# INVESTIGATION OF THE CONTRIBUTION OF DIRECTION FLUCTUATIONS INTO THE GONIOMETER MEASUREMENTS USING LASER BEAMS ALONG THE "LAND-TO-SEA" PATHS 

G.A. Kaloshin and V.V. Nosov<br>Institute of Atmospheric Optics, Siberian Branch of the Russian Academy of Sciences, Tomsk Received December 30, 1996


#### Abstract

The paper presents some results of theoretical and experimental investigations of regular and random refraction of laser beams when determining the spatial position of sea ships in the regime of measuring directions using the optoelectronic goniometer systems. The theoretical estimations are compared with the results of measurements of the beam rms deviations along the "land-to-sea" paths. Satisfactory agreement between theoretical estimates and experimental results is achieved when correctly choosing profiles of the underlying surface near the beam source.


## INTRODUCTION

The development of the present-day transportation facilities (TF) imposes heavy demands on the accuracy of determination of the spatial position, which is at present several tens of angular seconds. ${ }^{1}$ This requirement is due to the necessity of remotely measuring the directions in an automated mode. This problem is currently solved using the optoelectronic goniometer systems (OEGS) based mainly on the use of laser beams.

The requirements to OEGS are permanently increasing. Especially strict requirements are on the operation range, accuracy and the operation rate. In addition, these systems should be convenient and easy in operation. The simplest technical solutions to increase the accuracy and operation rate are connected with the availability of two practically equivalent information carriers, namely, laser beams, giving the most effective connection with a TF in the indicator mode, and an electric signal allowing realization of all the necessary logical and calculational operations.

Thus, in Refs. 1 and 2 a technical solution to this problem is proposed in the form of a line-scan conversion unit for a successive scanning of all parts of the orientation sector and isolation onboard a TF of the information signal with the subsequent determination of the azimuthal displacements. Photometric functions of the device ${ }^{1}$ are based on converting the measured linear coordinate $x$ in the information sector into a corresponding time interval $\Delta t$. Two laser beams are shaped using a coastal device. These laser beams rotate in space at different angular velocities $\omega_{1}$ and $\omega_{2}$ with the use of deflectors. In this case the zeroth spatial direction is denoted as the line of intersection of the abovementioned angles rigidly bound with the coastal device (CD) located at a "passive" direction keeper with a maximum error of 5 to $15^{\prime \prime}$ over a long period of time (one year and more).


FIG. 1. Diagram of OEGS: the coastal device (1), the photoreceiving device (2), and processing and indication block (3).

Time interval $\Delta t$ between the optical signals uniquely determines the azimuth position of a TF and the directional to it. The installation of an additional CD makes it possible to solve the navigation problem. Thus, the measurement process is the sequence of conversions $\alpha=\alpha[\Delta t(m)]$, where $\Delta t=t_{2}-t_{1} ; t_{1}$ and $t_{2}$ are the moments of passage of the reference and information laser beams through a photoreceiving device (PRD); $m$ is the number of pulses arriving at the PRD

The method, proposed in Ref. 1, makes it possible to increase $m$ (measurement frequency) up to
$25-30 \mathrm{~Hz}$. This enables one to record the running value of the angular drift of a TF within certain accuracy.

For a successful operation of the up-to-date OEGS a detailed account is needed of the components making up the sum measurement error $\alpha$. The rms deviation $\sigma_{\Sigma}$ when measuring $\alpha$ is determined as the sum of the rms deviation (RMSD) due to the instrumental error $\sigma_{i}$, and RMSD contributed by the turbulent atmosphere, $\sigma_{a}$ due to the lateral regular and random shifts of the laser beam,
$\sigma_{\Sigma}=\sqrt{\sigma_{\mathrm{i}}^{2}+\sigma_{\mathrm{a}}^{2}}$.
The primary goal of this paper was the theoretical and experimental investigation of $\sigma_{a}$ above the underlying surface of the "land-to-sea" type.

The paper presents some results of theoretical estimates of the lateral refraction and variance of random shifts of the laser beam performed for the paths over the ground and sea surface for zenith angles $90^{\circ}$, $87.5^{\circ}, 85^{\circ}$ and $75^{\circ}$ for the radiation source location under conditions of coastal mountainous country. The results of calculation are compared with the experimental data obtained using OEGS at the distances up to 5 km .

