

# Character of light intensity distribution over the width of a slit illuminated by a plane monochromatic wave at image formation by a bounded light beam. Part 2

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Received June 29, 1999

I obtain expressions that enable one to determine the position of fringes in the diffraction pattern within the image of a slit illuminated a bounded parallel beam of a monochromatic light. The expressions well agree with the experiments and thus confirm explanations of the fringes' origin.

The first part of this paper<sup>1</sup> presents experimental facts on the appearance of a maxima and minima of intensity  $J$  in the image of a slit  $Sl_1$  illuminated by a plane light wave ( $\lambda = 0.53 \mu\text{m}$ ) in the case when the slit width  $S$  is increased and the light beam which forms the image is bounded by a slit of variable width ( $Sl_0$ ).

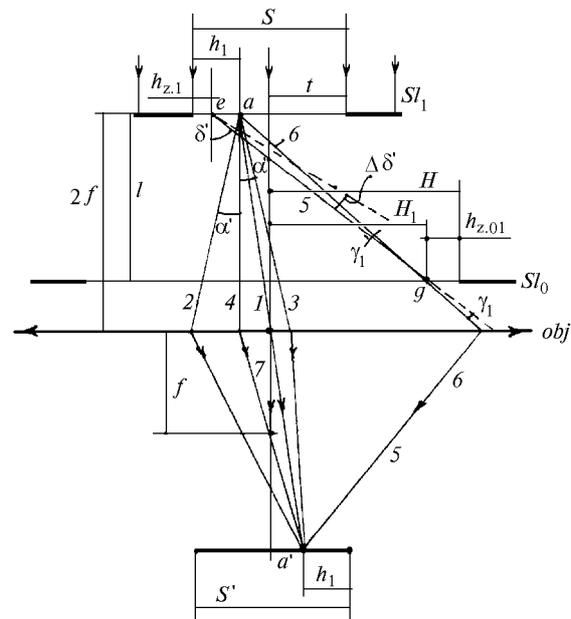
It was noted that experimentally observed variation of  $J$  across the  $S'$  segment of the  $Sl_1$  slit's image is a consequence of several factors: existence of zones over the screens' surface (deflection zones where light beams are deflected along the directions from the initial direction, both toward the screen and from the screen); increase of the light deflection with a decrease of the distance between their initial trajectories and the screen; appearance of the initial phase difference between the deflected (boundary) and incident light at the beam deflection; interference of beams deflected in the zones of  $Sl_1$  and  $Sl_0$ , with those deflected in the weak part of the zones of  $Sl_1$  or propagated without deflection.

Let us show that this is the real case.

In the experimental optical arrangement presented in Fig. 1, *obj.* is a Yupiter-8 lens objective forming the image of the slit  $Sl_1$  without magnification ( $S = S'$ );  $H$  is the half-width of the bounding slit  $Sl_0$  placed at a distance  $l$  from  $Sl_1$ ;  $t$  is the half-width of  $Sl_1$ ;  $H \gg t$ .

At each point  $a'$  of the image of  $Sl_1$ , at the distance  $h_1$  from the image's right-hand edge, the rays 1-6 converge. The beam 1 is the imaginary one; it is used to identify the conjugate point  $a$  in the plane of  $Sl_1$ ; 2 and 3 are the rays of light incident onto  $Sl_1$  and deflected by angles  $\alpha'$  from the initial direction in the deflection zones of its left-hand and right-hand screens at a point that is at the distance  $h_1$  from the left-hand screen; 5 is the ray deflected by an angle  $\delta'$  mainly in the zone of the left-hand screen of  $Sl_1$  from the distance  $h_{z,1}$  and by an angle  $\gamma_1$  mainly in the zone of the right-hand screen of  $Sl_0$  from the distance  $h_{z,01}$ ; 6 is the imaginary ray propagated from the point  $a$  toward the conjugate point  $a'$  without a deflection in the plane of

$Sl_0$ ; 4 is the ray of incident light beam passing through the point  $a$  without a deflection in the planes of  $Sl_1$  and  $Sl_0$ .



