ESTIMATION OF THE EFFICIENCY OF ASPIRATION OF AEROSOL PARTICLES IN THE TURBULENT ATMOSPHERE

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The process of aspiration of aerosol particles in the atmosphere is considered for the case, when typical linear scale of turbulence is much larger than typical diameter of a sampler inlet. Estimations of the efficiency of aspiration of aerosol particles in the turbulent atmosphere have been made for the surface atmospheric layer using known empirical equations for the laminar flows. We used the method of modeling the series of the instant values of wind velocity pulsation and pulsation of concentration with a preset distribution laws and correlation in calculations. It is shown that the corrections for the atmospheric turbulence are, as a rule, insignificant. Only in some cases, usually with high permissible Stokes numbers and intensity of concentration pulsation, they can reach 20%.

Determination of the efficiency of aspiration of aerosol particles in a turbulent medium is of a certain practical importance. At the same time, the number of papers devoted to this study is too small as compared with a large number of papers devoted to the solution of the problem on aspiration under conditions of laminar flows. Experimental investigations into the process of aspiration in the turbulent atmosphere have been carried out only under laboratory conditions (see, for example, Ref. 1). On the basis of this very poor information, the opinion has been formed that the influence of turbulence on the efficiency of aspiration in the atmosphere is only little.[2] Thus, we can conclude that this problem is still to be addressed.

The process of aspiration in the atmosphere is mainly determined by the ratio between the typical linear scale of turbulence and the diameter of a sampler inlet pipe. If the linear scale of turbulence is much smaller than the diameter of a sampler inlet, then turbulent medium can be considered as a fluid one consisting of small eddies sucked in into the inlet pipe.[3] The estimations done using the similarity theory for the surface atmospheric layer, give, for the height of 2 m above the underlying surface, that typical linear size of turbulent perturbation (displacements) of 1 m (Ref. 4). Typical diameter of the sampler inlet is, as a rule, 1 cm. Thus, the method considered in Ref. 3 is unacceptable for estimating the aerosol aspiration efficiency in the atmosphere.

In the other limiting case, when the linear scale of turbulence much exceeds the diameter of the sampler inlet, the running-on flow can be considered as chaotically changing its velocity vector flow.[3] Therefore the aspiration coefficient[2,3] keeps its meaning, but only for the instant values of the flow velocity vector. It is just this case which is of practical importance when estimating the aspiration efficiency in the turbulent atmosphere.

Let the sampler inlet be round-shaped with the radius R, straight, and have sharp edges. Let the increment of the number of aerosol particles dn, crossing the sampler inlet section during the time interval dt, be equal to $\mathrm{d}n = Q\,\eta_a\,C\,\mathrm{d}t$, where Q is the air flow rate; $Q = \pi R^2 U_s$; U_s is the average flow velocity in the tube; η_a is the aspiration coefficient for aerosol particles; C is the instant value of the admixture concentration.

The aspiration coefficient depends on the following parameters[2,3]: the absolute value of the wind velocity U, the angle between the flow velocity vector and the sampler inlet pipe axis, q, the average flow velocity in the pipe, the pipe radius, the aerodynamic diameter of aerosol particles D, as well as their density r$_\mathrm{p}$; $\eta_a = \eta_a(U, \theta; U_s, R; D, \rho_p)$. The first couple of these variables characterizes the flow, the second one characterizes the sampler, and the last one characterizes aerosol particles.

Mathematical expectation of the number of particles sampled during the time t is:

$$\bar{n} = Q \int_0^t \overline{\eta_a\,C}\,\mathrm{d}t_1 . \qquad (1)$$

The bar above the expression denotes the operation of averaging over the statistical ensemble. Let us introduce the representation of instant values as a sum of averaged values and pulsation: $\eta_a = \bar{\eta}_a + \hat{\eta}_a$, $C = \bar{C} + \hat{C}$, etc. After transformations, with the account for the stationarity of the variation processes of aerosol concentration and wind velocity vector, we can obtain the following equation for the measured concentration:

$$\bar{C}_m = \eta_t \bar{C} \; ; \quad \eta_t = \bar{\eta}_a + \overline{\hat{\eta}_a \frac{\hat{C}}{\bar{C}}} \; , \tag{2}$$

where $\bar{C}_m$ is the averaged value of measured concentration.

It follows from Eq. (2) that in the general case h_t can be found provided that one knows the dependence of the aspiration coefficient η_a on the instant values of wind velocity components $\eta_a = \eta_a(U_x, U_y, U_z)$ and that the common distribution function of instant wind velocity pulsation components and instant concentration of aerosols is known. Unfortunately, the exact determination of such a function is impossible because of the closure problem in application to the averaged equations of the theory of turbulence.[4] Let us focus our attention on the method of estimation of h_t, that uses simulation of these random processes. Such a procedure accounts for the distribution laws of instant pulsations of wind velocity components[5] and aerosol concentration,[6] as well as the correlation between the random values considered. Let h_t be calculated by averaging the terms in the right-hand side of Eq. (2), the instant values of which are determined on the basis of the modeled series $\hat{U}_x$, $\hat{U}_y$, $\hat{U}$, and $\hat{C}/\bar{C}$.

