FORMATION OF HOLOGRAPHIC AND SPECKLE LATERAL SHEAR INTERFEROGRAMS TO CONTROL THE QUALITY OF TELESCOPE OPTICAL SYSTEMS

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In this paper two types of interferometers are analyzed. The first is based on a two-exposure record of a hologram of an amplitude scatterer image by Gabor scheme focused with a Kepler telescopic system. The second is based on a two-exposure record of specklogram of a focused mat screen image. It was shown both theoretically and experimentally that when performing spatial filtration of a diffraction field, shear interferogram is formed in the hologram plane that characterizes axial wave aberrations of an object under control. In the case of a two-exposure specklogram reconstruction in the far diffraction zone a shear speckle-interferogram is formed that characterizes axial wave aberrations of an object under control with a doubled sensitivity at a fixed lateral shear.

It was shown in Ref. 1 that two-exposure record of a hologram of mat screen image focused with the help of a Kepler telescopic optical system using an off-axis quasiplane reference wave results in formation of a lateral shear interferogram in infinite bands in diffusively scattered fields. In this case, before repeatedly exposing the photographic plate, tilt angle of quasiplane wavefront of coherent radiation used for illumination of the mat screen and tilt angle of reference wave were changed. At the stage of hologram reconstruction, superposition of diffracted waves from two exposures gives rise to an interference pattern localized in far diffraction zone and characterizing wave aberrations of the optical system of a Kepler telescope used. For its recording, the spatial filtration of reconstructed field in the hologram plane should be performed. In its turn, the interference pattern characterizing wave aberrations of optical system in the channel of reference wave formation and in the channel of formation of wave front of radiation used for the mat screen illumination is localized in the hologram plane. For its recording, spatial filtration of the diffraction field at the optical axis in the plane of formation of Fourier transform of the mat screen image should be performed.

In this paper we analyze the peculiarities of the lateral shear interferogram formation in infinite bands for the case of two-exposure record, with a Kepler telescopic optical system, of a hologram of a focused image of an amplitude scatterer by Gabor method and two-exposure specklogram of focused image of the mat screen.

As shown in Fig. 1*a*, paraxial image of a amplitude scatterer *1* being in the plane (x_1, y_1) is formed with a telescopic optical system comprising two positive lenses L_1 (objective) and L_2 (ocular) in the plane of photographic plate *2*. Gabor hologram is recorded when illuminating the scatterer with a coherent radiation during the first exposure. Before the second exposure, the tilt angle α of the quasiplane wave front of radiation used for illumination of the amplitude scatterer, for example, in (x, z) plane was changed. After photographic processing, the wave from the coherent light source used at the stage of its recording comes to the hologram, and in the Fourier plane *3* (Fig. 1*b*) the interference pattern is recorded when performing spatial filtration of the diffraction field at the optical axis in the hologram plane using round hole in an opaque screen p_3.

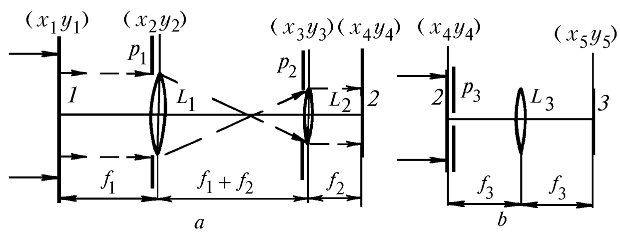

FIG. 1. Geometry of recording (a) and reconstruction (b) of two-exposure Gabor hologram: amplitude scatterer (1); photographic plate-hologram (2); plane of interferogram recording (3); lenses L_1, L_2, L_3; aperture diaphragms p_1, p_2; spatial filter p_3.