**Fig. 1.** Formation of the first diffraction pattern within the limits of the slit image formed by a bounded light beam.

Existence of the ray 4 was first supposed in the hypothesis<sup>2</sup> proposed to explain the experimental results. According to the hypothesis, light quanta propagated along the ray trajectories are in three states with respect to the direction of their deflection in the deflection zone. In the first state, they deflect onto the screen, in the second state from the screen; in the third state, they are propagated through the zone without deflection.

Propagation of a part of light rays through the zone without changes in direction seems to be confirmed in Ref. 3. In this paper, it was established that each flux of light rays coming from an arbitrary

part of the deflection zone and deflected toward the screen and out from it is equal to 0.143 of the light flux incident onto that part of the zone. Therefore, even in case of coincidence of the flows' phases before splitting into individual components, their sum  $(\sqrt{0.143} + \sqrt{0.143})^2 = 0.57$  appears to be less than the incident flux.

Based on findings from Ref. 4, the deflection angles  $\delta'$  and  $\alpha'$  of the rays 5, 2, and 3 are determined by the expression

$$\delta'(\alpha') = 259.5/[h_{z,1}(h_1) + 0.786], \quad (1)$$

where the values of the angles and  $h_{z,1}(h_1)$  are represented in minutes of arc and  $\mu\text{m}$ , respectively.

As is shown below, the rays 2-4 that take part in forming max and min of intensity within  $S'$  come from the points  $a$ , situated at  $h_1 \gg h_{z,1}$ , so  $\alpha' \ll \delta'$ . As a consequence, the rays 2 and 3 propagate at a long distance from the edges of  $Sl_0$ , in weak parts of its deflection zones which in fact do not make any obstacles for the rays' arrival at the conjugate points  $a'$ .

Because of the tautochronism, no phase difference occurs between the beams 2, 3, 4, and 6, but it appears between them and the ray 5, which propagates from the point  $e$  and comes toward the point  $a'$  due to its deflection in the right-hand screen's zone of the slit  $Sl_0$ .

The ray 5 and the imaginary ray 6 propagated along the same path from the plane of  $Sl_0$  toward the point  $a'$ . Therefore, the geometrical phase between the rays 5 and 6 (and between the rays 2-4) is

$$\Delta_{5,6} = (eg - ag); \quad eg = (l + \Delta_1); \quad ag = (l + \Delta_2),$$

so

$$\Delta_{5,6} = (\Delta_1 - \Delta_2).$$

The value

$$H_1 = (H - h_{z,01});$$

$$\Delta_1 = (H_1 + t - h_{z,1})^2/2l,$$

$$\Delta_2 = (H_1 + t - h_1)^2/2l,$$

$$\Delta_{5,6} = 2(H_1 + t - h_{z,1}) \cdot (h_1 - h_{z,1}) - (h_1 - h_{z,1})^2 = k'\lambda/2;$$

$$h_1 = (H_1 + t) - \sqrt{(H_1 + t - h_{z,1})^2 - k'\lambda l}.$$

According to Refs. 3 and 5, light rays get equal advance and delay with respect to the incident light at their deflection out from the screen and towards it, respectively. In the experiments, the advance and delay are mainly within the limits of  $\lambda/2$ .

It is still unclear whether this salient feature manifests itself only at the first deflection or takes place also at other deflections.

The ray 5 is deflected out from the screen in the deflection zones of the left-hand ( $Sl_1$ ) and right-hand ( $Sl_0$ ) screens. Therefore, during the deflection process, it undergoes an advance by  $k_{01}\lambda/2$  which is equal at least to the advance received in the zone of the left-hand screen  $Sl_1$ .

It is evident that the first max of intensity  $J$  is formed at the point  $a'$  where the initial advance of the ray 5 (i.e., advance acquired in the zone) appears to be equal to its geometrically explained phase lag from the beams 2-4 and 6.