These series are modeled using the method described in Ref. 7. To do this, it is necessary to set the following parameters: the intensity of concentration pulsation $I_c = \sigma_c/\bar{C}$, where $\bar{C}$ is the standard deviation of the concentration pulsation; the turbulent flows of the admixture are φ_x, φ_y, φ_z (those are proportional to the correlation coefficients of the concentration pulsations and pulsations of the wind velocity components r_{xc}, r_{yc}, r_{zc}), as well as the components of the Reynolds tensor of the viscous stress (those are proportional to the correlation coefficients of pulsations of the wind velocity components r_{xy}, r_{xz}, r_{yz}).

The total number of parameters for this problem is quite large. Therefore, we consider only some examples that are characteristic of sampling in the surface layer of the atmosphere under conditions of neutral temperature stratification. Let the axis X be directed along the vector of average wind velocity, the axis Y be perpendicular to the axis X in the horizontal plane, and the axis Z be directed upwards. Let us set the characteristics of the turbulence according to the theory of the surface atmospheric layer.[4] Let the roughness parameter of the underlying surface be equal to 0.014 m, and the friction rate be 0.2 m/s. As a result, the average wind velocity at the height of 4 m is equal to $\bar{U}_x = 4.24$ m/s, and standard deviations of the velocity pulsations are equal to: $\sigma_x = 0.48$ m/s, $\sigma_y = 0.34$ m/s, and $\sigma_z = 0.16$ m/s. In addition, we obtain $r_{xy} = 0$, $r_{xz} = -0.52$, and $r_{yz} = 0$. Let $R = 0.005$ m. If Q = 10, 20, and 40 l/min; then $\psi = \bar{U}_x/U_s$ = 2.0, 1.0, and 0.5.

Let us characterize the size of the aerosol particles, as accepted in the theory of aspiration, by the Stokes number (St). Its instant value $\mathrm{St} = U\tau/(2R)$, where $U^2 = (\bar{U}_x + \hat{U}_x)^2 + (\hat{U}_y)^2 + (\hat{U}_z)^2$; τ is the relaxation time of aerosol particles.[2,3] Below we also use the parameter $\mathrm{St}_0 = \bar{U}_x\tau/(2R)$.

There is no exact equation for the dependence of the aspiration coefficient on the parameters it depends on. We use the empirical relations,[8] that generalize the results of Ref. 9. The equation for coefficient h_a in the range $0° \le \theta \le 60°$ has the following form:

$$\eta_a = 1 + (\xi \cos q - 1) \times$$
$$\times \frac{1 - [1 + (2 + 0.617\,\xi^{-1})\,\mathrm{St}_1]^{-1}}{1 - (1 + 2.617\,\mathrm{St}_1)^{-1}} \times$$
$$\times \{1 - [1 + 0.55\,\mathrm{St}_1 \exp(0.25\,\mathrm{St}_1)]^{-1}\} \; ;$$
$$\xi = \frac{U}{U_s} \; ; \tag{3}$$
$$\mathrm{St}_1 = \mathrm{St}\exp(0.022\,\theta) \; ; \quad 0.02 \le \mathrm{St} \le 4.0 \; ;$$
$$0.5 \le \xi \le 2.0 \; .$$

For angles from the range $45 \le \theta \le 900$, we use the following equation:

$$\eta_a = 1 + 3\,(\xi \cos\theta - 1)\,(\mathrm{St})^{\xi - 0.5}. \tag{4}$$

being valid for $0.02 \le \mathrm{St} \le 0.2$ and $0.5 \le \xi \le 2.0$ (Ref. 8).

When using Eq. (4), we assume that the axis of the sampler inlet pipe is in the horizontal plane at an angle φ to the axis X. Then $\cos\theta = [(\bar{U}_x + \hat{U}_x)\cos\varphi + \hat{U}_y \sin\varphi]/U$. Obviously, Eq. (3) holds at $\varphi = 0°$.

The drawback of Eqs. (3) and (4) is that those do not transform continuously to each other as the angle θ changes from 45° to 60°, other conditions being the same. Moreover, in a number of cases, deviations exceed the 10-% level of the error in determination of η_a (Ref. 2). For example, if St = 0.2, $\psi = 2$, and $\theta = 45°$, the difference in values of the coefficient η_a calculated using Eqs. (3) and (4) is 30%.

The aspiration coefficient η_{av} was chosen as the parameter to be compared with the calculated values η_t. It has also been determined from Eqs. (3) and (4), where the values St_0 and $\bar{U}_x$ are substituted instead of the instant values St and U_x as well as corresponding values of the angle φ.

Calculations made for $\varphi = 0°$, $\varphi = 68°$; $\varphi_y \ne 0$, $\varphi_x = \varphi_z = 0$; $\varphi_z \ne 0$ and $\varphi_x = \varphi_y = 0$ in the entire range of the parameters I_c and ψ variation have shown that relative deviations of η_t from η_{av}, $\delta_1 = (\eta_t - \eta_{av})/\eta_{av}$, are smaller and often much smaller than 10-% level of the error in determination of the coefficient η_a by Eqs. (3) and (4).

