Optimization of a lidar receiving system. Analyzers of the polarization state

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Received April 9, 1999

In this paper we consider the approach to measuring the Stokes parameters of backscattered radiation based on matrix description of lidar equation and the instrumental vectors of analyzers used in receiving systems of polarization lidars. Polarization states of sounding radiation needed for measuring the backscattering phase matrix of a medium under study are presented, as well as the methods to obtain them. Principal types of analyzers based on beam splitter prisms, polaroids, and phase plates are listed along with their characteristics and efficiency criteria which are needed for comparative analysis. The Stratosfera–1M and Svetozar–3 lidars are considered as examples of optimal design of single-channel and multichannel receiving systems of polarization lidars.

1. Introduction

In the first two parts of our research on the optimization of lidar receiving systems,[1,2] we analyzed different types of objectives and spatial filters. In the present part we continue this research and consider the analyzer of the polarization state. This device allows separation of the lidar return components with different polarization that provide additional information on the properties of a scattering medium. Measuring the intensities of polarization components of the lidar returns allows estimation of non-sphericity of scattering particles in the case of single scattering or the contribution from multiple scattering when sounding optically dense aerosol formations.[3,4] Various prisms, polaroid films, and phase plates are used in lidars as analyzers of the polarization state of single-scattered radiation.[4–6]

In this paper we consider the principles of polarization lidar measurements and synthesis of the structure of a polarization lidar. Characteristics of the active polarization devices which change the state of the radiation beam are presented; general criteria to evaluate their quality are proposed. These criteria are needed when developing a polarization lidar.

2. Stokes vector. Matrix description of the procedure of measurement of the Stokes parameters

The field arising in the far zone as a result of electromagnetic wave scattering by a particle is described by the following equation[7]:

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_0\, e^{i\mathbf{k}_0\mathbf{r}} + \frac{e^{i\mathbf{k}_0\mathbf{r}}}{r}\, \hat{A}(\mathbf{k}, \mathbf{k}_0)\, \mathbf{E}_0\, e^{-ik_0 z_0}, \qquad (1)$$

where $\mathbf{E}_0$ is the complex amplitude of the plane wave incident on a particle from the half-space of negative z; the plane wave has the wave vector $\mathbf{k}_0$ directed along the axis z_0 of the coordinate system with the origin at an arbitrary point inside the particle; $\mathbf{r}$ is the radius vector of the point at which the field is observed; $\mathbf{k}$ is the wave vector of the scattered wave; $\hat{A}(\mathbf{k}, \mathbf{k}_0)$ is the amplitude scattering matrix (ASM) which depends on the direction of scattering and has the size 2×2. ASM relates the scattered optical wave defined in the coordinate system $\mathbf{e}_x \times \mathbf{e}_y = \mathbf{e}_r$ with the incident plane wave defined in the coordinate system $\mathbf{e}_{x_0} \times \mathbf{e}_{y_0} = \mathbf{e}_{z_0}$.

Normally detectors of optical radiation respond to the light intensity. That is why the parameters to be determined in optical measurements are either elements of the matrix of coherence[8] $J_{ij} = \frac{1}{2} \langle E_i\, E_j \rangle$ at $i, j \leftrightarrow x, y$ or the Stokes parameters which are linear combinations of the elements of the matrix of coherence, and the latter ones are used more widely.

There are several definitions of the Stokes parameters. In this paper we use the following definition[9]:

$$S_i = cn \langle \mathbf{E}^+\, \hat{\sigma}_i\, \mathbf{E} \rangle,\ \ i = 1, 2, 3, 4, \qquad (2)$$

where c is the speed of light in vacuum; n is the refractive index of a medium; $\mathbf{E}$ is the vector-column with the elements E_x and E_y; $\mathbf{E}^+$ is the vector-row which is Hermitian conjugate to vector $\mathbf{E}$; $\hat{\sigma}_1$ is the unit 2×2 matrix; $\hat{\sigma}_{2,3,4}$ are Pauli matrices:

0235-6880/99/07 619-11 $02.00

$$\hat{\sigma}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix};\ \hat{\sigma}_2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix};\ \hat{\sigma}_3 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix};\ \hat{\sigma}_4 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}.$$

To find the Stokes parameters of a scattered wave, one should apply definition (2) to the field described by the second term in the right-hand side of Eq. (1). The Stokes vector of the scattered wave is determined by the following vector–matrix equation[9]:

$$\mathbf{S}(\mathbf{r}, \mathbf{k}) = \frac{1}{r^2} \hat{M}(r, \mathbf{k}, \mathbf{k}_0)\, \mathbf{S}_0, \quad (3)$$

where $\mathbf{S}$ and $\mathbf{S}_0$ are the Stokes vectors of the scattered and incident waves, respectively; $\hat{M}(\mathbf{k}, \mathbf{k}_0)$ is the 4×4 scattering phase matrix (SPM) at the point $\mathbf{r}$. The elements of the matrix $\hat{M}$ have dimensionality of the area divided by solid angle, within which the radiation propagates because the wave diverges. The matrix $\hat{M}$ can be expressed through ASM from Eq. (1) by the following matrix equation[10]:

$$\hat{M} = \hat{T}(\hat{A} \oplus \hat{A}^*)\, \hat{T}^{-1},$$

where $\oplus$ denotes the Kronecker product of ASM by the complex conjugate matrix (for principles of matrix optics see Ref. 12);

$$\hat{T} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & -i & i & 0 \end{bmatrix};\ \hat{T}^{-1} = \frac{1}{2}\hat{T}^{+}.$$

