ABOUT FRAUNHOFER'S AND FRESNEL'S DIFFRACTION BY A SLIT

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The validity is established of the formula describing the relation between the intensity maxima of the Fraunhofer diffraction pattern from a slit. The formula is derived from the Huygens principle. Moreover, the absence of the secondary waves, which are necessary for application of this principle, is observed in the experiment. The formula for the intensities of maxima in the Fraunhofer diffraction pattern from the slit in the shadow zone, based on the interference of edge rays, has been additionally verified. Based on the edge ray interference, the formula has been derived describing the intensities of maxima of the diffraction pattern from the slit in the focal plane of an objective in terms of the intensity of light incident on the slit as functions of the focal distance.

As known, light diffraction by an opening is tentatively derived into the Fresnel and Fraunhofer diffraction types. In case of the Fraunhofer diffraction observed by the naked eye, a plane wave is incident on the slit S (Fig. 1) and secondary waves coming at the observation point P are also plane. In this case, the condition[1] $k' = (b\cos\varphi)^2/4\lambda L \ll 1$ should be satisfied.

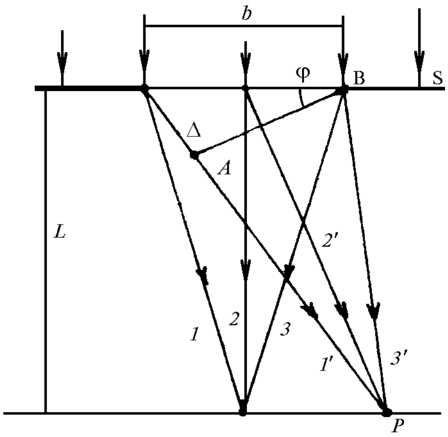

FIG. 1. Scheme of the Fraunhofer diffraction by the slit.

When the slit is illuminated by a linear source, the condition

$$k'' = b^2/4\lambda l \ll 1$$

should be satisfied in case of the Fraunhofer diffraction, where l is the distance from the source to the slit.

For $k' \ll 1$, the phases of elementary secondary waves coming at the point P from the entire surface of the slit are determined by the phase distribution in the plane AB perpendicular to the line connecting the center of the slit with the point[2] P.

As known, this circumstance in combination with identical amplitudes of the elementary waves, owing to practically identical tilts of these waves, simplifies a solution to the Fraunhofer diffraction problem, reducing it to a summation of the elementary secondary waves over the entire surface of the slit. In this case, the intensity distribution in the diffraction pattern is described by the formula[2]

$$I_\varphi = I_0\left[\sin^2\left(\frac{b\pi}{\lambda}\sin\varphi\right)\right] \Big/ \left(\frac{b\pi}{\lambda}\sin\varphi\right)^2, \quad (1)$$

where I_0 is the intensity of light coming from the slit in the direction of the incident beam.

Because I_0 is not the intensity of light incident on the slit and their relationship is unknown, this formula cannot be used to compare the calculated and experimental values of I_φ.

Experiments described in Refs. 3 and 4 showed that the wavefront surface is not the source of the secondary waves and the perturbation at the point P, located in the shadow zone, is determined by the interference of the edge waves coming from the opposite slit sides. However, the edge waves, according to the experimental data of Ref. 5, are emitted not only by the screen edge, as follows from Refs. 6–8, but also by the adjacent deflection zone as a result of light ray deflection on both sides of the incident light direction.

On these grounds, the formula for the diffraction intensity maxima in the diffraction pattern was derived in Ref. 4. It has the form

$$J_{\max} = 0.3274\,\lambda\,L\,b^2\,J_c/[(k_g\,\lambda\,L)^2 - 2\,b^4(1 + L/l)^2)], \quad (2)$$

where J_c is the intensity of incident light in the diffraction pattern plane at the shadow boundary for

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the remoted slit, being equal to the intensity of incident light at the slit edges when the incident wave is plane; k_g is the number of half-waves in the geometric path difference between the edge rays coming from the opposite slit sides.

