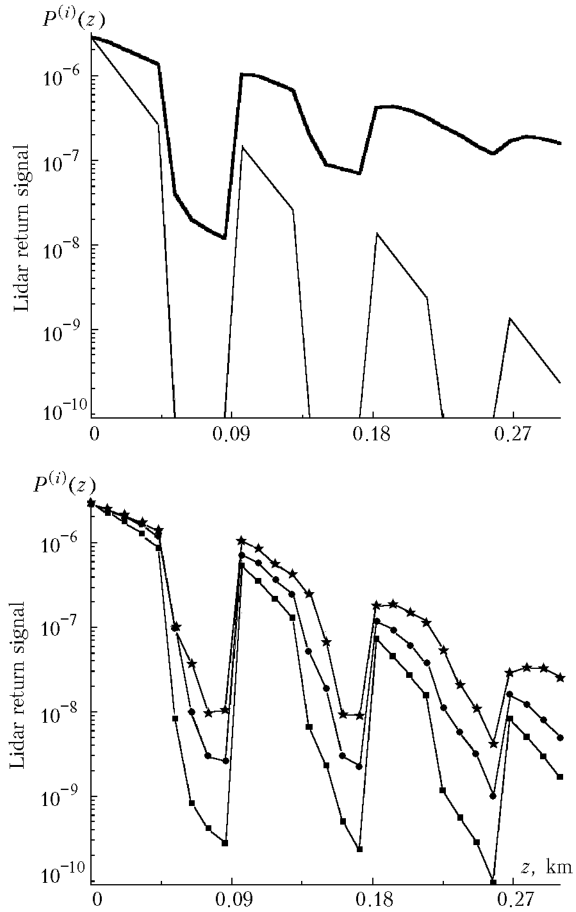

Fig. 6. Comparison of methods of calculating the lidar return signal for a cloud consisting of 7 layers (4 scattering and 3 non-scattering); Σ is the total signal calculated by the Monte Carlo method[5]; (1 + ...) is the signal formed by the sum of the corresponding scattering orders; λ = 0.53 μm; z_0 = 270 km; $\varphi_0 = 5 \cdot 10^{-3}$ rad; ——— Σ; —— 1; —■— 1 + 2; —●— 1 + 2 + 3; —★— 1 + 2 + 3 + 4.

Following the reasoning of Ref. 9 and Part 1.2 of this paper, the Stokes vector of the N-tuply scattered radiation flux arriving at the receiving system has the form

$$\mathbf{S}^{(N)}(z) = (2\pi)^{N-2} P_0\, kA\, \Delta z\, T^2(0, z) \int\limits_{\rho_1} \sigma(z_1)\, d\rho_1 \ldots \times$$

$$\times \int\limits_{\rho_i} \int\limits_{\varphi_{i-1}} \sin(\varphi_{i-1})\, \sigma(z_i)\, d\rho_i\, d\varphi_{i-1} \ldots \times$$

$$\times \int\limits_{\varphi_{N-1}} \frac{\sin(\varphi_{N-1})\, \sigma(z_N)\, d\varphi_{N-1}}{R^{(N)}} \times$$

$$\times \int\limits_0^{2\pi} K(-2\psi)\, A(\varphi'_N) \ldots A(\varphi'_1)\, K(2\psi)\, \mathbf{S}^{(0)}\, d\psi, \quad (10)$$

where ψ is the rotation angle of an arbitrary plane relative to Q; $K(2\psi)$ is the rotation matrix of the reference plane

$$K(2\psi) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\psi) & \sin(2\psi) & 0 \\ 0 & -\sin(2\psi) & \cos(2\psi) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

where $K(-2\psi)$ is the inverse rotation matrix, differing from $K(2\psi)$ only in the sign of the sine elements; $A(\varphi)$ is the normalized scattering phase matrix; $\mathbf{S}(0)$ is the dimensionless Stokes vector of the sensing beam in the coordinate system referenced to the plane Q. It follows from comparison of Eqs. (6) and (10) that the general formula for the Stokes vector of the N-tuply scattered radiation can be obtained from the corresponding formula for $P^{(N)}(z)$ by means of the formal substitution

$$2\pi\gamma(\varphi_1) \ldots \gamma(\varphi_N) \to$$

$$\to \int\limits_0^{2\pi} K(-2\psi)\, A(\varphi'_N) \ldots A(\varphi'_1)\, K(2\psi)\, \mathbf{S}^{(0)}\, d\psi.$$