Equation (3) describes scattering by an individual particle. For an ensemble of particles occupying the volume ΔV and containing N particles per unit volume, the following matrix:

$$\hat{\overline{M}}(\mathbf{k}, \mathbf{k}_0) = \frac{1}{N}\sum_{i=1}^{N} \hat{M}_i(\mathbf{k}, \mathbf{k}_0)$$

is introduced. The dimensions of the scattering volume ΔV are thought to be far less than r. The matrix elements are measured in $\mathrm{m}^{-1} \cdot \mathrm{sr}^{-1}$. The Stokes vector of the radiation scattered by an element of the volume ΔV in the direction $\mathbf{e}_r = \mathbf{k}/|k|$ at the distance r from the volume is described by the equation

$$\mathbf{S}(r) = \frac{1}{r^2} \hat{\overline{M}}(r, k, k_0)\, \mathbf{S}_0\, \Delta V. \quad (4)$$

The goal of measurements in the case of polarization monostatic laser sounding is to determine either all SPM elements or some linear combinations of these elements for the process of scattering in the direction opposite to the wave vector of the incident wave $-\hat{\overline{M}}(z, -\mathbf{k}_0, \mathbf{k}_0) = \hat{\overline{M}}_\pi(z)$, that is, the backscattering phase matrix (BSPM).

The measurement procedure resulting in determination of BSPM is described in Ref. 4. Its idea is in measuring the Stokes vector of the scattered radiation at different polarization states of sounding radiation. Consequently, in the most general form, the polarization sounding should provide for the possibility of varying the polarization state of laser radiation and to measure the Stokes parameters of sounding radiation. Let us consider the latter option. In the expanded form, definition (2) looks like:

$$\left.\begin{aligned} S_1 &= cn\,(\langle E_x^* E_x\rangle + \langle E_y^* E_y\rangle) = I \\ S_2 &= cn\,(\langle E_x^* E_x\rangle - \langle E_y^* E_y\rangle) = Q \\ S_3 &= cn\,(\langle E_x^* E_y\rangle + \langle E_y^* E_x\rangle) = U \\ S_4 &= cn\,(\langle E_x^* E_y\rangle - \langle E_y^* E_x\rangle) = V \end{aligned}\right\}. \quad (2')$$

Below, for a convenience, we shall use symbols S_i and I, Q, U, V (see Eq. (2′)) to denote Stokes parameters.

Here we present some designations and definitions used in the below discussion: $a_1 = (E_x E_x^*)^{1/2}$ and $a_2 = (E_y E_y^*)^{1/2}$ are the absolute values of amplitudes of x and y components of the vector of electric field; $\delta = \varphi_x - \varphi_y$, where φ_x and φ_y are the phases of the corresponding components of the vector $\mathbf{E}$ at the time $t = 0$. Depending on the relation between a_1 and a_2 and the values of δ, the following *types of polarization* are distinguished[8,11]: linear polarization with the azimuth $\alpha = \arctan(a_2/a_1)$ and the phase shift $\delta = m\pi$, the plus sign of the azimuth α corresponds to even values of m, while minus corresponds to odd values of m; circular polarization ($a_1 = a_2$ and $\delta = \pi/2 + m\pi$), plus corresponds to the right-hand polarization, and minus corresponds to the left-hand one; elliptical polarization. The last term describes the most general polarization state including the above two as particular cases. Elliptical polarization is characterized by the ratio of the minor semiaxis of an ellipse to the major one b/a, the azimuth of the major semiaxis α, and the direction of rotation (left or right). One more term is the reference plane (the plane of polarization basis), the azimuth α of the plane of oscillations for the linearly polarized wave or the angle of inclination of the ellipse's major axis is measured from. This plane is often related to the plane xOz. In this case, a plane wave is horizontally polarized if $a_2 = 0$ ($\alpha = 0°$) and vertically polarized if $a_1 = 0$ ($\alpha = 90°$). The degree of polarization of radiation p is defined as

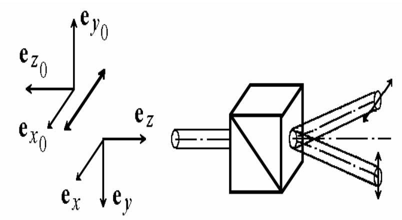
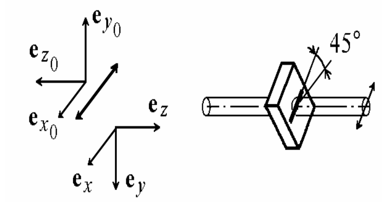
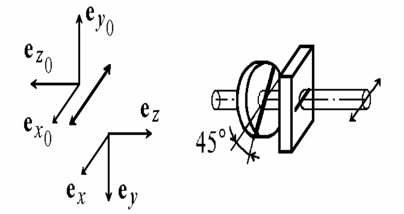
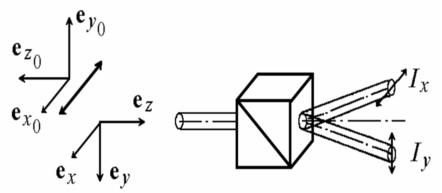
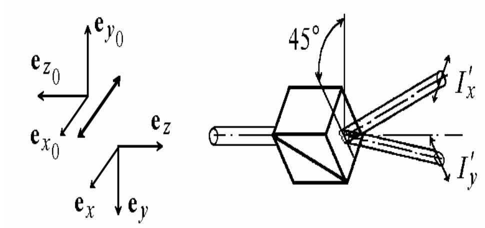
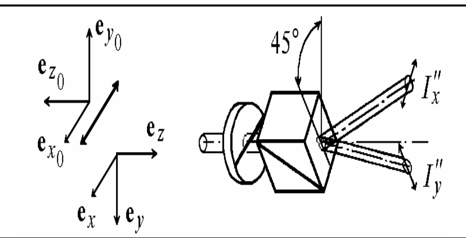
$$P = (Q^2 + U^2 + V^2)^{1/2}/I.$$

From Eq. (2′) it follows that to determine the first Stokes parameter, one should find the sum of intensities I_x and I_y of two components with orthogonal polarization, while to find the second Stokes parameter, one should subtract these two intensities (Table 1). To determine the parameter U, one should measure the intensities I'_x and I'_y in the coordinate system turned by 45° about the beam incidence direction (axis z). To determine the parameter V, one should measure I''_x and I''_y after passage of radiation beam through a phase $\lambda/4$ plate installed in front of an analyzer (see Table 1). It follows from the above-said that to determine the Stokes vector, one needs to measure six intensities of the beam at different positions of the polarization devices. This classical measurement procedure[8] proves to be excessive, because the condition

$$I_x + I_y = I'_x + I'_y = I''_x + I''_y$$

holds true for stationary measurement conditions. Therefore, if the beam parameters keep constant during the measurements, the procedure can be reduced to the measurement of four intensities. This point will be discussed below when considering possible schemes of polarization analysis of lidar signals.