## 1. THEORETICAL ESTIMATES

### 1.1. Regular lateral refraction

Experimental investigations ${ }^{3}$ have shown that vertical gradients of air density are, on the average, by two orders larger than the horizontal gradients. The refraction angles in the vertical plane are by two orders larger than the refraction angle in the horizontal plane. For the conditions of standard atmosphere the refraction in the vertical plane does not exceed several angular minutes at a distance up to 100 km , and the lateral refraction in this case does not exceed one second. Only in some extraordinary cases, when a laser beam propagates close to the walls heated by the sun under urban conditions, the values of lateral refraction up to $20^{\prime \prime}$ were recorded. ${ }^{3}$

The lateral refraction is usually calculated by the expression ${ }^{3}$
$r_{1}=P L /\left(T^{2} \cos \alpha\right) \tan \tau\left(k_{1}+k_{2} \gamma\right) \sin (A-Q)$,
where $k_{1}=0.28 ; k_{2}=8.1 ; p$ is the air pressure in mb ; $T$ is the absolute temperature at the observation point; $\gamma$ is the vertical gradient of temperature (degs $/ \mathrm{m}$ ); $\tau$ is the angle of inclination of the layers having the same index of refraction to the horizontal plane or the angle, calculated from the zenith point to the vector $\nabla n$, directed toward the decrease of the index of refraction $n$; $A$ and $Q$ are the azimuths of the observed direction and the vector $\nabla n ; L$ is the distance; $\alpha$ is the elevation angle of the observed source.

The inclination of layers of equal index of refraction, due to the horizontal gradients, can be calculated by the expression
$\tau=-2.068 \cdot 10^{3} \Gamma+58.85 G$,
where $\Gamma$ is the horizontal gradient of temperature (degs $/ \mathrm{m}$ ); $G$ is the horizontal gradient of pressure $(\mathrm{mb} / \mathrm{m})$, being on the average $(1-5) \cdot 10^{-5} \mathrm{mb} / \mathrm{m}$. The values of $\Gamma$ above the water surface at 2 m height, as a rule, are $\pm 1.5 \cdot 10^{-4}$ degs $/ \mathrm{m}$ and decrease sharply with the increase of the beam height.

Let us now determine, using Eqs. (1) and (2), the value of $r_{1}$ for the distance $L=30 \mathrm{~km}$. For some average conditions of (standard) atmosphere we have: $T=280 \mathrm{~K}, \quad p=1000 \mathrm{mb}, \quad \Gamma=+10^{-4} \mathrm{degs} / \mathrm{m}$, $G=-5 \cdot 10^{-5} \mathrm{mb} / \mathrm{m}, \quad \gamma=0.006 \mathrm{degs} / \mathrm{m}, \quad A-Q=90^{\circ}$, and $\alpha=1^{\circ}$. Then $\tau=-0.21^{\circ}$, and $r_{1}=-0.46^{\prime \prime}$. From Eq. (1) we also see that $r_{1}$ decreases proportionally to the decrease of $L$.

### 1.2. Random refraction

In the atmospheric boundary layer the turbulence structure is determined by the Monin-Obukhov similarity theory. ${ }^{4}$ The parameter $C_{n}^{2}$ characterizes the intensity of turbulent pulsations of the index of refraction and is connected with the meteorological parameters by the relationships:
$C_{n}^{2}=\left[\left(79 P / T^{2}\right) \cdot 10^{-6}\right]^{2} C_{T}^{2}, P[\mathrm{mb}], T[\mathrm{~K}]$,
$C_{T}^{2}=c^{2} \alpha^{2}\left(R_{i}\right)\left(\varkappa_{k} h\right)^{4 / 3}\left(\frac{\partial T(h)}{\partial h}\right)^{2}, R_{i}=\frac{g}{T} \frac{\partial T / \partial h}{(\partial u / \partial h)^{2}}$,
where $g$ is the acceleration of gravity; $p$ and $T$ are the pressure and temperature; $\partial T / \partial h$ and $\partial u / \partial h$ are the altitude gradients of temperature and wind velocity; $x_{k}=0.4 ; c^{2}=2.8$. The plot of $\alpha^{2}\left(R_{i}\right)$ may be found in Ref. 5. The distribution of $C_{n}^{2}$ values near a flat underlying ground surface is illustrated by Table I. ${ }^{7}$

TABLE I. Values of $C_{n}^{2}$ over steppe at $h=2.5 \mathrm{~m}$.