Taking this into account, the formula for  $h_1$  takes the form

$$h_1 = (H_1 + t) - \sqrt{(H_1 + t - h_{z,1})^2 - (k_{01} + k)\lambda l}, \quad (2)$$

where  $k = 0, 2, 4, \dots$  and  $k = 1, 3, 5, \dots$  are the numbers of  $J$  maxima and minima, respectively.

This formula is valid for calculated points  $a(a')$  in both of the  $S(S')$  halves.

If one restricts himself to studying variation of  $J$  at the center of  $S'$ , i.e., when  $h_1 = t$ , the formula (2) reduces to the following form:

$$t = \sqrt{H_1^2 + (k_{01} + k)\lambda l} - (H_1 - h_{z,1}), \quad (3)$$

where  $k = 0, 2, 4, \dots$  correspond to max at the center of  $S'$ ;  $k = 1, 3, 5, \dots$  correspond to min;  $t$  is the value of the half-width of  $Sl_1$  at the moments of max and min at the center of  $S'$ .

For  $l = 72 \text{ mm}$ ,  $\lambda = 0.53 \mu\text{m}$ ,  $H > 1 \text{ mm}$ , the fringes of different orders  $k$  on  $S'$  and in the center of it, on the scale of dimensions of  $S$ , have in fact similar width.

According to Eqs. (2) and (3), fringe width depends mainly on  $H$ ,  $l$ , and  $\lambda$  because  $H \gg t$ . With the increase of  $H$  and decrease of  $l$ , the fringes become narrower. Their number increases with the increasing  $t$  and  $H$ .

The diffraction pattern formed by the rays 5 jointly with the rays 2-4 (the first diffraction pattern) begins from the right-hand side edge of  $S'$ , i.e., the order of fringes in it increases as they approach the left-hand side edge of  $S'$ . Since the rays 5 in the plane of the slit  $Sl_0$  are deflected near its right-hand screen, the pattern still takes place when the left-hand side screen of  $Sl_0$  is removed.

From Eq. (2) it follows that

$$k_{01} = \{[2(H_1 + t - h_{z,1}) \cdot (h_1 - h_{z,1}) - (h_1 - h_{z,1})^2]/\lambda l - k\}. \quad (4)$$

In this formula,  $H_1$  and  $h_{z,1}$  are unknown values.

As it was established in Ref. 4, if light rays are deflected in the deflection zone of a screen by an angle  $\varepsilon(\delta' + \Delta\delta')$ ,

$$h_z = (259.5 - 0.786\varepsilon)/\varepsilon, \quad (5)$$

where  $h_z$  and  $\varepsilon$  are expressed in  $\mu\text{m}$  and minutes of arc, respectively.

Under conditions considered, the ray 5 propagate simultaneously through the overlapping deflection zones of the left-hand and right-hand screens of  $Sl_1$ , and is deflected out from those.

If we suppose that both of these zones effect the ray independently, we have, in accordance with Eq. (1), that

$$(\delta' + \Delta\delta') = 259.5/(h_{z,1} + 0.786);$$

$$\Delta\delta' = 259.5 / (S - h_{z,1} + 0.786).$$

In this case, the deflection angle of the ray 5 is

$$\delta' = \frac{259.5}{h_{z,1} + 0.786} - \frac{259.5}{S - h_{z,1} + 0.786}. \quad (6)$$

At the same time,

$$\delta' = 3438(H_1 + 0.5 \cdot 10^{-3} S - 10^{-3} h_{z,1}) / l, \quad (7)$$

where  $H_1$  and  $l$  are in mm;  $S$  and  $h_{z,1}$  in  $\mu\text{m}$ . As a result of joint transformation of these expressions, we obtain

$$h_{z,1} = A_1 - \sqrt{A_1^2 - B_1}, \quad (8)$$

where

$$A_1 = \frac{S(H_1 + 0.5 \cdot 10^{-3} S) + 0.1508 l}{2(H_1 + 1.5 \cdot 10^{-3} S)};$$

$$B_1 = \frac{S(0.0754 l - 0.786 H_1 - 0.393 \cdot 10^{-3} S) - 0.618 H_1}{H_1 + 1.5 \cdot 10^{-3} S};$$

$S$  and  $h_{z,1}$  are in  $\mu\text{m}$ ;  $l$  and  $H_1$  in mm.