Table 1. Possible versions of the design of an analyzer unit of a lidar receiving system.

Stokes parameters to be measured	Instrumental vector of the receiving channel and the algorithm of calculation	Scheme of the analyzer in the receiving channel
I; Q	$\mathbf{G}_1(0°) = \frac{1}{2}\{1\ 1\ 0\ 0\}$ $\mathbf{G}_2(90°) = \frac{1}{2}\{1\ -1\ 0\ 0\}$ $I = (\mathbf{G}_1 + \mathbf{G}_2)\ \mathbf{S}$; $Q = (\mathbf{G}_1 - \mathbf{G}_2)\ \mathbf{S}$	
U	$\mathbf{G}_3(45°) = \frac{1}{2}\{1\ 0\ 1\ 0\}$ $U = (2\mathbf{G}_3 - \mathbf{G}_1 - \mathbf{G}_2)\ \mathbf{S}$	
V	$\mathbf{G}_5(0°, 45°) = \frac{1}{2}\{1\ 0\ 0\ -1\}$ $V = -\ (2\mathbf{G}_5 - \mathbf{G}_1 - \mathbf{G}_2)\ \mathbf{S}$	
I; Q	$\mathbf{G}_1(0°) = \frac{1}{2}\{1\ 1\ 0\ 0\}$ $\mathbf{G}_2(90°) = \frac{1}{2}\{1\ -1\ 0\ 0\}$ $I = I_x + I_y$; $Q = I_x - I_y$	
I; U	$\mathbf{G}_3(45°) = \frac{1}{2}\{1\ 0\ 1\ 0\}$ $\mathbf{G}_4(-45°) = \frac{1}{2}\{1\ 0\ -1\ 0\}$ $I = I'_x + I'_y$; $U = I'_x - I'_y$	
I; V	$\mathbf{G}_5(45°, 0°) = \frac{1}{2}\{1\ 0\ 0\ -1\}$ $\mathbf{G}_6(-45°, 0°) = \frac{1}{2}\{1\ 0\ 0\ 1\}$ $I = I''_x + I''_y$; $V = I''_x - I''_y$	

In 1948, when describing the action of polarization devices, Muller introduced the operators, which are 4×4 matrices with the elements being real values. The device action is described by the linear transformation

$$\mathbf{S} = \hat{o}\ \mathbf{S}_0,$$

where $\mathbf{S}_0$ and $\mathbf{S}$ are the Stokes vectors of radiation incident on the device and having passed through it, respectively; $\hat{o}$ is Muller matrix describing the polarization device. If there are several devices the radiation must pass through, then their combined action is described by the product of matrix operators

$$\mathbf{S} = \hat{o}_n\ \hat{o}_{n-1} \dots \hat{o}_1\ \mathbf{S}_0.$$

Reference 12 gives a detailed review of the Muller method of calculation.

To describe the procedure of measurement of the Stokes parameters, it is convenient to introduce the concept of instrumental vector for the polarization device of the receiving system $\mathbf{G}_i$. The instrumental vector is the vector-row consisting of the elements of the first row of the matrix operator $\hat{o}$ of the polarization device or the matrix being a product of operators of several polarization devices $\hat{o} = \hat{o}_n\ \hat{o}_{n-1} \dots\ \hat{o}_1$. This problem is considered in detail in Ref. 4. Below we present the instrumental vectors corresponding to the measurement of the six above-mentioned intensities needed for determination of the Stokes vector.

Analyzers of linear polarization whose azimuth α makes 0, 90, 45°, and − 45° angles with the axis Ox of the polarization basis $\mathbf{e}_x \times \mathbf{e}_y = \mathbf{e}_z$ (rotation around the z-axis) can be written as follows[4,11]:

$$\mathbf{G}_1(0°) = \frac{1}{2}\{1\ 1\ 0\ 0\}; \tag{5a}$$

$$\mathbf{G}_2(90°) = \frac{1}{2}\{1\ -1\ 0\ 0\}; \tag{5b}$$

$$\mathbf{G}_3(45°) = \frac{1}{2}\{1\ 0\ 1\ 0\}; \tag{5c}$$

$$\mathbf{G}_4(-45°) = \frac{1}{2}\{1\ 0\ -1\ 0\}. \tag{5d}$$

The next couple of instrumental vectors describes the combined action of the $\lambda/4$ plate whose fast axis makes 45° angle with the axis Ox and the analyzer set at the angles of 0 and 90°

$$\mathbf{G}_5(0;\ 45°) = \frac{1}{2}\{1\ 0\ 0\ -1\}; \tag{5e}$$

$$\mathbf{G}_6(90;\ 45°) = \frac{1}{2}\{1\ 0\ 0\ 1\}. \tag{5f}$$

Then the presence of the second argument in the instrumental vector is indicative of the installed $\lambda/4$ plate with the above-specified azimuth of the fast axis. The matrix operator of the receiving objective and the spatial filter (diagram) are thought unit matrices. Besides, in the absence of active polarization elements in the receiving system, we believe that the instrumental vector of the form

$$\mathbf{G}_0 = \{1\ 0\ 0\ 0\} \tag{6}$$

combines with the Stokes vector of the incident radiation.