In connection with the fact that already at the first moment edge ray *1'*, deflecting from the slit edge, runs ahead of ray *3'*, deflecting from the slit edge[4–9] by $\lambda/2$, $k_g = 3$, 5,... correspond to the maxima of the diffraction pattern.

In case $2\,b^4\,(1 + L/l)^2 \ll (k_g\,\lambda\,L)^2$,

$$J_{\max} = B\,b^2\,J_c/(k_g^2\,\lambda\,L), \qquad (3)$$

$B = 0.3274$.

The validity of Eq. (2) under conditions of the Fraunhofer diffraction is confirmed by the experimental data reported in Ref. 4 in Tables IV ($b = 159$ μm, $l = 100$ mm, $L = 189$ mm, $k' = 0.063$, $k'' = 0.12$), VI ($b = 95.2$ μm, $l = 36.2$ mm, $L = 112$ mm, $k' = 0.036$, $k'' = 0.1$), VII ($b = 184$ μm, $L = 130.3$ mm, $l = \infty$, and $k' = 0.122$), and VIII ($b = 48$ μm, $L = 99.5$ mm, $l = \infty$, and $k' = 0.011$).

Proceeding from the conditions of formation of diffraction patterns from the slit in its shadow zone owing to the interference of the edge rays coming from the opposite edge zones, the difference between Fresnel's and Fraunhofer's diffraction patterns is that in the first case the interfering rays are coming at the points of the diffraction pattern approximately at two different angles and, because of the dependence of the edge light intensity on the angle of diffraction,[10] have different intensities. In case of the Fraunhofer diffraction, the interfering edge rays are parallel, have the same intensities, and therefore form more contrast pattern.

In contrast with Eq. (1), formulas (2) and (3) relate the band intensities with the incident light intensity and the distance from the slit to the diffraction pattern.

Let us reduce Eq. (1) to the form

$$I_\varphi = I_0\,\sin^2 0.5\,k_g\,\pi/(0.5\,k_g\,\pi)^2. \qquad (4)$$

In the maxima of the diffraction pattern, $\sin^2 0.5\,k_g\,\pi = 1$. Hence, the maximum intensities are inversely proportional to k_g^2.

As seen, formulas (3) and (4) demonstrate the same types of the dependence of the maximum intensities on k_g^2. This is a reason why formulas (1) and (4), in spite of their formal basis, characterize adequately the relationships of the intensities in the maxima.

Formula (2) is unsuitable for calculation of the intensity of the central maximum formed due to the interference of the edge rays with the directly transmitted light.[11]

In case of the Fraunhofer diffraction, the intensity of the central maximum cannot be calculated from formulas (3), (4), and (11) derived in Ref. 11, because of the violation of the inverse proportionality of the edge light amplitude to the deflection angles of the edge rays, underlying these formulas, for angles ≤0.07°.

According to Eq. (1),

$$I_0(J_{\max 1}) = 22.2\,J_{\max 2}. \qquad (5)$$

In case of plane wave diffraction by the slit 48 μm wide ($L = 99.5$ mm and $k' = 0.011$), $J_{\max 1}/J_{\max 2}$ was equal to 21.53. Hence Eq. (5), as applied to this example, characterizes correctly the relation between $J_{\max 1}$ and $J_{\max 2}$.

On the basis of Eq. (3), the intensity of the secondary maximum is proportional to b^2.

As seen from Table I, in case of Fraunhofer's diffraction, the intensity of the central maximum is also proportional to b^2. Therefore, Eq. (5) should be valid not only for the given example, but also for other values of b and k'.

TABLE I.

b_i/b_1	$(b_i/b_1)^2$	$J_{\max 1i}/J_{\max 11}$	k_i^1/k_1^1
75/50	2.25	57.2/25.7 = 2.23	–
100/50	4	105.5/25.7 = 4.1	–
150/50	9	236/25.7 = 9.18	–
225/50	20.25	509/25.7 = 19.8	–
300/50	36	917.5/25.7 = 35.7	0.193/0.005

(Here, b is in μm, $\lambda = 0.53$ μm, and J is in relative units.)

