# Transformation of the distribution law of the intensity of laser radiation fluctuations when changing the parameters of the corner-cube reflector array 

A.P. Rostov and O.A. Rubtsova<br>Institute of Atmospheric Optics, Siberian Branch of the Russian Academy of Sciences, Tomsk Received December 28, 1998


#### Abstract

We have studied experimentally the transformations of the probability distribution of the spherical wave intensity fluctuations reflected from an array of corner-cubes. It is shown, that as the spacing between the corner-cubes increases, the probability density approaches the lognormal distribution.


The principal characteristics of optical radiation in a turbulent medium have been studied by now quite well, including those in the region of strong intensity fluctuations, which is characterized by the saturation of the relative variance of fluctuations. At the same time, regardless of the fact that a lot of papers ${ }^{1-3}$ have been published on this problem and different models of the distribution density of saturated intensity fluctuations were proposed, the question is still open on the form of the intensity fluctuation distribution function under saturation conditions, when the multiple scattering effects dominate.

It is conventionally accepted that the scattered field is Gaussian in the range of saturation, and the probability distribution of the intensity fluctuations approaches the exponential form:

$$
\begin{equation*}
P(I)=<I>^{-1}-\exp (-I /<I>) . \tag{1}
\end{equation*}
$$

The conclusion on the applicability of the exponential distribution was also drawn on the basis of analysis of the normalized moments of the intensity 4 :

$$
<I^{n>} /<I>n=n!\left[1+0.21 \beta_{0}^{-4 / 5} n(n-1)\right]
$$

Here $\quad \beta_{0}^{2}=1.23 C_{n}^{2} k^{7 / 6} L^{11 / 6} \quad$ is the parameter characterizing the conditions of propagation along the path ( $C_{n}^{2}$ is the structure characteristic of the air refractive index; $k=2 \pi / \lambda$ is the wave number; and $L$ is the path length). In the limiting case $\left(\beta_{0} \rightarrow \infty\right)$ the expression (2) leads to the ratio between the moments, which corresponds to the exponential distribution. However, the exponential distribution was not obtained experimentally while, on the contrary, experimental data ${ }^{5}$ exhibit the distribution to be close to the lognormal one at $\beta_{0} \geq 25$ :

$$
\begin{aligned}
& P(I)=(\sqrt{2 \pi \sigma} I)^{-1} \exp \left[-\left(1 / 2 \sigma^{2}\right)(\ln I-\xi)^{2}\right] ; \\
& \sigma^{2}=\ln \left(1+\beta^{2}\right), \quad \xi=\ln \left[<I>/\left(1+\beta^{2}\right)^{1 / 2}\right],
\end{aligned}
$$

where $\left.\left.\beta^{2}=\left(<I^{2}\right\rangle-\langle I\rangle^{2}\right) /<I\right\rangle^{2}$ is the relative variance of the intensity fluctuations.

Deviation from the Gaussian statistics of the scattered field is explained by the fact ${ }^{1,3}$ that its components correlate only partially due to the influence of large inhomogeneities. Earlier ${ }^{6,7}$ we have already studied $K$-distribution ${ }^{2}$ proposed as a model of nonGaussian statistics of the scattered field:

$$
\begin{align*}
<I>P(I)= & (2 / \Gamma(y)) y^{(y+1) / 2} I^{(y-1) / 2} \times \\
& \times K_{y-1}\left[2(I y)^{1 / 2}\right] ;  \tag{4}\\
y & =2 /\left(\beta^{2}-1\right), \quad y>0 .
\end{align*}
$$

Our experimental data have confirmed the possibility of using such a distribution in describing the probability density of the intensity fluctuations in the following cases: 1) at the direct propagation under conditions of strong fluctuations; 2) at the reflection from an array of corner-cube reflectors. The $K$ distribution asymptotically tends to the exponential one as the parameter $\beta$ increases.

In this paper we try to follow up the evolution of the experimentally obtained histograms from the $K$-distribution to the exponential one or any another limiting distribution. Our reasoning is as follows. At a multibeam regime of the wave propagation not only one but several beams with different initial coordinates come to the same point of a space. If the number of such independent channels is large enough ( $\geq 12$ ), then the scattered field obeys the Gaussian statistics. In the case when the illuminated area is comparable or less than the spatial correlation of the field fluctuations, the mean number of independent channels is small, and the statistics of scattered field is not Gaussian, and the probability density in our case approaches the $K$ distribution.

This fact also explains the non-Gaussian statistics obtained at the reflection of a spherical wave from an array of corner-cube reflectors. ${ }^{6}$ After the reflection from the array of corner-cube reflectors, the field of the directed spherical wave is a superposition of partial waves coming from separate reflectors. Regardless the fact that the number of reflectors in the experiment was quite large (12), correlation of the field fluctuations still exists at the distance of the order of the array size, so the mean number of independent propagation channels contributing to the recorded field is less than the number of corner-cubes. We tried to obtain the limiting distribution by moving the elements of the array to the distance greater than the correlation radius of the wave field, in order to make the beams coming from individual corners to be independent.