According to Ref. 13, the lidar equation in the vector-matrix form can be written as follows:

$$P(z)\ \mathbf{s}(z) = \frac{1}{2}\, cW\, Az^{-2}\ \hat{\overline{M}}_\pi(z)\ \mathbf{s}_0 \times$$

$$\times \exp\left\{-\,2\int\limits_0^z \alpha(z')\ \mathrm{d}z'\right\}, \tag{7}$$

where $P(z)$ is the power of the scattered radiation incident on the receiving objective at the time $t = 2z/c$; c is the speed of light; W is the energy of a laser pulse; A is the area of the receiving objective; $\hat{\overline{M}}_\pi(z)$ is the backscattering phase matrix; $\alpha(z)$ is the extinction coefficient of the scattering medium; $\mathbf{s}_0$ and $\mathbf{s}(z)$ are the Stokes vectors of the incident and scattered radiation, respectively. These Stokes vectors are scaled to the intensities; they have the form of vector-columns with the components $\{1\ q_0\ u_0\ v_0\}$ and $\{1\ q(r)\ u(r)\ v(r)\}$.

Upon multiplication of both sides of Eq. (7) by the instrumental vector $\mathbf{G}_i$ and the coefficients characterizing the transmission of optical elements η_i, and the ampere–watt sensitivity of the detector μ_i, through which this instrumental vector is realized, Eq. (7) can be written in the form

$$F_i(z) = F_0(z)\ \gamma_i\ \mathbf{G}_i\ \mathbf{s}(z) = \gamma_i\ \mathbf{G}_i\ \mathbf{S}'(z)\ KT^2(z), \tag{8}$$

where

$$\gamma_i = \frac{\eta_i\ \mu_i}{\eta_0\ \mu_0}\,;\ \mathbf{S}'(z) = \frac{1}{z^2}\ \hat{\overline{M}}_\pi(z)\ \mathbf{s}_0;\ K = \frac{1}{2}\, cWA\ \eta_0\ \mu_0.$$

Here the vector $\mathbf{S}'(z)$ with the parameters $I'(z)$, $Q'(z)$, $U'(z)$, and $V'(z)$ has the meaning of the Stokes vector of scattered radiation normalized to the product $KT^2(z)$; $F_i(z)$ has the meaning of a current response of the detector at the time moment $t = 2z/c$. It is just this parameter that is the measured value. The parameter $F_0(z)$ has the meaning of a response of the detector through which the vector $\mathbf{G}_0$ is realized. The latter statement does not mean that it is necessarily used in measurements. This is only a convenient method for scaling the products $\eta_i\ \mu_i$ in the cases when the instrumental vectors of the receiving system represent different receiving channels. The term “receiving channelB means the set of a receiving objective, a spatial filter, an analyzer, and a detector. Thus, one receiving system of a lidar may have several receiving channels.[5]

The parameter to be determined is the vector $\mathbf{S}(z)$. So far as we are dealing with measurements of the Stokes parameters, their presentation through the elements of the backscattering phase matrix is not of our concern. Because the Stokes parameters are determined by applying several measurement operations, the following requirements should be satisfied. First, either all operations are executed simultaneously, or the vector $\mathbf{S}(z)$, i.e. $\hat{\overline{M}}_\pi(z)$, and the conditions of atmospheric transmittance $T(z)$ for the radiation should be constant during the measurements. Second, either all operations are executed with a single-channel receiving system by sequentially changing the vectors $\mathbf{G}_i$ in it, and then $\hat{\overline{M}}_\pi(z)$ and $T(z)$ must necessarily be constant; or several receiving channels are used in the receiving system, and then one should know the ratio of efficiencies γ_i/γ_j of these channels.

Let us use an example to clarify the above-said. Assume that the parameter $Q(z)$ of the vector-column $\{I(z), Q(z), U(z), V(z)\}$ is measured by sequential application of the instrumental vectors $\mathbf{G}_1$ and $\mathbf{G}_2$ (see Eqs. (5a) and (5b)). Using Eq. (8) we have

$$F_1(z) = \frac{1}{2}\gamma_1 \, [I'(z) + Q'(z)] \, KT_1^2,$$

$$F_2(z) = \frac{1}{2}\gamma_2 \, [(I'(z) - Q'(z))] \, KT_2^2, \tag{9}$$

where T_1 and T_2 are the transmittance at the time t_1 and t_2, wherefore we have

$$I'(z) = \frac{1}{K} \, [F_1(z)/\gamma_1 \, T_1^2 + F_2(z)/\gamma_2 \, T_2^2];$$

$$Q'(z) = \frac{1}{K} \, [F_1(z)/\gamma_1 \, T_1^2 - F_2(z)/\gamma_2 \, T_2^2]. \tag{10}$$

As follows from Eq. (10), if measurements are performed at different time, one needs to know the profile of transmittance $T(z)$ at different time also. This significantly complicates the problem, because $T(z)$ can be determined in lidar measurements only with low accuracy. If $t_2 = t_1$ or $t_2 - t_1$ is less than the time of "frozen turbulenceB so that $T_1 = T_2$, then we can determine the normalized Stokes parameter

$$q(z) = Q'(z)/I'(z) = \frac{F_1(z)/\gamma_1 - F_2(z)/\gamma_2}{F_1(z)/\gamma_1 + F_2(z)/\gamma_2}. \tag{11}$$

To do that, we have no need to know the transmittance $T(z)$ and the instrumental constant K. The normalized Stokes parameters $u(z)$ and $v(z)$ can be found in a similar way.