In connection with this, to calculate $J_{\max 1}$, $J_{\max 2}$ should be determined first from Eq. (3) and then it should be multiplied by 22.2.

In this method of I_0 determination, the opportunity arises to compare this value for different values of b and k' with the intensity of light incident at the same points without slit.

Let us substitute $k_g = 3$ corresponding to $\max_2$ in Eq. (3) and multiply it by 22.2 in order to proceed to I_0; after that, let us set this expression equal to J_c.

On the basis of the obtained equality, the intensity of light on the axis of Fraunhofer's pattern turns out to be equal to the intensity of the light incident on the slit for

$$b = b_0 = \sqrt{1.2384\,\lambda L}. \qquad (6)$$

In the experiment with $b_1 = 0.192$ mm, $L = 130.3$ mm, and $\lambda = 0.53$ μm, $J_c/J_{\max 1} = 2.235$. To increase $J_{\max 1}$ to J_c, it is necessary to widen the slit, considering the proportionality of $J_{\max 1}$ to the parameter b^2, up to $b_2 = \sqrt{2.235 b_1^2} = 0.287$ mm. This value is roughly equal to $b_0 = \sqrt{1.238{\cdot}0.53{\cdot}10^{-3}{\cdot}130.3} = 0.292$ mm.

Hence Eq. (6) is valid. This formula, combined with the fact of proportionality between I_0 and b^2, can be used to determine simply the variation of the light intensity on the axis of a parallel beam transmitted through the slit.

To check additionally formulas (2) and (3), we compare the calculated light intensity, J_b with the experimental one ($J_{\rm be}$) at points b lying on both sides of the central maximum with $k_g = 1$ and spaced at $h_b = \lambda L/2b$ from the pattern axis. As follows from Ref. 4, at these points $\Delta_g = \lambda/2$ between the edge rays coming from opposite sides of the slit is compensated by the initial path difference $\Delta_{ig} = \lambda/2$.

According to Eq. (3), the light intensity at these points is 9 times larger than $J_{\rm max2}$. Presented in Table II are the experimental data that confirm this. (Here, $J_{\rm max2e}$ is the experimentally measured intensity $J_{\rm max2}$.)

TABLE II.

b, μm	L, mm	$J_{\rm c}$, rel. units.	$J_{\rm be}$, rel. units.	$J_{\rm max2e}$,	$J_{\rm be}/J_{\rm max2e}$	$h_{\rm b}$, mm
48	99.5	7132	15.51	1.74	8.9	0.6
184	130.3	2560	33.5	3.8	8.82	0.192

Practically all experimental results reported in Ref. 4 correspond to Fraunhofer's diffraction. According to them, points b and the secondary maxima are located in the shadow zone of the slit.

In the case of Fresnel's diffraction, with the increase of k' and k'', the point b and increasing number of maxima formed by the edge rays are located within the limits of slit projection, where the intensity distribution is determined by the interference of edge rays *1* and *2* (*1*′ and *2*′) not only with each other, but also with directly transmitted rays *3* (Fig. 2). Therefore, formula (2) is valid only for maxima in the shadow zone.

Within the limits of the slit projection, the main roles in the interference with the directly transmitted rays play the least deflected edge rays (*1*′ and *2*). For large k' and k'', the contribution from rays *1* and *2*′ becomes so small that the diffraction pattern from the slit is transformed into two mirror patterns from screens forming the slit.[10]

Based on the interference of the edge waves coming from the opposite sides of the slit, we now derive the formulas characterizing the Fraunhofer diffraction in the focal plane of the objective (Fig. 3).

Because the incident rays passing through the slit out of its edge deflection zones are collected by the objective on the beam axis practically entire diffraction pattern in the focal plane of the objective is caused by the interference of edge rays *1* and *2* with each other.

As indicated above, at the first moment ray *1* runs ahead of ray *2* by $\Delta_{ig} = \lambda/2$.