As follows from Ref. 1, in the Fresnel approximation, neglecting constant amplitude and phase factors, the complex field amplitudes corresponding to the first and second exposures in the (x_4, y_4) plane of photographic plate take the form:

$$u_1(x_4, y_4) \sim [1 - t\,(-\mu_1 x_4, -\mu_1 y_4)] \exp i\varphi_0\,(-\mu_1 x_4, -\mu_1 y_4) \otimes P_1(x_4, y_4) \otimes P_2(x_4, y_4)\ , \quad (1)$$

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$$u_2(x_4, y_4) \sim \exp(-ik\,\mu_1 x_4 \sin\alpha)\,\{[1 - t(-\mu_1 x_4, -\mu_1 y_4)]\ \exp i\varphi_0(-\mu_1 x_4 + a, -\mu_1 y_4) \otimes \exp(ik\,\mu_1 x_4 \sin\alpha) \times \times P_1(x_4, y_4) \otimes \exp(ik\,\mu_1 x_4 \sin\alpha)\, P_2(x_4, y_4)\}\ , \quad (2)$$

where $\otimes$ is the symbol of the convolution operation; k is the wave number; $\mu_1 = f_1/f_2$ is the scale transformation coefficient; f_1 is the lens L_1 focal length; f_2 is the lens L_2 focal length; $t(x_1, y_1)$ is the real random function of coordinates characterizing the amplitude of scattered absorption; $\varphi_0(x_1, y_1)$ is the determinate function characterizing phase distortions of the wave front of radiation used for illuminating the amplitude scattered due to aberrations of an optical system generating it; a is the wave front shift due to change of its tilt before the second exposure of the photographic plate;

$$P_1(x_4, y_4) = \iint\limits_{-\infty}^{\infty} p_1(x_2, y_2)\, \exp i\varphi_1(x_2, y_2) \times$$

$\times \exp[-ik(x_2 x_4 + y_2 y_4)/f_2]\, \mathrm{d}x_2\, \mathrm{d}y_2$ is the Fourier transform of generalized function $p_1(x_2, y_2) \exp i\varphi_1(x_2, y_2)$ of the pupil[2] of the lens L_1 allowing for its axial wave aberrations;

$$P_2(x_4, y_4) = \iint\limits_{-\infty}^{\infty} p_2(x_3, y_3)\, \exp i\varphi_2(x_3, y_3) \times$$

$\times \exp[-ik(x_3 x_4 + y_3 y_4)/f_2]\, \mathrm{d}x_3\, \mathrm{d}y_3$ is the Fourier transform of the generalized function of pupil of the lens L_2.

Let the photographic layer being exposed to light with the intensity $I(x_4, y_4) = u_1(x_4, y_4) \times \times u_1^*(x_4, y_4) + u_2(x_4, y_4)\, u_2^*(x_4, y_4)$, be processed with obtaining the negative at a straight-line portion of the characteristic curve of blackening. Then under the condition $t(x_1, y_1) \ll 1$ (Ref. 3) the transmission amplitude $\tau(x_4, y_4)$ of the hologram shown in Fig. 1*b* is determined from the expression

$$\begin{aligned}
\tau(x_4, y_4) \sim\ & [\exp i\varphi_0(-\mu_1 x_4, -\mu_1 y_4) \otimes P_1(x_4, y_4) \otimes P_2(x_4, y_4)]\,[t(-\mu_1 x_4, -\mu_1 y_4)\ \exp -i\varphi_0(-\mu_1 x_4, -\mu_1 y_4) \otimes \\
& \otimes P_1^*(x_4, y_4) \otimes P_2^*(x_4, y_4)] + [\exp -i\varphi_0(-\mu_1 x_4, -\mu_1 y_4) \otimes P_1^*(x_4, y_4) \otimes P_2^*(x_4, y_4)]\ \ [t(-\mu_1 x_4, -\mu_1 y_4) \times \\
& \times \exp i\varphi_0(-\mu_1 x_4, -\mu_1 y_4) \otimes P_1(x_4, y_4) \otimes P_2(x_4, y_4)] + [\exp i\varphi_0(-\mu_1 x_4 + a, -\mu_1 y_4) \otimes \\
& \otimes \exp(ik\,\mu_1 x_4 \sin\alpha)\, P_1(x_4, y_4) \otimes \exp(ik\,\mu_1 x_4 \sin\alpha) \times P_2(x_4, y_4)]\,[t(-\mu_1 x_4, -\mu_1 y_4)\, \exp -i\varphi_0(-\mu_1 x_4, -\mu_1 y_4) \otimes \\
& \otimes \exp(-ik\,\mu_1 x_4 \sin\alpha)\, P_1^*(x_4, y_4) \otimes \exp(-ik\,\mu_1 x_4 \sin\alpha)\, P_2^*(x_4, y_4)] + [\exp -i\varphi_0(-\mu_1 x_4 + a, -\mu_1 y_4) \otimes \\
& \otimes \exp(-ik\,\mu_1 x_4 \sin\alpha)\ P_1^*(x_4, y_4) \otimes \exp(-ik\,\mu_1 x_4 \sin\alpha)\, P_2^*(x_4, y_4)]\,[t(-\mu_1 x_4, -\mu_1 y_4) \times \\
& \times \exp i\varphi_0(-\mu_1 x_4 + a, -\mu_1 y_4) \otimes \exp(ik\,\mu_1 x_4 \sin\alpha)\ \ P_1(x_4, y_4) \otimes \exp(ik\,\mu_1 x_4 \sin\alpha)\, P_2(x_4, y_4)]\ , \qquad (3)
\end{aligned}$$