| Time | Range of most <br> probable values, <br> $C_{n}^{2}, \mathrm{~m}^{-2 / 3}$ | Probability <br> of being in the <br> range |
| :--- | :---: | :---: |
| Daytime | $5.4 \cdot 10^{-14}-5.4 \cdot 10^{-13}$ | 84 |
| Nighttime | $5.4 \cdot 10^{-15}-5.4 \cdot 10^{-14}$ | 69 |
| Evening, morning | $5.4 \cdot 10^{-16}-5.4 \cdot 10^{-15}$ | 66 |

The typical range of values of $C_{n}^{2}$ during a summer day under clear sky conditions is determined by the inequality ${ }^{5}$ :
$10^{-17} \mathrm{~m}^{-2 / 3} \leq C_{n}^{2} \leq 10^{-13} \mathrm{~m}^{-2 / 3}$.
During the daytime because of the difference in heating of the dry land and sea surface the values of
$C_{n}^{2}$ close to the sea surface are usually less than those near the dry land surface.

In the estimates we used an average model of the altitude profile $C_{n}^{2}(h)$,
$C_{n}^{2}(h)=C_{n_{0}}^{2}\left(h / h_{0}\right)^{-2 / 3} \mathrm{e}^{-h / \hbar}$,
$C_{n_{0}}^{2}=C_{n}^{2}\left(h_{0}\right), \hbar=3200 \mathrm{~m}$,
where $h_{0}$ is the altitude of the measurement source over the underlying surface. For the most probable values of $C_{n_{0}}^{2}$ this model provides a good agreement with the experimental data. 6,7

In accordance with Ref. 8 the rms deviation of the laser beam sighting axis $\sigma_{\mathrm{a}}$ for horizontal and oblique paths was calculated by the following formula:
$\sigma_{\mathrm{a}}^{2}=4.1 L a^{-1 / 3} \int_{0}^{1} \mathrm{~d} \xi(1-\xi)^{2} C_{n}^{2}(h(\xi L)) \times$
$\times\left\{C^{-1 / 3}(\xi L)-\left[C^{2}(\xi L)+\beta(h(\xi L))\right]^{-1 / 6}\right\}$.
Here $\quad h(\xi L)=\sqrt{\xi^{2} L^{2}+\left(R+h_{0}\right)^{2}+2\left(R+h_{0}\right) \xi L \cos \theta}-R$ is the running height of the path point; $\xi L$ is the distance from a running point of the path to the emitter; $h(0)=h_{0} ; R$ is the Earth's radius; $\theta$ is the receiver zenith angle at which the receiver is visible from the source location; $0 \leq \theta \leq \pi / 2$, $\beta(h)=2[0.4 h /(2 \pi a)]^{2} ; a$ is the radius of the source emitting aperture; $L$ is the path length; $C(\xi)=$ $=\left[1+(\xi L / a) \tan \left(\varphi_{0} / 2\right)\right]^{2}+\Omega^{-2} \xi^{2}+8 \sigma^{12 / 5}(\xi) \Omega^{-1} \xi^{16 / 5}$, $\Omega=k a^{2} / L ; \sqrt{O(\xi)}$ is the running, along the path, mean laser beam radius normalized to its value at the source
output $(C(0)=1) ; \varphi_{0}$ is the full angle of the initial divergence of an optical beam; $k=2 \pi / \lambda$ is the wave number, $\sigma^{2}(\xi)=0.82 k^{7 / 6} L^{11 / 6} \int_{0}^{1} \mathrm{~d} t(1-t)^{5 / 3} C_{n}^{2}(h(\xi t L))$.

Based on the algorithm (3) the angular variances of the laser beam random shifts $\sigma_{a}$ were estimated with the account of the path passing over the ground and sea surface for different source heights (coastal equipment) and photoreceiving device on a TF and taking into account the altitude profile of $C_{n}^{2}$.

