Intensity of a signal due to reflection from a phase-conjugating mirror in the absorbing turbulent atmosphere

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The influence of fluctuations in the real and imaginary components of the dielectric constant of a random dissipative medium on the intensity of a signal reflected from a phase-conjugating mirror is studied. Manifestation of the effect of backscattering intensification with respect to the mean radiation intensity is analyzed. It is shown that in the considered case the effect of backscattering intensification is caused only by large-scale fluctuations of the imaginary component of the dielectric constant.

In recent years, quite a large number of papers have been published (see, for example, Refs. 1 and 2 and references therein) on the study of transformation of the parameters of radiation passed through the atmosphere and reflected from mirrors having different properties. These papers mostly consider detection and ranging problems assuming transparent random media. However, in some papers (Refs. 3–5) the problem of propagation of electromagnetic waves (including optical ones) through randomly inhomogeneous media characterized by fluctuations of not only the real component of the dielectric constant ε, but of the imaginary one also, as well as correlations of these components, is formulated and studied. Pulsations of the imaginary component of ε in the medium cause some non-trivial peculiarities both in the method of solution of lidar problems and in the behavior of characteristics of the reflected wave as compared to the case of radiation propagation through transparent random media.

In Refs. 3–5, the schemes that include an ordinary mirror as a reflecting object were considered. This mirror modeled passive scatterers of natural or artificial origin. However, phase-conjugating mirrors are widely used in adaptive optics for controlling distortions introduced by a random medium in a signal (in particular, as optical systems operate through the turbulent atmosphere). The application of phase-conjugation mirrors under some conditions allows complete compensation for the medium-induced fluctuations of the phase and, consequently, pulsations of the reflected wave amplitude caused by them.

The efficiency of using phase-conjugating mirrors to decrease the influence of random distortions by a transparent medium of radiation propagation was checked not only in laboratories, but in the field experiments as well. In this connection the question arises on how does a lidar system with a phase-conjugating mirror operate in a random absorbing medium under conditions that the medium dielectric constant has a random component not only in the real part, but in the imaginary one as well? How do the statistical characteristics of the reflected signal change? In this paper, we consider one aspect of this problem, namely, joint effect of fluctuations in the real and imaginary components of the dielectric constant of a random dissipative medium on the intensity of a wave reflected from the phase-conjugating mirror and analysis of the peculiarities in manifestation of the effect of backscattering intensification.

Let an optical wave pass through a random dissipative medium along an open path arranged as follows: a radiation source with a given amplitude-phase distribution be placed at the plane $z = 0$; the wave from the source passes along the positive direction of the z axis; a reflecting object – phase-conjugating mirror with a given characteristics – is at the plane $z = L$; a receiver of the reflected signal is at the same plane as the source ($z = 0$).

Since the scales of random inhomogeneities of the dielectric constant in the turbulent atmosphere far exceed the wavelength of optical radiation, we can use, for describing the propagation of the optical radiation in such a medium, the parabolic equation of quasi-optics for the complex amplitude of both the forward propagated wave

$$2ik\frac{\partial}{\partial z}U_+ + \Delta_\perp U_+ + k^2\Delta\varepsilon U_+(\rho, z) = 0 \qquad (1)$$

under boundary condition

$$U_+(\rho, z)\big|_{z=0} = U_0(\rho),$$

and the reflected wave

$$-2ik\frac{\partial}{\partial z}U_- + \Delta_\perp U_- + k^2\Delta\varepsilon U_-(\rho, z) = 0 \qquad (2)$$

under the boundary condition

$$U_-(\rho, z)\big|_{z=L} = U_+^*(\rho, L)f(\mathbf{r}),$$

where $U_\pm(\rho, z)$ is the complex amplitude of the forward (subscript +) and reflected (subscript −) waves; $\rho = \{x, y\}$ is the cross radius-vector;

$k = 2\pi\sqrt{\varepsilon_{0R}}/\lambda$ is the wave number; λ is the wavelength; $\Delta_\perp$ is the Laplace operator for the variables x and y; $\Delta\varepsilon = (\varepsilon - \varepsilon_{0R})/\varepsilon_{0R}$ is the relative change of the medium dielectric constant; ε_{0R} is the mean value of the real part of ε of the unperturbed medium (for optical radiation in the atmosphere $\varepsilon_{0R} \approx 1$); $f(\rho)$ is the complex reflection coefficient of the mirror; the asterisk means complex conjugation.