in which the regular component of light transmission is missing, because in the following consideration it will result only in distribution of illumination over a small spot in the observation plane.

As follows from Ref. 4, distribution of the complex amplitude of diffusely scattered field component in the rear focal plane (x_5, y_5) of the lens L_3 with the focal length f_3 (see Fig. 1*b*) can be expressed as

$$u(x_5, y_5) \sim \iint\limits_{-\infty}^{\infty} \tau(x_4, y_4)\, \exp\left[-\frac{ik}{f_3}(x_4 x_5 + y_4 y_5)\right] \mathrm{d}x_4\, \mathrm{d}y_4 \ \otimes P_3(x_5, y_5)\ , \qquad (4)$$

where $P_3(x_5, y_5) = \iint\limits_{-\infty}^{\infty} p_3(x_4, y_4)\, \exp[-ik(x_4 x_5 + y_4 y_5)/f_3]\, \mathrm{d}x_4\, \mathrm{d}y_4$

is the Fourier transform of the transmission function of the opaque screen p_3 with a round hole.[5]

Having substituted Eq. (3) into Eq. (4), we obtain

$$\begin{aligned}
u(x_5, y_5) \sim\ & \{\{\Phi_1(x_5, y_5)\, p_1(\mu_2 x_5, \mu_2 y_5)\, p_2(\mu_2 x_5, \mu_2 y_5)\ \exp i[\varphi_1(-\mu_2 x_5, -\mu_2 y_5) + \varphi_2(-\mu_2 x_5, -\mu_2 y_5)]\} \otimes \{[F(x_5, y_5) \otimes \\
& \otimes \Phi_2(x_5, y_5)]\, p_1(\mu_2 x_5, \mu_2 y_5)\, p_2(\mu_2 x_5, \mu_2 y_5)\ \exp -i[\varphi_1(\mu_2 x_5, \mu_2 y_5) + \varphi_2(\mu_2 x_5, \mu_2 y_5)]\} + \{\Phi_2(x_5, y_5)\, p_1(\mu_2 x_5, \mu_2 y_5) \times \\
& \times p_2(\mu_2 x_5, \mu_2 y_5)\, \exp -i[\varphi_1(\mu_2 x_5, \mu_2 y_5) + \varphi_2(\mu_2 x_5, \mu_2 y_5)]\} \otimes \{[F(x_5, y_5) \otimes \Phi_1(x_5, y_5)]\, p_1(\mu_2 x_5, \mu_2 y_5) \times \\
& \times p_2(\mu_2 x_5, \mu_2 y_5)\, \exp i[\varphi_1(-\mu_2 x_5, -\mu_2 y_5) + \varphi_2(-\mu_2 x_5, -\mu_2 y_5)]\} + \{\Phi_3(x_5, y_5)\, p_1(\mu_2 x_5 - b, \mu_2 y_5)\, p_2(\mu_2 x_5 - b, \mu_2 y_5) \times \\
& \times \exp i[\varphi_1(-\mu_2 x_5 + b, -\mu_2 y_5) + \varphi_2(-\mu_2 x_5 + b, -\mu_2 y_5)]\} \otimes \{[F(x_5, y_5) \otimes \Phi_4(x_5, y_5)]\ \ p_1(\mu_2 x_5 + b, \mu_2 y_5) \times \\
& \times p_2(\mu_2 x_5 + b, \mu_2 y_5)\, \exp -i[\varphi_1(\mu_2 x_5 + b, \mu_2 y_5) + \varphi_2(\mu_2 x_5 + b, \mu_2 y_5)]\} + \{\Phi_4(x_5, y_5)\, p_1(\mu_2 x_5 + b, \mu_2 y_5) \times
\end{aligned}$$