The integration is performed over the same limits as when calculating $P^{(N)}(z)$.

Equation (10) can be made concrete if we substitute the corresponding scattering phase matrix into it. We assume that the normalized scattering phase matrix has the following form for the problem under consideration:

$$A_{ij} = \begin{pmatrix} a_{11} & a_{12} & 0 & 0 \\ a_{12} & a_{22} & 0 & 0 \\ 0 & 0 & a_{33} & -a_{43} \\ 0 & 0 & a_{43} & a_{44} \end{pmatrix}. \qquad (11)$$

This assumption is valid both for liquid-droplet clouds on the condition that $a_{33} = a_{44}$ and $a_{12} = 0$ (see Ref. 10) and for crystal clouds containing one type of symmetric particles with random orientation in space.[11] The matrix of the polarizer has block-diagonal form for $N > 2$, and is defined by the recursion formula

We assume for definiteness that the source radiation is polarized in the plane coinciding with the reference plane, and we represent the Stokes vector of the sensing beam as

$$\mathbf{S}^{(0)} = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \qquad (13)$$

Substituting Eqs. (12) and (13) into Eq. (10) and integrating over the variable ψ, we obtain the following formulas for the components of the Stokes vector:

$$\mathbf{S}_j^{(N)}(z) = (2\pi)^{N-1} P_0\, kA\, \Delta z\, T^2(0, z) \int\limits_{z_0}^{z} \sigma(z_1)\, dz_1 \ldots \times$$

$$\times \int\limits_{z_0}^{z} \sigma(z_i) \int\limits_0^{\varphi^*_{i-1}} \tan(\varphi_{i-1})\, \gamma(\varphi'_{i-1})\, dz_i\, d\varphi_{i-1} \ldots \times$$

$$\times \int\limits_0^{\varphi^*_{N-1}} \frac{\sin(\varphi_{N-1})\, \sigma(z_N)\, G_j^{(N)}(\varphi_1, \ldots, \varphi_N)}{R^{(N)}} d\varphi_{N-1},\ (j = 1, 2),$$

$$\mathbf{S}_3^{(N)}(z) = \mathbf{S}_4^{(N)}(z) \equiv 0,$$

where

$$G_1^{(N)}(\varphi_1, \ldots, \varphi_N) = a_{11}(\varphi_N)\, a_{11}^{N-1} + a_{12}(\varphi_N)\, a_{21}^{N-1};$$

$$G_2^{(N)}(\varphi_1, \ldots, \varphi_N) = \frac{1}{2}\, [a_{12}(\varphi_N)\, a_{12}^{N-1} + a_{22}(\varphi_N)\, a_{22}^{N-1} +$$

$$+ a_{33}(\varphi_N)\, a_{33}^{N-1} - a_{43}(\varphi_N)\, a_{43}^{N-1}].$$

In analogy with Ref. 9 it is possible to assert that for reflection of a pulse from an atmospheric formation with scattering phase matrix of the form (11) multiple scattering can be represented as a mixture of linearly polarized and unpolarized radiation, and the plane of predominant polarization coincides with the plane of polarization of the sensing beam.