Passing through the  $Sl_0$ , in the zones of the right-hand and left-hand side screens of the slit the ray 5 is deflected from them by the angles  $\gamma_1 + \Delta\gamma_1$  and  $\Delta\gamma_1$ :

$$(\gamma_1 + \Delta\gamma_1) = 259.5 / (h_{z,01} + 0.786);$$

$$\Delta\gamma_1 = 259.5 / (2H - h_{z,01} + 0.786).$$

Therefore, the angle of the resulting deflection of a ray within the limits of  $Sl_0$  is

$$\gamma_1 = \frac{259.5}{h_{z,01} + 0.786} - \frac{259.5}{2H - h_{z,01} + 0.786} =$$

$$= \frac{3.438(h_1 - h_{z,1})}{l},$$

where  $H$ ,  $h_{z,01}$ ,  $h_1$ , and  $h_{z,1}$  are in  $\mu\text{m}$ ;  $l$  in mm. It follows that

$$h_{z,01} = \left( \frac{75.48l}{h_1 - h_{z,1}} + H \right) - \sqrt{\left( \frac{75.48l}{h_1 - h_{z,1}} + H \right)^2 - \left( \frac{75.48l}{h_1 - h_{z,1}} - 0.786 \right) 2H - 0.62}. \quad (9)$$

To determine  $k_{01}$  for different  $H$  and  $l$  from the experimental values of  $h_1 - h_e$ , one can apply the following technique:

1. Find the underestimated value of  $h_{z,01}$  by Eq. (9) ignoring  $h_{z,1}$ .
2. Determine  $H_1 = (H - h_{z,01})$ .
3. Determine  $h_{z,1}$  by Eq. (8).
4. Repeat the calculations 1–3 with the account of obtained  $h_{z,1}$  until  $H_1$  is almost constant.
5. Find  $k_{01}$  by Eq. (4).

The results of calculations of  $k_{01}$  for the bands of different orders are presented in Table 1, where  $\alpha'$  is determined by substitution of  $h_e$  for  $h_{z,1}$  in Eq. (6).

As one can see from these results, the values ( $k_{01} - 0.5$ ) are small as compared with  $(0.5 + k)$ . Therefore, Eq. (2) describes the position of the diffraction bands in  $S'$  quite reliably.

The presented values of  $k_{01}$  are within the limits of the values of  $k_0$  observed in the experiments<sup>3,5</sup> on light diffraction on a screen with a rectangular edge.