The estimates were tested using the following parameters characterizing the source, the path geometry and the structure characteristic of $C_{n}^{2}$ fluctuations, namely, the source emitting the laser beam of 4,5, $6^{\circ}$ divergence; and 1 mm radius, at the wavelength of $0.63 \mu \mathrm{~m}$. The path geometry enabled the distances over the sea $\left(L_{\mathrm{s}}\right)$ of $5 ; 10 ; 15 ; 20 ; 25 ; 35 \mathrm{~km} ; 10 ; 20 ; 50$; $100 ; 200 \mathrm{~m}$ over the land $\left(L_{1}\right)$. The source height $\left(H_{\mathrm{s}}=h_{0}\right)$ and receiver height $\left(H_{\mathrm{r}}\right)$ are $2.5 ; 5 ; 10 ; 20$; 30 m ; the zenith angle is $90^{\circ}$. The structure characteristics of the dielectric constant over the sea is $C_{n}^{2}=10^{-14} ; 5 \cdot 10^{-15} ; 10^{-15} \mathrm{~m}^{-2 / 3}$.

Below a portion of the table is presented (see Table II) of the variance of the laser beam random shifts $\sigma_{\mathrm{a}}^{2}=\sigma_{\mathrm{s}}^{2}+\sigma_{1}^{2}$, where $\sigma_{\mathrm{s}}^{2}$ and $\sigma_{1}^{2}$ are the values of the variances for segments of the path over the sea and the land, respectively. Here, as an illustration, some values of $\sigma$ are given for the divergence $4^{\prime}$, the beam radius 1 mm and the value $C_{n}^{2}=10^{-13}-10^{-15} \mathrm{~m}^{-2 / 3}$, being most suitable to the conditions of experimental tests, given in Table II as a fraction. The numerator corresponds to the zenith angle of $90^{\circ}$, and the denominator corresponds to the zenith angle of $87.5^{\circ}$.

TABLE II. The part of table of dispersions of a laser beam depending the path geometry, beam parameters and the medium.

| $\lambda=0.63 \mu \mathrm{~m}$,beam divergenceis $4^{\prime}$, beam radiusis 2 mm,$C_{n_{0}}^{2}=10^{-13}-10^{-15}$,$\mathrm{m}^{-2 / 3}$ |  |  | $\sigma_{1}^{2} \cdot 10^{10}, \operatorname{rad}^{2}$ |  |  |  |  |  | $\sigma_{\mathrm{s}}^{2} \cdot 10^{10}, \mathrm{rad}^{2}$ |  |  |  |  |  | $\sigma_{a} \cdot 10^{5}, \mathrm{rad}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $L_{1}, \mathrm{~m}$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $H_{\mathrm{S}}, \mathrm{m}$ | $H_{\mathrm{r}}, \mathrm{m}$ | $L_{\text {s }}$, km | 5 | 10 | 15 | 20 | 25 | 35 | 5 | 10 | 15 | 20 | 25 | 35 | 5 | 10 | 15 | 20 | 25 | 35 |
|  | 3 | 200 100 50 20 10 | $0.0$ |  |  |  |  | $(0.35$ |  |  |  |  |  | $0.67$ |  |  |  |  |  |  |
|  |  | 200 100 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

For zenith angles less than $90^{\circ}$ the value $\sigma_{\mathrm{a}}$ was estimated on the basis of the algorithm (3) for the following parameters characterizing the conditions of propagation.

The source parameters:
the radiation wavelength is $0.63 \mu \mathrm{~m}$;
the beam divergence is $2^{\prime} ; 4^{\prime}$;
the beam diameter is $2 ; 5 \mathrm{~mm}$.
The path geometry
a) coastal region:
the distance over the sea is $1,4,10,20,35 \mathrm{~km}$;
the distance over land is $10,100,300 \mathrm{~m}$;
b) land:
the distance over the land is $1,4,10,20,35 \mathrm{~km}$; the zenith angle is $87.5^{\circ} ; 85^{\circ} ; 75^{\circ}$;
c) sea:
the distance over the sea is $1,4,10,20,35 \mathrm{~km}$;
the zenith angle is $87.5^{\circ} ; 85^{\circ}$; $75^{\circ}$;
the source height is 1.5 m .
The value of $C_{n}^{2}$ :
over the land at 2.5 m height $-10^{-12} ; 10^{-14}$; $10^{-15} \mathrm{~m}^{-2 / 3}$;
over the sea at 2.5 m height $-10^{-14} ; 5 \cdot 10^{-14}$; $10^{-15} \mathrm{~m}^{-2 / 3}$

In this case the results of calculations of $\sigma_{a}$ are also presented in the tables similar to Table II, with the difference that the columns $m_{\mathrm{s}}$ and $m_{\mathrm{r}}$ are replaced by the columns "divergence" and "source radius" and the column "zenith angle" is added.