The normalized Stokes vector well determines the polarization state of radiation. Combined measurements of the Stokes parameters at different polarization states of the laser radiation allows determination of the normalized scattering phase matrix which, in turn, fully characterizes the transformation of polarization at backscattering.[4,14]

3. Polarization state of the sounding radiation

As was mentioned above, the measurements of BSPM imply the possibility of changing the polarization state of sounding radiation. Therefore, one of the basic functions of the lidar transmitting system which emits sounding pulses is to provide a required polarization state of sounding radiation, as well as the feasibility of changing it. In so doing, either the laser itself or special polarization devices should be used. In the general case, the polarization state of a laser beam depends on the type of the active medium, characteristics of the cavity and the laser operating mode and can vary widely.[15,16]

For gas and liquid-state lasers operating in the multimode regime, polarization of the output radiation depends on the orientation of the exit windows of the laser cell set at the Brewster angle.[8,15] In this case, the loss in the laser cavity is minimum for the radiation polarized linearly in the plane of incidence; and the radiation with this polarization becomes predominant. If the exit windows of a cell are normal to the optic axis of the laser cavity, the output radiation has arbitrary polarization independent of its mode structure.

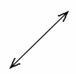
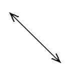
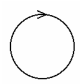
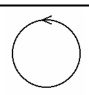
In solid-state lasers most widely used in lidars, the polarization state of the output radiation depends on both the structure of the active medium itself and on the elements inserted in the laser cavity for modulation of its Q-factor.[15] For active crystal media, the highest probability of the stimulated transitions and, consequently, the largest gain are connected with certain directions relative to the symmetry axes of the crystals.[17] The output radiation of solid-state lasers has mainly the linear polarization controlled by the elements of the modulation cell.

The polarization state of radiation from semiconductor lasers depends on numerous factors: crystal structure of a semiconductor material, geometry of the laser cavity formed by the crystal faces, pump current, etc. Besides, in view of high divergence of the radiation, the degree of its polarization can vary within the directional pattern.[16]

Polarization of sounding radiation and methods to control it[4–6] are shown in Table 2. In this case the initial linear polarization has the azimuth $\alpha = 0°$. Versions of the methods to change the polarization state are given in the last column of the table. The angles ψ determine the azimuth (relative to the reference plane xOz of the polarization basis of a laser) of the direction of the highest transmission of a Glan prism or the fast axis of a phase plate. It should be noted that insertion of additional polarization elements into the laser receiving channel causes loss of the radiation power. In the phase plates, it depends on the Fresnel reflection at the medium interfaces and absorption in the material of the plate. When using a Glan prism, a half of the flux is additionally lost due to suppression of one of the orthogonal components.

As seen from Table 2, in order to obtain radiation with a circular polarization (states *6* and *7*), the linearly polarized radiation should be passed through a $\lambda/4$ plate. If an additional Glan prism is set in the path of a circularly polarized radiation, then the circular polarization can be transformed back into the linear one. By rotating the Glan prism, one can change the azimuth of the linearly polarized sounding radiation in the medium under study without changing the position of the laser itself. We have used this technical solution in the Svetozar–3 airborne polarization lidar.[5] Another solution is the use of a combination of two $\lambda/4$ plates whose fast axes are directed at the azimuths listed in Table 2 (Ref. 4). Application of two sequential phase plates for rotation of the azimuth of the linearly polarized radiation introduces minimal power loss, however the error in the azimuth increases because of imperfect manufacture of phase plates.

Table 2. Possible polarization states of sounding radiation.

Type of polarization	Projection diagram	Polarization: azimuth	b/a	a_2/a_1	δ	Normalized Stokes vector $\mathbf{s}_0$	External polarizer
Linear	*1*	0	0	0	0	{1 1 0 0}	No
	2	$\pi/2$	0	∞	0	{1 −1 0 0}	single $\lambda/4$ plate with ψ = 45° and Glan prism with c ψ = 90° two $\lambda/4$ plates, each with ψ = 45°
	3 	$\pi/4$	0	1	0	{1 0 1 0}	single $\lambda/4$ plate with ψ = 45° and Glan prism with ψ = 45° two $\lambda/4$ plates with ψ = 0° and ψ = 45°
	4 	$-\pi/4$	0	1	$\pm\pi$	{1 0 −1 0}	single $\lambda/4$ plate with ψ = 45° and Glan prism with ψ = −45° two $\lambda/4$ plates with ψ = 0° and ψ = −45°
	5 Any direction	α	0	$0 \div \infty$	0 or $\pm\pi$	{1 cos2α sin2α 0}	single $\lambda/4$ plate with ψ = 45° and Glan prism with any ψ
Circular	*6* 	–	1	1	$\pi/2$	{1 0 0 1}	single $\lambda/4$ plate at ψ = 45°
	7 	–	1	1	$-\pi/2$	{1 0 0 −1}	single $\lambda/4$ plate at ψ = −45°

4. Basic characteristics and criteria of efficiency of polarization elements in a lidar

By the term “polarizerB we mean an optical device which separates out the radiation with a preset polarization from the incident beam. Polarizers can be linear, circular, and elliptical depending on the type of polarization to be separated. As was mentioned above, such devices are used in lidar transmitting systems. We use the term “analyzersB for polarization devices applied in the receiving system for analysis of the polarization state of incident radiation.

An analyzer breaks the incident beam into two components with orthogonal polarization and passes one component, while absorbing or deflecting the other. Four physical phenomena are involved in this process: dichroism, birefringence, reflection, and scattering.[8,11] Analyzers applied in lidars employ the first two phenomena. The dichroic analyzer selectively absorbs one polarization, while passing the another (orthogonal) one. A refracting analyzer, introducing different phase shifts into orthogonal polarization, breaks the incident beam into two beams having orthogonal polarization that propagate in different directions.