**Table 1. Light intensity variation across the slit's image ( $l = 72; 71.25$  mm)**

$S$ , $\mu\text{m}$	$H$ , mm	$h_e$ , $\mu\text{m}$	Fringe	$k$	$h_{z,1}$ , $\mu\text{m}$	$\gamma_1$ , min	$\delta'$ , min	$\alpha'$ , min	$h_{z,01}$ , mm	$k_{01}$	$k_{02}$	$h_{z,2}$ , $\mu\text{m}$
141	0.55	35.5	max <sub>1</sub>	0	12.04	1.12	18	4.7	0.231	0.451	0.524	15.6
		70.5	min <sub>1</sub>	1	8.64	2.95	24.8	0	0.087	0.604	0.355	11.3
81	0.975	20.5	max <sub>1</sub>	0	7.15	0.637	28.75	7.96	0.406	0.417	–	–
		40.5	min <sub>1</sub>	1	4.93	1.72	41.1	0	0.15	0.596	–	–
121	0.975	20.25	max <sub>1</sub>	0	7.36	0.616	29	9.8	0.421	0.406	0.372	7.84
		39	min <sub>1</sub>	1	5.12	1.618	41.5	3.4	0.16	0.517	0.31	5.85
		60.5	max <sub>2</sub>	2	4.75	2.66	44.6	0	0.097	0.652	0.465	5.43
161	0.975	19.25	max <sub>1</sub>	0	8.21	0.527	26.55	11.1	0.49	0.317	–	–
		38	min <sub>1</sub>	1	5.1	1.57	42.3	4.6	0.164	0.5	–	–
		59.9	max <sub>2</sub>	2	4.71	2.63	45.5	1.73	0.098	0.678	–	–
		80.5	min <sub>2</sub>	3	4.54	3.63	46.8	0	0.071	0.753	–	–
116	2.05	10.5	max <sub>1</sub>	0	2.99	0.36	66	20.5	0.723	0.542	0.55	2.96
		19.9	min <sub>1</sub>	1	2.15	0.85	86	9.9	0.305	0.67	0.583	2.3
		29.25	max <sub>2</sub>	2	1.99	1.3	91.1	5.7	0.2	0.71	0.58	2.15
		39.25	min <sub>2</sub>	3	1.92	1.78	93.7	3.14	0.145	0.8	0.648	2.07
		48.25	max <sub>3</sub>	4	1.87	2.22	95	1.51	0.116	0.78	0.607	2
		58	min <sub>3</sub>	5	1.85	2.68	96	0	0.096	0.84	0.66	2
$Sl_0$ without the left-hand side screen; $l = 71.25$ mm												
90	2.05	10	max <sub>1</sub>	0	3	0.34	65.2	20.8	0.766	0.5	–	–
		20	min <sub>1</sub>	1	2.07	0.87	87.8	8.8	0.3	0.713	–	–
		27.6	max <sub>2</sub>	2	1.94	1.24	92.1	5	0.209	0.582	–	–
		37	min <sub>2</sub>	3	1.86	1.7	94.9	2	0.152	0.623	–	–
		45	max <sub>3</sub>	4	1.82	2.08	96.2	0	0.124	0.515	–	–



$k_{03,4}\lambda/2$  acquired by the rays  $5'$  and  $8'$  in the zone of the right-hand side screen of  $Sl_1$  somewhat differ from  $k_{01,2}\lambda/2$  and the stronger the smaller is  $h_z$ .

This supposition is quite well explained by the facts considered in Ref. 6 (influence of the absorptivity, thickness, shape of the screen upon an edge wave), as well as by an incomplete identity of the screens of  $Sl_1$  and because of ignoring the edge light component arising

as a result of reflection of incident beams off the screen edge.

As it was mentioned in the first part of the paper, the rays 2–4 form the mean illumination in the image of  $Sl_1$ . As follows from Table 1, their deflection angles  $\alpha' \ll \delta'$ , so the light power in the optical arrangement considered, while  $Sl_1$  is illuminated by a parallel beam, does not depend on the width of  $Sl_0$ .

**Table 2. Light intensity variation at the center of the slit image as a function of  $t$  and  $H$**

$l$ , mm	$H$ , mm	$t$ , $\mu\text{m}$	Fringe	$k$	$h_{z,1}$ , $\mu\text{m}$	$\gamma_1$ , min	$\delta'$ , min	$h_{z,01}$ , mm	$k_{01}$	
72	0.55	35.5	max <sub>1</sub>	0	11.2	1.16	16.8	0.223	0.432	
		70.5	min <sub>1</sub>	1	8.64	2.95	24.75	0.087	0.604	
		108	max <sub>2</sub>	2	7.87	4.78	28.5	0.054	0.867	
		22	max <sub>1</sub>	0	5.96	0.766	31.8	0.338	0.541	
		41	min <sub>1</sub>	1	4.93	1.72	41.1	0.15	0.596	
		59.5	max <sub>2</sub>	2	4.72	2.62	44.5	0.099	0.597	
	72	0.975	79.5	min <sub>2</sub>	3	4.55	3.58	46.7	0.072	0.698
			101	max <sub>3</sub>	4	4.4	4.61	48.5	0.056	0.9
			10.5	max <sub>1</sub>	0	2.44	0.39	66.1	0.673	0.63
			20.5	min <sub>1</sub>	1	2.05	0.88	84.7	0.294	0.712
			30	max <sub>2</sub>	2	1.94	1.34	90	0.193	0.754
			40	min <sub>2</sub>	3	1.9	1.82	92.8	0.142	0.847
72	2.05	49.25	max <sub>3</sub>	4	1.87	2.26	94.7	0.114	0.87	
		58	min <sub>3</sub>	5	1.85	2.68	96	0.096	0.838	
		67.4	max <sub>4</sub>	6	1.83	3.13	97.1	0.082	0.88	
		8	max <sub>1</sub>	0	1.31	0.32	106.3	0.803	0.804	
		15	min <sub>1</sub>	1	1.11	0.67	127.8	0.386	0.93	
		21.1	max <sub>2</sub>	2	1.07	0.97	132.8	0.268	0.92	
	71.25	3	28.6	min <sub>2</sub>	3	1.05	1.33	136.7	0.194	1.12
			35	max <sub>3</sub>	4	1.03	1.64	139	0.158	1.15
			41.25	min <sub>3</sub>	5	1.02	1.94	140.3	0.133	1.15
			49.25	max <sub>4</sub>	6	1.01	2.33	141.8	0.111	1.44
			54.25	min <sub>4</sub>	7	1	2.57	142.6	0.1	1.24
			60.5	max <sub>5</sub>	8	1	2.87	143.3	0.09	1.26