$$\times\, p_2(\mu_2 x_5 + b, \mu_2 y_5)\ \exp -i\,[\varphi_1(\mu_2 x_5 + b, \mu_2 y_5) + \varphi_2(\mu_2 x_5 + b, \mu_2 y_5)]\} \otimes \{[F(x_5, y_5) \otimes \Phi_3(x_5, y_5)] \times$$

$$\times\, p_1(\mu_2 x_5 - b, \mu_2 y_5)\, p_2(\mu_2 x_5 - b, \mu_2 y_5)\ \exp i\,[\varphi_1(-\mu_2 x_5 + b, -\mu_2 y_5) + \varphi_2(-\mu_2 x_5 + b, -\mu_2 y_5)]\}\} \otimes P_3(x_5, y_5)\,, \quad (5)$$

where $\mu_2 = f_2/f_3$ is the scale transformation coefficient; $b = f_1 \sin\alpha$ is the shift value;

$$F(x_5, y_5) = \iint\limits_{-\infty}^{\infty} t\,(-\mu_1 x_4, -\mu_1 y_4)\, \exp[-ik\,(x_4 x_5 + y_4 y_5)/f_3]\, dx_4\, dy_4\ ;$$

$$\Phi_1(x_5, y_5) = \iint\limits_{-\infty}^{\infty} \exp i\varphi_0\,(-\mu_1 x_4, -\mu_1 y_4)\, \exp[-ik\,(x_4 x_5 + y_4 y_5)/f_3]\, dx_4\, dy_4\ ;$$

$$\Phi_2(x_5, y_5) = \iint\limits_{-\infty}^{\infty} \exp -i\varphi_0\,(-\mu_1 x_4, -\mu_1 y_4)\, \exp[-ik\,(x_4 x_5 + y_4 y_5)/f_3]\, dx_4\, dy_4\ ;$$

$$\Phi_3(x_5, y_5) = \iint\limits_{-\infty}^{\infty} \exp i\varphi_0\,(-\mu_1 x_4 + a, -\mu_1 y_4)\, \exp[-ik\,(x_4 x_5 + y_4 y_5)/f_3]\, dx_4\, dy_4\ ;$$

$$\Phi_4(x_5, y_5) = \iint\limits_{-\infty}^{\infty} \exp -i\varphi_0\,(-\mu_1 x_4 + a, -\mu_1 y_4)\, \exp[-ik\,(x_4 x_5 + y_4 y_5)/f_3]\, dx_4\, dy_4$$

are Fourier transforms of the corresponding functions.

If the period of the function $\exp i\varphi_0(-\mu_1 x_4, -\mu_1 y_4)$ is greater than the size of a subjective speckle, in the photographic plate plane determined by the width of the function $P_1(x_4, y_4) \otimes P_2(x_4, y_4)$, then we can assume that $\Phi_1 = \Phi_2 = \Phi_3 = \Phi_4 = \delta(x_5, y_5)$, where $\delta(x_5, y_5)$ is the Dirac delta-function. Then, taking into account the condition $d_1 = \mu_1 d_2$ where d_1 and d_2 are the diameters of the aperture diaphragms of lenses L_1 and L_2, respectively (Fig. 1*a*), Eq. (5) takes the form

$$u(x_5, y_5) \sim \{\, p_2(\mu_2 x_5, \mu_2 y_5)\, \exp -i\,[\varphi_1(\mu_2 x_5, \mu_2 y_5) + \varphi_2(\mu_2 x_5, \mu_2 y_5)] + p_2(\mu_2 x_5, \mu_2 y_5) \times$$

$$\times \exp i\,[\varphi_1(-\mu_2 x_5, -\mu_2 y_5) + \varphi_2(-\mu_2 x_5, -\mu_2 y_5)] + p_2(\mu_2 x_5 + b, \mu_2 y_5)\, \exp -i\,[\varphi_1(\mu_2 x_5 + b, \mu_2 y_5) +$$

$$+ \varphi_2(\mu_2 x_5 + b, \mu_2 y_5)] + p_2(\mu_2 x_5 - b, \mu_2 y_5)\, \exp i\,[\varphi_1(-\mu_2 x_5 + b, -\mu_2 y_5) +$$

$$+ \varphi_2(-\mu_2 x_5 + b, -\mu_2 y_5)]\}\, F(x_5, y_5) \otimes P_3(x_5, y_5)\,. \quad (6)$$