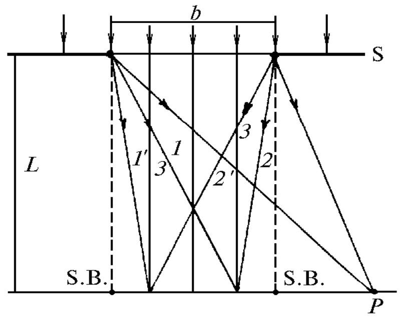

FIG. 2. Scheme of diffraction of the edge and directly transmitted rays for Fresnel's diffraction by the slit.

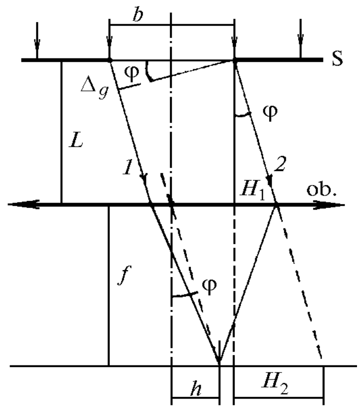

FIG. 3. Scheme of formation of Fraunhofer's diffraction pattern from a slit in the objective focal plane.

The geometric path difference between these rays is $\Delta_g = b\sin\varphi$, $\tan\varphi = \Delta_g/b\cos\varphi$.

The distance from the pattern axis to the bands is $h = f\tan\varphi = \Delta_g f/b\cos\varphi$. From this, $\Delta_g = hb\cos\varphi/f$. The total path difference between the edge rays $\Delta_t = (\Delta_g - \Delta_{ig}) = (\Delta_g - \lambda/2) = k_t\lambda/2$, where k_t is the number of half waves that fall on the total path difference. Therefore, $\Delta_g = (k_t + 1)\lambda/2$ and

$$h = (k_{\rm t} + 1)\ \lambda\ f/2b\ \cos\varphi; \tag{7}$$

$k_t = 0$, 2, 4... correspond to the secondary maxima. Because $k_t + 1 = k_g$, the secondary maxima are at $k = 1$, 3, 5... and $k_g = 1$ correspond to points b on the sides of the central maximum observed by the naked eye.

Because Δ_g gradually decreases from $\lambda/2$ to zero as the points of ray incidence are displaced from points b toward the pattern axis, under condition that $\Delta_{ig} = \lambda/2$ remains unchanged for small φ, Δ_t will increase from 0 to $\lambda/2$ and rays *1* and *2* will increasingly destroy each other. Therefore, instead of the central maximum corresponding to the Fraunhofer diffraction observed by the naked eye, $\max_0$ must be formed due to focusing of the directly transmitted rays on the beam axis, and on both sides of it, at the points b ($k_t = 0$), $\max_b$ must be formed. Between these maxima and $\max_0$, $\min_0$ must be formed. They do not follow from Eq. (1).

Their absence for perfectly plane incident wave and highly corrected objective may be explained only by the decrease of Δ_{ig} for small φ.

Because $\cos\varphi = \sqrt{b^2 - \Delta_g^2}/b$,

$$h = k_g \lambda f / [2 \sqrt{b^2 - (k_g \lambda/2)^2}]. \tag{8}$$

On the basis of Eq. (10) from Ref. 2, the intensity of edge rays *2* in the objective plane is $J_{ob} = 0.0205 \lambda L J_c / H_1^2$. In accordance with the inverse proportionality of the intensity of edge rays *2* to L in the focal plane of the objective, without it the intensity is $J_f = J_{\rm ob} L/(L + f)$.

Without objective, the edge light propagating within the angular limits 0–φ, spreads in the focal plane at the width H_2, but owing to the objective, the width is contracted to h.

Because $H_2 = (L + f) \tan\varphi$ and $h = f \tan\varphi$, $H_2/h = (L + f)/f$. As a result, the intensity of edge rays *2* in the focal plane is $J_{2f} = J_{ob} [L/(L + f)](L + f)/f = J_{ob} L/f$. Considering that $H_1/L = h/f$, we obtain $H_1 = Lh/f$ and

$$J_{2\rm f} = \frac{0.0205 \lambda L f^2 L J_{\rm c}}{L^2 h^2 f} = \frac{0.0205 \lambda f J_{\rm c}}{h^2} .$$

In this case, the amplitude of edge light *2* is $a_2 = \sqrt{0.0205\lambda f J_c}/h$.