As in Ref. 11, we described the use of the following characteristics for description of a linear analyzer used in a lidar receiving system:

1) *the largest principal transmittance* k_1 which is defined as a ratio of the radiation intensity after propagation to the intensity of incident radiation when the incident beam is linearly polarized at the azimuth corresponding to the maximum transmittance; *the least*

principal transmittance k_2 is determined at the minimum transmittance of the analyzer for the linearly polarized radiation. For high-quality elements $k_1 \approx 1$ and k_2 is close to zero;

2) *the principal surface* is the surface through which the beam under analysis enters because there is no the reciprocity law for most analyzers;

3) *direction of the transmission axis* of the analyzer at which the vector of electric oscillations of the linearly polarized radiation incident normally on the principal surface of the analyzer has maximum transmittance;

4) *spectral band* $(\lambda_1 - \lambda_2)$ within which the analyzer keeps its characteristics constant;

5) *tolerable angle of grazing incidence* γ on the principal surface of the analyzer at which it keeps its characteristics;

6) *eigenvector of the analyzer* which characterizes the analyzer taking into account its input and output properties. This is the polarization satisfying the following condition: if the incident fully polarized beam has this polarization, then the outcoming beam also has this polarization, that is, the eigenvector can be thought as determined by Eqs. (5);

7) *maximum diameter of the clear aperture* D_0.

Phase plates used in analyzers to change polarization are characterized by two parameters[11]:

(a) *phase shift* δ for waves with orthogonal polarization;

(b) *azimuth of the fast axis* ψ usually measured counterclockwise with respect to the reference plane when viewed opposite to the beam.

Following Ref. 8, we may present some relationships. The intensity of a linearly polarized radiation passed through a linear analyzer depends on the angle χ between the direction of the transmission axis of the analyzer and the direction of oscillations of the vector **E**:

$$I = a^2 \cos^2\chi. \qquad (12)$$

The phase plate of thickness h introduces the phase shift equal to

$$\delta = (2\pi/\lambda)\,(n'' - n')\,h, \qquad (13)$$

where $n'' - n'$ is the difference between the refractive indices for the ordinary and extraordinary rays. The device consisting of such a phase plate and the linear analyzer in the general case ensures the intensity of the passed radiation equal to

$$I = a^2\,[\cos^2\chi - \sin 2\alpha \,\sin 2(\alpha - \chi)\,\sin^2(\delta/2)], \qquad (14)$$

where α is the azimuth of linear polarization with respect to the reference plane.

Besides the above-listed characteristics of an analyzer, one should take into account the following characteristics, which also can be considered as criteria of efficiency of applying this analyzer in a lidar receiving system:

8) *capability of simultaneously separating beams with orthogonal polarization*;

9) *angle* φ between the orthogonal components at the analyzer output;

10) *cost* of the analyzer.

5. Comparative analysis of polarization elements

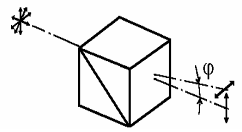
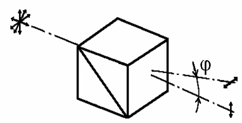
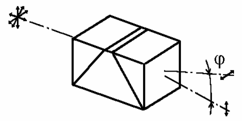
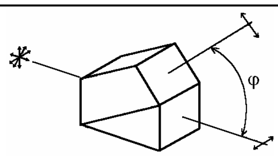
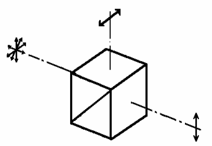
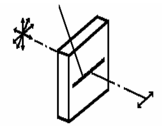
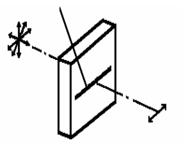
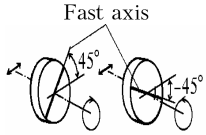
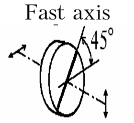
With the use of some data borrowed from Refs. 11, 18–20, Table 3 presents the polarization elements, which are most promising for application in lidar receiving systems. They allow realization of the instrumental vectors described by Eqs. (5). The use of birefringent prisms is preferable because they separate two orthogonal components of the linearly polarized radiation simultaneously. From the design consideration, it is worth using prisms providing for the largest value of the angle φ. In this case, radiation detectors are arranged maximally close to the analyzer output, and the Wollaston prism is the instrument of choice because it provides for practically symmetric spread of beams at the output with respect to the incident beam axis. This is more convenient when rotating the analyzer in order to sequentially measure all the Stokes parameters.[4–6]

It should be noted that beam splitting analyzers manufactured from natural crystals are rather expensive, and their cost increases proportionally to the third power of the diameter D_0. Therefore, current foreign lidars often use thin-film polarization cubes (see, for example, Refs. 21 and 22), which split the orthogonal components at the angle of 90° due to different reflection and refraction indices on a thin-film composite.

6. Synthesis of receiving systems for polarization lidars

The receiving system of a polarization lidar should enable the polarization state of the recorded radiation to be unchanged as the radiation passes from the entrance aperture of the objective to the analyzer. When designing a polarization lidar, one should take into account that in spite of active polarization elements (polarizers, analyzers, and phase plates) other optical elements (lenses, mirrors, prisms) having interfaces and coatings also influence the polarization state of the radiation beam.[8,23] Therefore, as applied to the instrumental vector of the lidar receiving system having no polarization elements, Eq. (6) is not fully correct even in the case of the radial symmetry of the system because of possible depolarization of radiation passed through the system.[23,24]

Table 3. Comparative characteristics of analyzers and phase plates.