As follows from Eq. (6), within the region where the functions $p_2(\mu_2 x_5 - b, \mu_2 y_5)$ and $p_2(\mu_2 x_5 + b, \mu_2 y_5)$ overlap, identical subjective speckles of two exposures of (+1) and (–1) diffraction orders overlap that gives rise to correlation of speckle fields in the observation plane and to formation of an interference pattern.[6] Really, if the period of the function $\exp -i\,[\varphi_1(\mu_2 x_5, \mu_2 y_5) + \varphi_2(\mu_2 x_5, \mu_2 y_5)] +$ $+ \exp i\,[\varphi_1(-\mu_2 x_5, -\mu_2 y_5) + \varphi_2(-\mu_2 x_5, -\mu_2 y_5)] +$ $+ \exp -i\,[\varphi_1(\mu_2 x_5 + b, \mu_2 y_5) + \varphi_2(\mu_2 x_5 + b, \mu_2 y_5)] +$ $+ \exp i\,[\varphi_1(-\mu_2 x_5 + b, -\mu_2 y_5) + \varphi_2(-\mu_2 x_5 + b, -\mu_2 y_5)]$ is greater than the subjective speckle size determined by the width of the function $P_3(x_5, y_5)$, then it can be removed from the convolution integral. Then within the region of images overlapping of output pupil of the telescopic optical system the complex field amplitude in the observation plane is determined by the expression

$$u(x_5, y_5) \sim \{\exp -i[\varphi_1(\mu_2 x_5, \mu_2 y_5) + \varphi_2(\mu_2 x_5, \mu_2 y_5)] + \exp i[\varphi_1(-\mu_2 x_5, -\mu_2 y_5) + \varphi_2(-\mu_2 x_5, -\mu_2 y_5)] +$$

$$+ \exp -i[\varphi_1(\mu_2 x_5 + b, \mu_2 y_5) + \varphi_2(\mu_2 x_5 + b, \mu_2 y_5)] + \exp i\,[\varphi_1(-\mu_2 x_5 + b, -\mu_2 y_5) + \varphi_2(-\mu_2 x_5 + b, -\mu_2 y_5)]\} \times$$

$$\times [F(x_5, y_5) \otimes P_3(x_5, y_5)]\,. \quad (7)$$

Since the functions $\varphi_1(x_2, y_2)$ and $\varphi_1(x_3, y_3)$ are even, illumination distribution in the observation plane *3* (see Fig. 1*b*) takes the form

$$I(x_5, y_5) \sim \left\{ \{1 + \cos[2\varphi_1(\mu_2 x_5, \mu_2 y_5) + 2\varphi_2(\mu_2 x_5, \mu_2 y_5)]\} \times \right.$$
$$\left. \times \left\{ 1 + \cos\left[\frac{\partial\varphi_1(\mu_2 x_5, \mu_2 y_5)}{\partial \mu_2 x_5} b + \frac{\partial\varphi_2(\mu_2 x_5, \mu_2 y_5)}{\partial \mu_2 x_5} b \right] \right\} \right\} \times |F(x_5, y_5) \otimes P_3(x_5, y_5)|^2, \tag{8}$$

where

$$\frac{\partial\varphi_1(\mu_2 x_5, \mu_2 y_5)}{\partial \mu_2 x_5} b = \varphi_1(\mu_2 x_5 + b, \mu_2 y_5) - \varphi_1(\mu_2 x_5, \mu_2 y_5),$$

$$\frac{\partial\varphi_2(\mu_2 x_5, \mu_2 y_5)}{\partial \mu_2 x_5} b = \varphi_2(\mu_2 x_5 + b, \mu_2 y_5) - \varphi_2(\mu_2 x_5, \mu_2 y_5).$$

It follows from Eq. (8) that the subjective speckle structure is modulated by the interference bands. Interference pattern consists of equally wide bands[7] characterizing spherical aberrations of the telescopic optical system of Kepler type and lateral shear bands corresponding to differential interferometry and also characterizing spherical aberrations of an object under control.