Because rays *1* and *2* propagate in the same direction, $a = a_1 = a_2$. In the maxima of the diffraction pattern, we have $a_{\max} = 2a$.
Hence,

$$J_{\max} = a_{\max}^2 = \frac{0.08184 \lambda f J_{\rm c}}{h^2} = \frac{0.32736 [b^2 - (k_g \lambda/2)^2] J_{\rm c}}{k_g^2 \lambda f} \tag{9}$$

or approximately

$$J_{\max} = \frac{0.3274 b^2 J_{\rm c}}{k_g^2 \lambda f} . \tag{10}$$

From comparison of Eq. (3) with Eq. (10), we see that to proceed from the first formula to the second formula, it is suffice to substitute L by f in Eq. (3).

The absence of the secondary waves propagating in the directions different from the direction of incident light propagation may be proved on the basis of the following considerations.

In accordance with the integral Kirchhoff theorem,[6] perturbation $U(P)$ at the observation point P, caused by a point source P_0, is determined by a surface integral along on arbitrary closed surface S_0 that includes the point P. At the same time, if S is the reference surface, the field at the point P is equal to the field of geometrical optics (GO) e^{ikR}/R, where R is the distance from the source to the point P.

If the reference surface S is divided into parts A, B, and E, as was made by Kirchhoff,[6] in accordance with the preceding, we have

$$U(p) = \int_A + \int_b + \int_E = {\rm e}^{ikR}/R. \tag{11}$$

Let us neglect $\int_E$ on the basis of the Kirchhoff consideration and place an opaque screen on the surface B. Because $\int_b = 0$, to meet equality (11), it is necessary to decrease the right side by the integral along the surface B, when it is uncovered with the screen. As a result,

$$U(p) = \int_A = {\rm e}^{ikR}/R - \int_b , \tag{12}$$

that is, the perturbation at the point P is determined by the integral along the surface of the opening A in the screen.

Further, without removal of the screen, we choose the surface B, similar to Rabinovich.[16] Under these conditions, the closed surface $S = A + B + E$ is uncovered with the screen. Therefore, in accordance with Eq. (11), $U(P)$ must be equal to $\int_A + \int_b$. However, when the opening A remains unchanged, $U(P)$ is equal to $\int_A$. Hence, in the Rabinovich scheme the surface B also does not contribute to $U(P)$, which is the clear evidence of the fact that the Huygens sources of secondary waves are virtual.

Because the point sources located on the surface of the opening are of the same kind as the sources on the surface B, they are also virtual. Nevertheless, the perturbation from the opening A at the point P is nonzero and according to the experiment is determined by $\int_A$. Hence the effect of the virtual sources on the surface A, when they play roles of real sources, is equivalent to the effect of real sources at the point P, namely, of the sources of incident and edge waves.

When the virtual point sources on the surfaces A and B are considered as the real sources, the surface integral along A, according to Eq. (12), is equal to a sum of the GO field and the field determined as $\int\limits_{b}$.

As established by Rabinovich, the last integral is equal to a curvilinear integral along the opening edge. However, this does not prove the existence of the edge wave. Laue[13] considered the Rabinovich transform as a calculation procedure that allows one to determine the edge wave type such that the result of its effect with the incident light coincides with the result of Kirchhoff's theory.

CONCLUSION

Our results, together with the facts published earlier, are the evidence of the validity of the Huygens principle formalism in optical wavelength range and the validity of conclusions from the strict theory of diffraction phenomena caused by the interference of the edge waves with each other or with the incident light.

The formulas derived here on this basis have allowed us to express the intensity maxima of the diffraction pattern from the slit not only through the intensity of the central maximum, but also through the intensity of the light incident on the slit.

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