In the random dissipative medium, the distribution of $\Delta\varepsilon$ has the form

$$\Delta\varepsilon = \frac{\bar{\varepsilon}_R - \varepsilon_{0R}}{\varepsilon_{0R}} + i\,\frac{\bar{\varepsilon}_I}{\varepsilon_{0R}} + \frac{\tilde{\varepsilon}_R(\mathbf{r}, t)}{\varepsilon_{0R}} + i\,\frac{\tilde{\varepsilon}_I(\mathbf{r}, t)}{\varepsilon_{0R}},$$

where $\bar{\varepsilon}$ and $\tilde{\varepsilon}$ are the mean and pulsation (with given statistical characteristics) components of the complex dielectric constant of the medium; $\mathbf{r} = \{\rho, z\}$ is the radius-vector; t is time; the subscripts R and I of $\bar{\varepsilon}$ and $\tilde{\varepsilon}$ mean the real (R) and imaginary (I) components of ε.

In some cases, especially, in the case of a nonlinear propagation of a laser beam, the mean value of ε depends on spatial coordinates. The scales of inhomogeneity of the mean values of the medium parameters can be comparable both with the cross dimensions of the beam and with the scales of random inhomogeneities.

For solution of lidar problems, it is very useful to present U_+ and U_- meeting Eqs. (1) and (2) in the integral form using the Huygens–Kirchhoff principle:

$$U_+(\rho, L) = \int d^2\rho' U_0(\rho') G_+(\rho, L; \rho', 0), \qquad (3)$$

$$U_-(\rho, 0) = \iint d^2\rho' d^2\rho'' f(\rho'') U_0(\rho') \times$$
$$\times\, G_-(\rho, 0; \rho'', L)\, G_+^*(\rho'', L; \rho', 0), \qquad (4)$$

where $G_\pm$ is the Green's function of Eqs. (1) and (2) for the forward (+) and reflected (−) waves.

At such a presentation of the wave field, we can form various combinations of the complex amplitudes of the reflected signal and, after averaging over the ensemble of realizations, write any moment of the field.

Since this paper is devoted to studying the behavior of the mean intensity of the wave reflected from a phase-conjugating mirror, we present here only the equation for the second moment of the field U_-:

$$\langle I_-(\rho, 0)\rangle = \frac{c}{8\pi}\langle U_-(\rho, 0) U_-^*(\rho, 0)\rangle =$$
$$= \frac{c}{8\pi}\iiiint d^2\rho' d^2\rho'' d^2t' d^2t'' \Gamma_{20}(\mathbf{t}', \rho') \times$$
$$\times\, f(\rho'') f^*(t'') \langle G_4(\rho'', L; \mathbf{t}'', L \,|\, \rho, 0; \rho', 0; \mathbf{t}', 0)\rangle, \qquad (5)$$

where $\Gamma_{20}(\mathbf{t}', \rho') = U_0(\rho') U_0^*(\mathbf{t}')$ is the coherence function of the source;

$$\langle G_4(\rho'', L; \mathbf{t}'', L \,|\, \rho, 0; \rho', 0; \mathbf{t}', 0)\rangle =$$
$$= \langle G_+(\rho'', L; \rho, 0) G_+^*(\rho'', L; \rho', 0) \times$$
$$\times\, G_+^*(\mathbf{t}'', L; \rho, 0) G_+(\mathbf{t}'', L; t', 0)\rangle,$$

and the angular brackets denote averaging over the ensemble of realizations of the random field of ε.

Note that the equation for G_4 is written with the allowance made for the relation between the Green's functions of the forward and backward going waves that follows from the reciprocity principle transformed for the case of absorbing media (see Ref. 5).

To calculate the mean intensity of the reflected wave in the random absorbing medium, we present the Green's function in the form of the Feynman integral over trajectories

$$G_+(\mathbf{r}, \mathbf{r}') = \int_{\rho'}^{\rho} D^2\rho(\xi) \exp\left\{\frac{ik}{2}\int_{z'}^{z} d\xi \left[\dot{\rho}^2(\xi) + \Delta\varepsilon(\rho(\xi), \xi)\right]\right\}, \qquad (6)$$

where the operator $\int_{\rho'}^{\rho} D^2\rho(\xi)$ means continuous integration, i.e., integration over all trajectories $\rho(\xi)$ that have their beginning at the point $\rho(z') = \rho'$ and end at the point $\rho(z) = \rho$; $\dot{\rho}(\xi) = \frac{d}{d\xi}\rho(\xi)$.