For the case of a two-exposure recording of a specklogram of a focused image of a mat screen being at the plane (x_1, y_1) (see Fig. 1*a*) and characterized by the complex transmission amplitude $t(x_1, y_1)$, which is random function of coordinates, before the second exposure of the photographic plate *2*, tilt angle of the quasiplane wave front of radiation used for its illumination also changes as in the former case. After photographic processing, the coherent wave from the light source used at the stage of its recording comes to the specklogram *2* (see Fig. 1*b*), and speckle-interferogram is recorded in the Fourier plane *3*.

Based on the assumptions that the period of the function $\exp i\varphi_0(-\mu_1 x_4, -\mu_1 y_4)$ is greater than the size of a subjective speckle, in the plane of the photographic plate, determined by the width of the function $P_1(x_4, y_4) \otimes P_2(x_4, y_4)$ and that the negative was processed at a straight-line portion of the characteristic curve of blackening, the transmission amplitude of a two-exposure specklogram can be written as

$$\tau'(x_4, y_4) \sim [t(-\mu_1 x_4, -\mu_1 y_4) \otimes P_1(x_4, y_4) \otimes P_2(x_4, y_4)]\,[t^*(-\mu_1 x_4, -\mu_1 y_4) \otimes P_1^*(x_4, y_4) \otimes P_2^*(x_4, y_4)] +$$
$$+ [t(-\mu_1 x_4, -\mu_1 y_4) \otimes \exp(ik\,\mu_1 x_4 \sin\alpha)\, P_1(x_4, y_4) \otimes \exp(ik\,\mu_1 x_4 \sin\alpha)\, P_2(x_4, y_4)]\,[t^*(-\mu_1 x_4, -\mu_1 y_4) \otimes$$
$$\otimes \exp(-ik\,\mu_1 x_4 \sin\alpha)\, P_1^*(x_4, y_4) \otimes \exp(-ik\,\mu_1 x_4 \sin\alpha)\, P_2^*(x_4, y_4)], \tag{9}$$

where the regular component of light transmission is missing. Then the distribution of complex amplitude of a diffusely scattered field component in the rear focal plane of the lens L_3 (see Fig. 1*b*) is determined by the expression

$$u'(x_5, y_5) \sim \{\{F_1(x_5, y_5)\, p_1(\mu_2 x_5, \mu_2 y_5)\, p_2(\mu_2 x_5, \mu_2 y_5) \exp i[\varphi_1(-\mu_2 x_5, -\mu_2 y_5) + \varphi_2(-\mu_2 x_5, -\mu_2 y_5)]\} \otimes \{F_2(x_5, y_5) \times$$
$$\times p_1(\mu_2 x_5, \mu_2 y_5)\, p_2(\mu_2 x_5, \mu_2 y_5) \exp -i[\varphi_1(\mu_2 x_5, \mu_2 y_5) + \varphi_2(\mu_2 x_5, \mu_2 y_5)]\} + \{F_1(x_5, y_5)\, p_1(\mu_2 x_5 - b, \mu_2 y_5) \times$$
$$\times p_2(\mu_2 x_5 - b, \mu_2 y_5) \exp i[\varphi_1(-\mu_2 x_5 + b, -\mu_2 y_5) + \varphi_2(-\mu_2 x_5 + b, -\mu_2 y_5)]\} \otimes \{F_2(x_5, y_5)\, p_1(\mu_2 x_5 + b, \mu_2 y_5) \times$$
$$\times p_2(\mu_2 x_5 + b, \mu_2 y_5) \exp -i[\varphi_1(\mu_2 x_5 + b, \mu_2 y_5) + \varphi_2(\mu_2 x_5 + b, \mu_2 y_5)]\}\} \otimes P_3(x_5, y_5), \tag{10}$$

where

$$F_1(x_5, y_5) = \iint\limits_{-\infty}^{\infty} t\,(-\mu_1 x_4, -\mu_1 y_4) \exp[-ik\,(x_4 x_5 + y_4 y_5)/f_3]\, dx_4\, dy_4 ;$$

$$F_2(x_5, y_5) = \iint\limits_{-\infty}^{\infty} t^*\,(-\mu_1 x_4, -\mu_1 y_4) \exp[-ik\,(x_4 x_5 + y_4 y_5)/f_3]\, dx_4\, dy_4$$

are Fourier transforms of a the corresponding functions.