Calculations made for $\varphi = 0°$, $\varphi = 68°$, $\varphi_x \ne 0$, $\varphi_y = \varphi_z = 0$ in the entire range of ψ variation at $I_c < 1$

allow us to conclude that relative deviations of h_t from η_{av} are also smaller than 10-% level of the error in determination of η_a using Eqs. (3) and (4). Analysis of the results obtained shows that the values δ_1 gradually increase from practically zero up to their maximum value as St_0 varies from its minimum to the maximum value.

As was established from calculations the deviations δ_1 that have the same order of magnitude as the errors of Eqs. (3) and (4) or above are observed only at: $\varphi_x \neq 0$, $\varphi_y = \varphi_z = 0$; $I_c > 1$; $0.5 \leq \psi \leq 2.0$; $\varphi = 0$ and $\varphi = 68°$. The calculated values of δ_1 are given in the Table I. We can see that the maximum value of δ_1 does not exceed 20%. It is seen that at $\varphi = 68°$ and $1.0 < \psi < 2.0$ the deviation δ_1 changes its sign, i.e. in this range there is the value ψ where $\delta_1 = 0$.

TABLE I. Normalized deviations δ_1 and δ_2, that are outside the limits of the relative errors of the equations for aspiration index.

		$I_c = 1$			$I_c = 2$		
		$\bar{U}_x/U_s$					
Angle, degs.	St_0	2.0	1.0	0.5	2.0	1.0	0.5
$\varphi = 0$	0.99	*	*	*	0.09/0.09	*	*
	1.33	*	*	*	0.10/0.11	*	*
	1.80	*	*	*	0.11/0.11	*	*
	2.42	0.09/0.09	*	*	0.12/0.12	0.09/0.09	*
	3.27	0.09/0.09	*	*	0.13/0.12	0.09/0.10	*
	4.41	0.09/0.09	*	*	0.13/0.12	0.09/0.10	0.09/*
	5.95	0.09/0.09	*	*	0.13/0.12	0.09/0.10	0.10/0.09
$\varphi = 68$	0.19	*	*	*	*	*	−0.09/−0.09
	0.22	*	*	0.09/*	*	−0.10/−0.09	−0.11/−0.11
	0.25	*	−0.09/−0.09	−0.11/−0.09	0.09/*	−0.12/−0.11	−0.15/−0.14
	0.29	*	−0.11/−0.11	−0.14/−0.13	0.10/0.09	−0.15/−0.11	−0.19/−0.18

Comments: the values δ_1/δ_2 are given in the rows; * denote the case when to $|\delta_1| < 0.09$ or $|\delta_2| < 0.09$.

Analysis of contributions coming from the two terms that make up the quantity η_t (see Eq. (2)) is also of practical interest. Let us write the equation for the aspiration coefficient in the turbulent atmosphere in the following form $\eta_t = \bar{\eta}_a(1 + \delta_2)$. The obtained values of δ_2 are given in the Table. We can see that the contribution coming from the second term to h_t is small. Therefore, the second term in the equation for η_t can be neglected. In this case, η_t is equal to $\bar{\eta}_a$ by its order of magnitude. The Table also shows that the values δ_1 and δ_2 are practically equal to each other. From equations for δ_1 and δ_2 it follows that

$$\bar{\eta}_a = \eta_{a0} \, [(1 + \delta_1)/(1 + \delta_2)] \, .$$

Therefore, the approximate equality $\eta_t = \eta_{a0}$ is valid.

Based on the above said, we obtain quite simple method for estimation of the aspiration coefficient of aerosol particles in the turbulent atmosphere. Certainly, such a simple conclusion is not trivial, because it requires the use of quite a complicated method for modeling the pulsation series.[7]

Thus, the method for estimation of efficiency of aerosol particles aspiration in the turbulent atmosphere is proposed for the cases when the diameter of a sampler inlet pipe is much larger than typical linear scale of turbulence. Estimations of aspiration efficiency of particles in the surface atmospheric layer are obtained on the basis of known empirical equations for aspiration efficiency in laminar flows.

The calculated results for a number of particular cases show that the influence of atmospheric turbulence on the aspiration efficiency is weak. Therefore, for estimation of the mathematical expectation of concentration the classical relations for aspiration efficiency in the laminar atmosphere can be used where the average absolute value of wind velocity should be substituted with the account for its position relative the axis of the sampler inlet pipe.

However, one should not consider these conclusions absolute ones, because those have been drawn based on analysis of a very limited array of input data and using rather rough empirical equations for the aspiration efficiency. Nevertheless, modern numerical methods for solution of hydrodynamics equations allow one to obtain a solution of the exactly formulated three-dimensional problem of aspiration into a sampling pipe that is arbitrarily oriented with respect to the running-on laminar air flow with any desired degree of accuracy. Such calculations enable one to refine equations for estimation of the aspiration efficiency and to widen the range of their applicability. This will be the time to return to the method proposed here for obtaining more reliable estimations.

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