Measurements were carried out in August and September 1998, in the afternoon or almost at noon along the horizontal path above the flat underlying surface. Radiation of a $\mathrm{He}-\mathrm{Ne}$ laser $(\lambda=0.63 \mu \mathrm{~m})$ was directed to the reflector through the diaphragm of 1 mm diameter. The reflector was placed 1000 m far from the source. The 2D array made of 13 prism corners (Fig. 1) was used as the reflector. The diameter of one corner was 26 mm . The distance between the cornercube reflector centers $\Delta$ changed from 26 (array) to maximum of 208 mm . The reflected radiation was recorded with a FEU-79 PMT with the input diaphragm of 0.3 mm diameter at the distance of 5 mm from the optical axis of the wave.


Fig. 1. The array of the corner-cube reflectors.
Electric signal from the output amplifier of the PMT entered one of the channels of the digital recording hardware and software complex especially created for this experiment. After the low-frequency filtration with the low-frequency Baterwart filter of 8th order (cut-off frequency of 1 kHz with the attenuation of 56 dB per octave), it was transformed to the digital form with a 12-digit analog-to-digital converter with the accuracy and the linearity equal to a half of the lowest bit. Then the signal was recorded as a digital data flow to the RAM of the computer by the real-time software written on the ASSEMBLER for this experiment in order to obtain the maximum fast operation and accuracy of the discretization period. When sampling stoops, the acquired data array, of a little bit in excess of 9 Mbyte, is transferred from RAM to a hard disk. Such a recording technique reduces the probability of the instrumentation malfunctioning to
zero in contrast to the previous complex, where the error was about $10^{-6}$.

The signals were recorded at a 5 kHz discretization frequency during 5 minutes. Before and after each record the laser beam was chopped and the background illumination was measured, then the linear trend was subtracted from the recorded realization. Besides, the PMT signal was recorded in the absence of the reflector. When processing the data, the PMT noise was excluded by means of the convolution of the intensity fluctuation histogram with the PMT noise histogram, in order to obtain more certain probability values in the deep fading range:

$$
\begin{gathered}
I=I_{\mathrm{s}}-I_{\mathrm{n}} \\
f(I)=\int_{\infty}^{-\infty} f_{\mathrm{s}}\left(I_{\mathrm{s}}\right) f_{\mathrm{n}}\left(I_{\mathrm{s}}-1\right) \mathrm{d} I_{\mathrm{s}}
\end{gathered}
$$

where $I_{\mathrm{S}}$ is the recorded signal and $I_{\mathrm{n}}$ is the PMT noise.
The turbulent state of the atmosphere was checked by the intensity fluctuations on the $100-\mathrm{m}$ long path. Additional check up of the turbulence stationary state was performed by means of the ultrasonic anemometerthermometer placed 50 m far from the measurement van.

All in all, 17 measurement sessions have been carried out. Each series included 3-4 realizations with different arrangement of the corner-cube reflectors. The values of the parameter $\beta_{0}(L)$ were in the range 1 to 7 .

Figures 2 and 3 show the characteristic histograms of the instantaneous values $\Delta$ under different turbulent conditions: a) for low and medium intensity values; b) for high intensity values. The model probability densities (1), (3), and (4) are also presented here for a comparison. As is seen from the data presented, the histogram at the values $\Delta=26 \mathrm{~mm}$ is well approximated by $K$-distribution for $I /<I>$ greater than the modal value. The deviation is quite noticeable for $I /<I>$ values below the mode. However, this region of deep fading does not significantly affect the values of the higher moments. If such details as the deep fading statistics are not of special interest, the $K$-distribution can be accepted as a model for the case of small $\Delta$. As $\Delta$ increases, the histogram is transformed to a more symmetric shape, and at the maximum $\Delta=208 \mathrm{~mm}$ for $\beta_{0}(L)=1$, and $\Delta=120 \mathrm{~mm}$ for $\beta_{0}(L)=7$, the lognormal distribution better agrees with the experimental data than the exponential one.

The latter fact also results from the dependence of the higher normalized moments on the second normalized moment of the intensity (Fig. 4). The curves are also presented here that correspond to the moments of the distributions (3) and (4) calculated taking into account the displacement due to the limitation of the dynamic range under the experimental conditions ( $I_{\max }=4095,\langle I\rangle=75$ ). 8,9 As $\Delta$ increases, the experimental moments deviate from the $K$ distribution moments and approaches the lognormal dependence.


Fig. 2. Histograms of the normalized intensity values for $\beta_{0}(L)=1 . \Delta=26 \mathrm{~mm}$ (1); 36 mm (2); 208 mm (3); $K$-distribution corresponding to the histogram 1 (4); lognormal distribution corresponding to the histogram 3 (5); exponential distribution (6).



Fig. 3. Histograms of the normalized intensity values for $\beta_{0}(L)=7 . \Delta=26 \mathrm{~mm}$ (1); 36 mm (2); 120 mm (3); $K$-distribution corresponding to the histogram 1 (4); lognormal distribution corresponding to the histogram 3 (5); exponential distribution (6).


Fig. 4. Normalized moments of the intensity of 3rd, 4th, and 5th orders: $\Delta=26 \mathrm{~mm}$ (1); 36 mm (2); 120 mm (3); 208 mm (4); moments of the lognormal distribution (5); moments of the $K$-distribution (6).

Thus, the experiment has shown that as the number of independent channels increases, the probability density of the laser beam intensity fluctuations in the turbulent atmosphere does not approach the exponential distribution, as it was supposed, but tends to the lognormal distribution.

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