$$A(\varphi_N) \ldots A(\varphi_1) = \begin{pmatrix} a_{11}^N & a_{12}^N & 0 & 0 \\ a_{21}^N & a_{22}^N & 0 & 0 \\ 0 & 0 & a_{33}^N & a_{34}^N \\ 0 & 0 & a_{43}^N & a_{44}^N \end{pmatrix} =$$

$$= \begin{bmatrix} a_{11}(\varphi_N)\, a_{11}^{N-1} + a_{12}(\varphi_N)\, a_{21}^{N-1} & a_{11}(\varphi_N)\, a_{12}^{N-1} + a_{12}(\varphi_N)\, a_{22}^{N-1} & 0 & 0 \\ a_{12}(\varphi_N)\, a_{11}^{N-1} + a_{22}(\varphi_N)\, a_{21}^{N-1} & a_{12}(\varphi_N)\, a_{12}^{N-1} + a_{22}(\varphi_N)\, a_{22}^{N-1} & 0 & 0 \\ 0 & 0 & a_{33}(\varphi_N)\, a_{33}^{N-1} - a_{43}(\varphi_N)\, a_{34}^{N-1} & a_{33}(\varphi_N)\, a_{34}^{N-1} - a_{43}(\varphi_N)\, a_{44}^{N-1} \\ 0 & 0 & a_{43}(\varphi_N)\, a_{33}^{N-1} + a_{44}(\varphi_N)\, a_{43}^{N-1} & a_{43}(\varphi_N)\, a_{34}^{N-1} + a_{44}(\varphi_N)\, a_{44}^{N-1} \end{bmatrix}. \qquad (12)$$

The results of our calculation of the degree of depolarization of the lidar return are shown in Fig. 7. The calculations were carried out for the scheme in which the locator is situated 200 km from the cloud C_1. It was assumed that the source emits isotropically at the wavelength $\lambda = 0.69$ μm and the return signal was detected by a receiver with field-of-view angle $\varphi_0 = 0.28$ mrad. In addition, the results of Monte Carlo calculations[13] in the cloud and our calculations of the signal formed by the sum of lower scattering orders in the cloud and beyond it are presented.

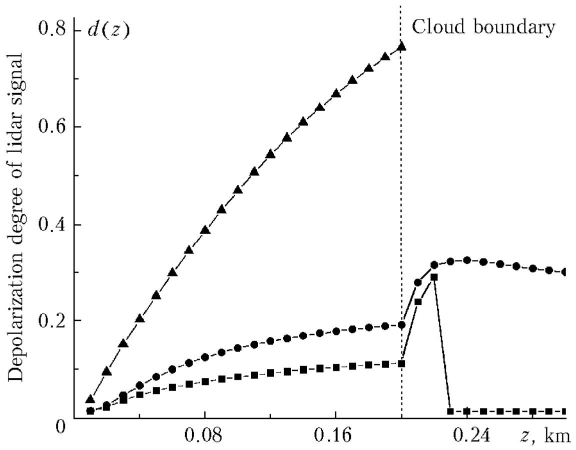

Fig. 7. Comparison of methods for calculating the depolarization of a lidar return signal: Σ is the depolarization of the total signal[13] (the calculations were not carried out beyond the cloud boundary); (1 + ...) is the depolarization of the signal formed by the sum of the corresponding scattering orders; $\lambda = 0.69$ μm; $z_0 = 200$ km; $\sigma = 25$ km^{-1}; $\varphi_0 = 2.8 \cdot 10^{-5}$ rad; —■— 1 + 2; —●— 1 + 2 + 3; — — Σ.

Analysis of the results shown in Fig. 7 shows that maximum values of the degree of depolarization can be observed not in the cloud, but beyond its boundary. One can explain this phenomenon by a delay of the cross-polarized component of the lidar return relative to the parallel one. This phenomenon must be taken it into account when interpreting polarization measurement data.

Conclusion

The calculations performed here are a continuation of the analytical theory of double scattering expounded in Refs. 8 and 9. The results of this paper represent a modification of the laser sensing equation (formulas (6) and (12)) for signals formed by arbitrary scattering orders. The adequacy of the equation is confirmed by comparison with Monte Carlo calculations for cases of sensing different types of cloudiness.

Acknowledgments

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