Type	View, name	Characteristics	Notes
Birefringent	 Rochon prism	$k_1/k_2 = 10^5$; $\gamma = 5°$; $D_0 = 15$ mm; $\lambda_1 - \lambda_2 = 0.35 - 2.2$ μm; $\varphi_{max} = 15°$	The directions of the ordinary and incident beams coincide
	 Wollaston prism	$k_1/k_2 = 10^5$; $\gamma = 6°$; $D_0 = 20$ mm; $\lambda_1 - \lambda_2 = 0.3 - 2.2$ μm; $\varphi_{max} = 20°$	Asymmetry in deflection of beams no more than 1°
	 Wollaston three-element prism	$k_1/k_2 = 2 \cdot 10^4$; $\gamma = 5°$; $D_0 = 12$ mm; $\lambda_1 - \lambda_2 = 0.3 - 2.2$ μm; $\varphi_{max} = 30°$	The same
	 Thompson beam splitter	$k_1/k_2 = 10^5$; $\gamma = 15°$; $D_0 = 12$ mm; $\lambda_1 - \lambda_2 = 0.3 - 2.2$ μm; $\varphi_{max} = 45°$	–
Reflecting	 KLC thin-film cube	$k_1/k_2 = 10^2$; $\gamma = 5°$; $D_0 = 25$ mm; $\lambda_1 - \lambda_2 = 0.4 - 0.7$ μm; $\varphi_{max} = 90°$	–
Dichroic	Transmission axis  K-type Polaroid	$k_1 = 0.5 - 0.85$; $k_2 = 10^{-4} - 10^{-2}$; $D_0 = 75$ mm; $\lambda_1 - \lambda_2 = 0.38 - 0.65$ μm; $\gamma = 35°$	The values of k_1 and k_2 vary within $\lambda_1 - \lambda_2$
	Transmission axis  H-type Polaroid	$k_1 = 0.5 - 0.85$; $k_2 = 10^{-4} - 10^{-2}$; $D_0 = 75$ mm; $\lambda_1 - \lambda_2 = 0.38 - 0.75$ μm; $\gamma = 35°$	The same. Less resistant to external actions
Phase-shifting	Fast axis  $\lambda/4$ phase plate	$D_{0\,max} = 40$ mm; $\gamma = 5°$; $\lambda_1 - \lambda_2 = 0.38 - 0.78$ μm for achromatic ones	–
	Fast axis  $\lambda/2$ phase plate	$D_{0\,max} = 40$ mm; $\gamma = 5°$; $\lambda_1 - \lambda_2 = 0.38 - 0.78$ μm for achromatic ones	–

Designing the polarization lidar requires optimization of the receiving system in the number of used polarization devices and their orientation in receiving channels. Let us first present the classification of possible procedures of measurement of the Stokes parameters. For compactness, let us take temporarily that γ_i in Eq. (8) for all detectors (if any) is equal to unity, and the vector **S** keeps unchanged in the process of measurements (if the measurements are not conducted simultaneously). Besides, we assume that the instrumental vectors can be summed up by the rules of the vector algebra. Then the classical measurement procedure considered in Section 2 can be written as follows:

$$\begin{aligned}(\mathbf{G}_1 + \mathbf{G}_2)\,\mathbf{S} = I,\ \ (\mathbf{G}_1 - \mathbf{G}_2)\,\mathbf{S} = Q,\\ (\mathbf{G}_3 + \mathbf{G}_4)\,\mathbf{S} = I,\ \ (\mathbf{G}_3 - \mathbf{G}_4)\,\mathbf{S} = U,\\ (\mathbf{G}_5 + \mathbf{G}_6)\,\mathbf{S} = I,\ \ (\mathbf{G}_6 - \mathbf{G}_5)\,\mathbf{S} = V.\end{aligned} \quad (15)$$

It should be emphasized that summation and subtraction do not produce new instrumental vectors. This is only a convenient form of presentation of, for example, the fact that subtraction of results of action of the instrumental vectors $\mathbf{G}_1$ and $\mathbf{G}_2$ on the vector **S** gives the parameter Q, etc.

As was mentioned above, the procedure similar to that described by Eqs. (15) is excessive because the parameter I is determined three times. Using the accepted formalism, below we demonstrate how the number of measurements can be decreased from six to four. It is easy to see that

$$\mathbf{G}_1 + \mathbf{G}_2 = \mathbf{G}_3 + \mathbf{G}_4 = \mathbf{G}_5 + \mathbf{G}_6 = \mathbf{G}_0, \quad (16)$$

therefore

$$\mathbf{G}_2 = \mathbf{G}_0 - \mathbf{G}_1,\ \mathbf{G}_4 = \mathbf{G}_0 - \mathbf{G}_3,\ \mathbf{G}_6 = \mathbf{G}_0 - \mathbf{G}_5. \quad (17)$$

Then one of the possible procedures for measuring I is $I = \mathbf{G}_0\mathbf{S}$, and upon substitution of Eq. (17) into Eq. (15), we have

$$\begin{aligned}Q &= (2\,\mathbf{G}_1 - \mathbf{G}_0)\,\mathbf{S};\\ U &= (2\,\mathbf{G}_3 - \mathbf{G}_0)\,\mathbf{S};\\ V &= -\,(2\,\mathbf{G}_5 - \mathbf{G}_0)\,\mathbf{S}.\end{aligned} \quad (18)$$

In this procedure, the Stokes vector is determined from measurements with the four instrumental vectors, $\mathbf{G}_0$, $\mathbf{G}_1$, $\mathbf{G}_3$, and $\mathbf{G}_5$. Under stationary atmospheric conditions, measurements can be conducted sequentially with a single detector. In contrast, under nonstationary conditions, all four instrumental vectors should be realized simultaneously that requires four receiving channels with correspondingly four detectors.