## 2. EXPERIMENTAL MEASUREMENTS OF $\sigma_{a}$

The idea of the experiment is based on the indirect measurements of the variance of random shifts of a laser beam $\sigma_{a}^{2}$ and is as follows. The rms deviations were determined when measuring the $\sigma_{\Sigma}$ directions to CD using a TF, whose position was controlled by independent methods with a high precision. The CD position was also fixed with a high precision. Owing to a considerable bulk of data on $\sigma_{\mathrm{i}}$, obtained during static tests in climatic chambers, the value of $\sigma_{a}$ was determined as
$\sigma_{a}=\sqrt{\sigma_{\Sigma}^{2}-\sigma_{i}^{2}}$.
This is equivalent to a prompt measurement of the beam center of gravity coordinates on a plane screen in the immediate vicinity of PRD. The coastal device with the lasers of LGN104 with the radiation wavelength $0.63 \mu \mathrm{~m}$ and the output power of 40 mW OEGS was positioned on a fixed "massive" keeper of direction with the rms error of $5^{\prime \prime}$ during a long period at 10 m hight. The PRD of OEGS was located on a TF, whose position was controlled with a high precision by two independent methods, namely, using a stationary phase radio range finder GRAS ${ }^{9}$ with the rms error of positioning, being equal to 0.7 m at a distance of $0.2-60 \mathrm{~km}$, and by means of two 1 km
spaced range finders of the ST-5 "Blesk" type with the rms error of distance measurement, not exceeding 0.20 m at a distance of $0.002-5 \mathrm{~km}$. The PRD was focused on the CD and during $10-15 \mathrm{~min}$ at a frequency 30 Hz the values of $\sigma_{\Sigma}^{2}$ were measured. Multiple measurements have made it possible to remove the influence of sea waves and regular refraction. The measurement accuracy of $\sigma_{\mathrm{a}}$ was $3 \cdot 2 \cdot 10^{-5} \mathrm{rad}$. Simultaneously the following meteorological parameters were controlled: the wind velocity and direction, temperature and temperature gradient. The PRD potentialities at the energy potential of CD available enabled us to operate at distances up to 5 km .

The results of generalized measurements of $\sigma_{\Sigma}$ for the two values of $C_{n}^{2}$ are given in Table III. The rms deviations of a beam shown in the table are due to the equipment $\sigma_{\mathrm{i}}$ at similar temperatures and the beam rms deviations, obtained in the experiment $\sigma_{a}^{e}$ and $\sigma_{a}^{c}$ calculated at the path parameters $L_{\mathrm{S}}=5 \mathrm{~km}$, $L_{1}=0.2 \mathrm{~km}, m_{\mathrm{s}}=10 \mathrm{~m}, m_{\mathrm{r}}=3 \mathrm{~m}$, corresponding to the measurement conditions of $\alpha$ and the beam radius being 2 mm .

TABLE III. Generalized measurement data on $\sigma_{\Sigma}$.

| $\sigma \cdot 10^{-5}, \mathrm{rad}$ | $C_{n_{0}}^{2}=5 \cdot 10^{-15} \mathrm{~m}^{-2 / 3}$, <br> $t=10^{\circ} \mathrm{q}$ | $C_{n_{0}}^{2}=10^{-15} \mathrm{~m}^{-2 / 3}$, <br> $t=18^{\circ} \mathrm{q}$ |
| :---: | :---: | :---: |
| $\sigma_{\Sigma}$ | 6.7 | 5.2 |
| $\sigma_{\mathrm{i}}$ | 2.7 | 2 |
| $\sigma_{\mathrm{a}}^{\mathrm{e}}$ | 6.1 | 4.8 |
| $=\sqrt{\sigma_{\Sigma}^{2}-\sigma_{\mathrm{i}}^{2}}$ |  |  |
| $\sigma_{\mathrm{a}}^{\mathrm{c}}$ | 2.5 | 1.8 |

## DISCUSSION

The results of measurements show that the proposed method of $\sigma_{a}$ measurements gives quite satisfactory results for the experiment in a turbulent atmosphere. As follows from the theoretical estimates, the basic contribution to the variance value of random shifts of the laser beam comes from the path segment adjacent to the source.

Therefore, a correct choice of the profile model of underlying surface close to the source makes it possible to decrease the error of theoretical estimates of $\sigma_{i}$. The contribution from regular lateral refraction is by an order of magnitude less than the value of the random refraction, and at goniometric measurements can be neglected.

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