**Table 3. Light intensity variation at the center of the slit image as a function of  $t$ ,  $H$  in removing the left-hand screen of  $Sl_0$**

$H$ , mm	$t$ , $\mu\text{m}$	Fringe	$k$	$h_{z,1}$ , $\mu\text{m}$	$\gamma_1$ , min	$\delta'$ , min	$h_{z,01}$ , mm	$k_{01}$
1.075	19.25	max <sub>1</sub>	0	5.45	0.67	33.8	0.39	0.506
	35.5	min <sub>1</sub>	1	4.5	1.5	45	0.173	0.507
	53	max <sub>2</sub>	2	4.2	2.35	48.9	0.11	0.558
	71.75	min <sub>2</sub>	3	4.07	3.27	51.3	0.079	0.694
	87	max <sub>3</sub>	4	4	4	52.8	0.64	0.627
	10	max <sub>1</sub>	0	2.43	0.37	66.2	0.71	0.551
2.075	17.4	min <sub>1</sub>	1	2.05	0.74	84	0.35	0.414
	27.4	max <sub>2</sub>	2	1.91	1.23	91.2	0.21	0.536
	35.5	min <sub>2</sub>	3	1.87	1.62	94.1	0.16	0.44
	45	max <sub>3</sub>	4	1.82	2.08	96.2	0.124	0.514
	53	min <sub>3</sub>	5	1.8	2.47	97.6	0.104	0.413
	61.1	max <sub>4</sub>	6	1.78	2.86	98.7	0.09	0.33
3.025	7	max <sub>1</sub>	0	1.364	0.27	100.24	0.95	0.615
	12.4	min <sub>1</sub>	1	1.12	0.544	123.5	0.476	0.53
	19.5	max <sub>2</sub>	2	1.08	0.89	133	0.291	0.673
	27	min <sub>2</sub>	3	1.03	1.25	137.3	0.206	0.9
	32.5	max <sub>3</sub>	4	1.02	1.52	139.3	0.17	0.79
	38.6	min <sub>3</sub>	5	1.01	1.81	141	0.142	0.78
	44.25	max <sub>4</sub>	6	1	2.09	142.1	0.124	0.7

Tables 2 and 3 present the values of  $k_0$  at the moments of max and min of intensity  $J$  at the axis of the image of the slit  $Sl_1$ . These were calculated by Eq. (4) for  $h_1 = t$  and different values of  $H$ ,  $t$ , and  $l$  in the experiments with  $Sl_0$  and  $Sl_0$  without its left-hand side screen.

When  $H \leq 2.1$  mm, they are approximately equal to those given in Table 1. With the increase of the  $Sl_0$  width up to 6 mm, additional increase of  $k_0$  took place. In the case of  $Sl_0$  without its left-hand side screen ( $l = 71.25$  mm),  $k_0$  is almost independent of  $k$  and in principle insignificantly differs from 0.5.

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