The number of channels can be decreased down to three, if one takes into account that any pair of instrumental vectors $\mathbf{G}_1$, $\mathbf{G}_2$; $\mathbf{G}_3$, $\mathbf{G}_4$; $\mathbf{G}_5$, $\mathbf{G}_6$ can be realized in one channel with a Wollaston prism, for instance.[4–6] Assume that the instrumental vectors $\mathbf{G}_1$ and $\mathbf{G}_2$ are formed with such a device. Then the detectors of the receiving channel in which the device is set can be used to determine

$$\begin{aligned}I &= (\mathbf{G}_1 + \mathbf{G}_2)\,\mathbf{S} = \mathbf{G}_0\,\mathbf{S};\\ Q &= (\mathbf{G}_1 - \mathbf{G}_2)\,\mathbf{S}.\end{aligned} \quad (19)$$

If the vectors $\mathbf{G}_3$ and $\mathbf{G}_5$ are realized in other two receiving channels, then it is clear that

$$\begin{aligned}U &= (2\,\mathbf{G}_3 - \mathbf{G}_1 - \mathbf{G}_2)\,\mathbf{S};\\ V &= -\,(2\,\mathbf{G}_5 - \mathbf{G}_1 - \mathbf{G}_2)\,\mathbf{S}.\end{aligned} \quad (20)$$

Starting from Eq. (16), one can see that there exist several similar measurement schemes, but they are equivalent to the scheme considered above. Their advantage over scheme (15), which also can be realized using three receiving channels, is the less number of detectors and, correspondingly, the less number of expensive elements such as Wollaston prisms (one prism instead of the three needed for realization of measurement scheme (15)). Therefore, these schemes are preferable as far as concerned the instrumental realization. Nevertheless, excessive character of scheme (15) gives some methodical advantages, but they are not discussed in this paper.

Table 1 in its first half gives the version of a design of the analyzers for realization of measurements by algorithm (19) and (20) at a horizontal linear polarization of sounding radiation with the zero azimuth. The schemes also show the orientation of the unit vectors of coordinate systems of the sounding radiation $\mathbf{e}_{x_0}$, $\mathbf{e}_{y_0}$, $\mathbf{e}_{z_0}$ and backscattered radiation $\mathbf{e}_x$, $\mathbf{e}_y$, $\mathbf{e}_z$. In this case the pairs of vectors $\mathbf{e}_{x_0}$ and $\mathbf{e}_x$ and $\mathbf{e}_{z_0}$ and $\mathbf{e}_z$ are collinear, and the reference plane for both beams lies in the plane formed by these vectors. The set of the coordinate systems chosen forms the lidar polarization basis which, if necessary, can be related to the geodesic coordinate system by determining the orientation of the lidar in space.

If, besides the above-described polarization state, one uses other states given in Table 2, namely, states *2*, *3*, and *6*, it is possible (according to Ref. 4) to conduct measurements for determining all 16 BSPM elements $\hat{\overline{M}}_\pi(z)$. However, when applying algorithm (20), the beams realized by the instrumental vectors $\mathbf{G}_4$ and $\mathbf{G}_6$ are not used.

The bottom part of Table 1 presents the design of the analyzers in the three receiving channels of the Svetozar–3 lidar.[5,6] As was mentioned above, this design is excessive, but it proves useful for mutual calibration of the channels.

Synthesis of the transceiving system of the polarization lidar consists in seeking optimal (best) relations between (i) lidar operating conditions if the parameters describing the state of the sounded object are decisive; (ii) lidar structure and parameters; (iii) cost of the lidar.

Most important operating conditions of the polarization lidar include those determining power relations between the sounding and received radiation fluxes and giving the power and size parameters of the lidar. Dynamic variability of the object of sounding for a stationary lidar or the speed of an aircraft (spacecraft) for an airborne (spaceborne) lidar with a preset restriction to the spectrum of resolvable spatial frequencies of the sounded object governs the needed repetition rate of lidar measurements, which is limited by the repetition rate of sounding events. The condition providing for such a limit is synchronous measurement of the Stokes parameters. However, as was demonstrated above, synchronous measurement requires parallel recording of, at least, four independent components in three independent receiving channels. Such lidars have a low power potential because entrance apertures of their receiving channels do not exceed, as a rule, 100–150 mm. Each channel includes prism analyzers[5,6] which separate the components with orthogonal polarization and azimuth satisfying Eqs. (5a–b) and (5c–d). One channel also includes a phase plate providing the fulfillment of Eqs. (5e–f). Six photodetectors simultaneously record the backscattering signal thus giving some excessive information on the Stokes parameter I. The main advantage of such a lidar is the limiting temporal resolution governed by the repetition rate of sounding pulses.

If temporal resolution is not a decisive factor, sequential analysis of the polarization state of backscattered signals becomes acceptable. This solution is used in lidars with a significant power potential. The increase in the number of receiving channels of such lidars is rather difficult and expensive. Such a lidar analyzes the polarization state in its receiving system by setting the azimuth of the prism analyzer in the positions corresponding to Eqs. (5a)–(5b) and (5c) and recording the signals in turn. Condition (5e) is fulfilled be inserting a phase plate. Application of the prism, which separates the components with orthogonal polarization, requires the use of two photodetectors rigidly connected to it. Fast and accurate change of the azimuth of such a unit is a technically complicated problem. The use of an analyzer, which separates out a single linearly polarized component, requires the change of the azimuth of only the analyzer rather than the entire unit. Technically, this is much simpler especially as the repetition rate of the sounding pulses increases. Insertion and removal of the phase plate is also easy to do because the plate has light weight. The frequency of measurements with such lidars becomes multiple of the repetition rate of sounding pulses. Other versions of the structure of polarization lidars do no need a discussion because they are not fundamentally different.

Conclusion

A brief analysis of elements of a polarization lidar presented in this paper lays the grounds for further systematization of polarizers and analyzers applied in lidars. The matrix description of the procedure of measurement of the Stokes parameters is given for different combinations of the instrumental vectors of the receiving system. The methods to obtain different polarization states of sounding radiation with external polarizers set in the lidar transmitting system are presented. Principal characteristics of the active polarization elements used as analyzers and phase-shifting plates are listed along with the criteria of efficiency of their usage. Some versions of the design of the analyzing part of the lidar receiving system are considered for both simultaneous (in parallel in several channels) and sequential (in a single receiving channel) measurement of the Stokes parameters which are used for determination of the elements of the backscattering phase matrix of the medium under study. Some critical points to be kept in mind when designing polarization lidars are noted.

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