Let us make a specific calculation for the case of propagation of a plane wave through a random dissipative medium along a path with the reflection from a plane unlimited phase-conjugating mirror.

Assuming the statistical characteristics of the medium to be known and using Eq. (6), for the function G_4 we have the following equation:

$$\langle G_4(\rho'', L; \mathbf{t}'', L \,|\, \rho, 0; \rho', 0; \mathbf{t}', 0)\rangle =$$
$$= \langle G_0(\rho'', L; \rho, 0) G_0^*(\rho'', L; \rho', 0) G_0^*(\mathbf{t}'', L; \rho, 0) \times$$
$$\times\, G_0(\mathbf{t}'', L; \mathbf{t}', 0)\rangle \exp\{2k^2 A_{II}(0) L\} \left\{1 - \frac{\pi k^2}{2}\int_0^L d\xi \iint d^2q \times\right.$$
$$\times (\Phi_{RR}(\mathbf{q}) + \Phi_{II}(\mathbf{q}))(2 - e^{i\mathbf{q}\rho_{12}(\xi)} - e^{i\mathbf{q}\rho_{14}(\xi)} -$$
$$- e^{i\mathbf{q}\rho_{23}(\xi)} - e^{i\mathbf{q}\rho_{43}(\xi)} + \exp\{i\mathbf{q}[\rho_{13}(\xi) - \frac{\mathbf{q}}{k}S(\xi)]\} +$$
$$+ \exp\{i\mathbf{q}[\rho_{24}(\xi) + \frac{\mathbf{q}}{k}S(\xi)]\}) - \pi k^2 \int_0^L d\xi \iint d^2q\, \Phi_{II}(\mathbf{q}) \times$$
$$\times (2 - \exp\{i\mathbf{q}[\rho_{13}(\xi) - \frac{\mathbf{q}}{k}S(\xi)]\} - \exp\{i\mathbf{q}[\rho_{24}(\xi) +$$
$$+ \frac{\mathbf{q}}{k}S(\xi)]\}) - i\pi k^2 \int_0^L d\xi \iint d^2q\, \Phi_{RI}(\mathbf{q}) \Bigg(\exp\{i\mathbf{q}[\rho_{13}(\xi) -$$
$$\left. - \frac{\mathbf{q}}{k}S(\xi)]\} - \exp\{i\mathbf{q}[\rho_{24}(\xi) + \frac{\mathbf{q}}{k}S(\xi)]\}\Bigg)\right\}, \qquad (7)$$

where $\Phi_{RR}(\mathbf{q})$, $\Phi_{II}(\mathbf{q})$, and $\Phi_{RI}(\mathbf{q})$ are the spectra of fluctuations of the real and imaginary components of ε and their correlations; the parameter $A_{II}(0)$ is described, based on the assumption of δ-correlated fluctuations of ε, by the equation $\langle\tilde{\varepsilon}_I(\xi_1, \rho_1)\tilde{\varepsilon}_I(\xi_2, \rho_2)\rangle = \delta(\xi_1 - \xi_2)A_{II}(\rho_1 - \rho_2)$, and the Green's function $G_0(\mathbf{r}; \mathbf{r}')$ of the homogeneous medium ($\tilde{\varepsilon} = 0$), and the functions $S(\xi)$ and $\rho_{ij}(\xi)$ are determined by the following equations:

$$G_0(\mathbf{r}; \mathbf{r}') = G(\mathbf{r}; \mathbf{r}')_{\tilde{\varepsilon}=0} = \frac{k}{2\pi i(z - z')} \times$$

$$\times \exp\left\{\frac{ik|\rho - \rho'|^2}{2(z - z')} - \frac{k}{2}\,\bar{\varepsilon}_I(z - z')\right\}; \qquad (8)$$

$$S(\xi) = \xi(L - \xi)/L;\ \rho_{12}(\xi) = (\rho - \rho')\frac{(L - \xi)}{L},$$

$$\rho_{13}(\xi) = \frac{(\rho'' - t'')\xi + (\rho - t')(L - \xi)}{L},$$

$$\rho_{14}(\xi) = (\rho'' - t'')\frac{\xi}{L};$$

$$\rho_{23}(\xi) = \frac{(\rho'' - t'')\xi + (\rho' - t')(L - \xi)}{L},$$

$$\rho_{43}(\xi) = (\rho - t')\frac{(L - \xi)}{L};$$

$$\rho_{24}(\xi) = \frac{(\rho'' - t'')\xi + (\rho' - \rho)(L - \xi)}{L}.$$

To find the role of absorption fluctuations in the formation of the wave reflected from the phase-conjugating mirror, let us calculate the coefficient of backscattering intensification with respect to the mean intensity. This coefficient is determined from the following equation:

$$\bar{N}_{WFC}(L) = \langle I_-(\rho, 0)\rangle / \langle I_+(\rho, 2L)\rangle,$$

where $\langle I_-(\rho, 0)\rangle$ is the mean intensity of the reflected signal having passed the path from the source ($z = 0$) to the mirror ($z = L$) and, after reflection, from the mirror ($z = L$) to the receiver ($z = 0$), $\langle I_+(\rho, 2L)\rangle$ is the mean intensity of radiation having passed the direct path of the length $2L$ from the same source ($z = 0$) to the receiver placed at the plane $z = 2L$.

We believe that fluctuations of the complex dielectric constant of the medium are described by the Kolmogorov–Obukhov spectrum corrected in the region of large and small scales:

$$\Phi_{\alpha\alpha'}(\mathbf{q}) = 0.033C^2_{\alpha\alpha'}(\kappa_0^2 + q^2)^{-11/6}\exp\left(-\frac{q^2}{\kappa_m^2}\right), \qquad (9)$$

where $C^2_{\alpha\alpha'} = C^2_{\alpha'\alpha}$ is the structure constant of fluctuations of the real ($\alpha = \alpha' = R$) and imaginary ($\alpha = \alpha' = I$) components of ε, as well as their correlations ($\alpha = R$, $\alpha' = I$); $\kappa_0 = 2\pi/L_0$, L_0 is the outer scale of turbulence; $\kappa_m = 5.92/l_0$, l_0 is the inner scale of turbulence.

Then, using Eqs. (5)–(8) for the coefficient of backscattering intensification we obtain

$$\bar{N}_{WFC}(L) = \exp\{k^2 A_{II}(0)L\}. \qquad (10)$$

As is seen from Eq. (10), as the electromagnetic wave travels along the path with the phase-conjugating mirror, the mean intensity of the reflected wave increases. This intensification occurs exceptionally due to the contribution of large-scale fluctuations of the imaginary component of ε because of wave attenuation by turbulent vortices having the characteristic size of the same order of magnitude as the outer scale of turbulence. The role of the phase-conjugating mirror in the considered case is in the suppression of small-scale fluctuations of the wave phase due to diffraction on both transparent and absorbing inhomogeneities of the medium dielectric constant.

To emphasize the role of the phase-conjugating mirror at radiation transfer through a turbulent medium, let us compare Eq. (10) with the equation for the coefficient of backscattering intensification in the case of wave propagation along the path with the reflection from an ordinary mirror. In the latter case (see Ref. 5) we have for $\bar{N}$

$$\bar{N}_{pl}(L) = \exp\{k^2 A_{II}(0)L - 2\sigma^2_{II}(L) - 7.4\sigma^2_{RI}(L)\}, \qquad (11)$$

where σ^2_{II} and σ^2_{RI} are the variances of fluctuations of the level of intensity due to pulsations of the imaginary component of ε and correlation of ε_R and ε_I.

From the comparison of Eqs. (10) and (11) we can see that $\bar{N}_{WFC} \geq \bar{N}_{pl}$, and if in the case of the phase-conjugating mirror in the absorbing random medium $\bar{N}_{WFC}$ is always greater than unity (effect of backscattering intensification), then in the case of the ordinary mirror the coefficient $\bar{N}$ can be less than unity, i.e., fluctuations of the imaginary component of ε have a double effect on $\bar{N}_{pl}$. On the one hand, the presence of ε_I pulsations on the path causes a decrease in $\bar{N}_{pl}$ due to the influence of small-scale inhomogeneities on the wave (what is determined by σ^2_{II} and σ^2_{RI}). On the other hand, the influence of large-scale inhomogeneities of ε_I, that manifests itself via the parameter A_{II} in the exponent, causes an increase in the $\bar{N}_{pl}$. Such a different behavior of $\bar{N}_{WFC}$ and $\bar{N}_{pl}$ is connected with the fact that the contribution of waves diffracted on turbulent vortices, whose sizes are within the inertial interval ($l_0 \leq r \leq L_0$), is significant on the path with the ordinary mirror, whereas in the case of the phase-conjugating mirror it is compensated for and does not affect fluctuations of the amplitude of the reflected wave.

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