Since Fourier transform of a complex-conjugate function equals the complex-conjugate Fourier transform of the initial function with the opposite sign of the argument, $F_2(x_5, y_5) = F_1^*(-x_5, -y_5)$, for maximum value of autocorrelation in Eq. (10) we obtain

$$u'(x_5, y_5) \sim \left\{ \left\{1 + \exp{-i}\left[\frac{\partial\varphi_1\,(\mu_2 x_5,\ \mu_2 y_5)}{\partial\,\mu_2 x_5}\,2b + \frac{\partial\varphi_2\,(\mu_2 x_5,\ \mu_2 y_5)}{\partial\,\mu_2 x_5}\,2b\right]\right\} \times \right.$$
$$\left. \times\, [F_1(x_5, y_5) \otimes F_1^*(-x_5, -y_5)] \right\} \otimes P_3(x_5, y_5)\ . \tag{11}$$

If the period of the function $1 + \exp{-i}\left[\frac{\partial\varphi_1\,(\mu_2 x_5,\ \mu_2 y_5)}{\partial\mu_2 x_5}\,2b + \frac{\partial\varphi_2\,(\mu_2 x_5,\ \mu_2 y_5)}{\partial\,\mu_2 x_5}\,2b\right]$ is at least by an order of magnitude greater than the size of a subjective speckle in the observation plane determined by the width of the function $P_3(x_5, y_5)$, then it can be removed from the integral in Eq. (11). Then the distribution of illumination in the plane of recording *3* (see Fig. 1*b*) is determined by the expression

$$I'\,(x_5, y_5) \sim \left\{1 + \cos\left[\frac{\partial\varphi_1\,(\mu_2 x_5,\ \mu_2 y_5)}{\partial\,\mu_2 x_5}\,2b + \frac{\partial\varphi_2\,(\mu_2 x_5,\ \mu_2 y_5)}{\partial\,\mu_2 x_5}\,2b\right]\right\}\, |F_1(x_5, y_5) \otimes F_1^*(-x_5, -y_5) \otimes P_3(x_5, y_5)|^2\ , \tag{12}$$

which describes the subjective speckle structure within the diffraction halo modulated by the interference fringes. Interference pattern looks like a lateral shear interferogram in infinite bands which characterizes spherical wave aberrations of an object under control. And the sensitivity of the speckle interferometer increases twice for a fixed value of a lateral shear.

The mechanism of formation of a lateral shear speckle interferograms with increase in sensitivity is in a two fold increase of the width of spatial frequency spectrum of the waves diffusely scattered by the specklogram.[8] Naturally, at specklogram recording due to quadratic character of the detection, phase information is lost but its part remains that allows reconstructing the direction of the components of the scattered field.[9] Since the region of variable values, at which autocorrelation interval in Eq. (10) does not equal zero, is determined by a doubled width of the output pupil of the Kepler telescope, the length of the diffraction halo in the observation plane ($\mu_2 = 1$) is doubled. Consequently, an increase in the angular spectrum of waves scattered by the specklogram results in doubling of the tilt (angular frequency) that follows from Eq. (9) at diffraction on speckle structure corresponding to the second exposure.

In the experiment, the two-exposure recording of holograms and specklograms of a focused image was performed on photographic plates of Mikrat VRL type with a He-Ne laser radiation at 0.63 μm. The technique of experimental study was in comparing of the results of two-exposure recording of holograms by Gabor method and specklograms of a focused image with the results of a two-exposure recording of holograms with the use of an off-axis reference wave.

For this, the real image of a mat screen, as in Ref. 7, was constructed in the photographic plate plane with a centered telescopic optical system comprising two identical positive lenses with the focal length $f_1 = f_2 = 180$ mm, and the pupil diameter of 25 mm. The diameter of illuminated area of a mat screen was 35 mm. Before the second exposure, tilt angle of quasiplane wave front of a radiation used for illuminating the mat screen was changed as well as the tilt angle of the off-axis quasiplane reference wave.[1]

Figure 2*a* shows shear interferogram in infinite bands which was recorded in the focal plane of camera's lens with focal length of 50 mm when performing spatial filtration of diffraction field at the optical axis in the hologram plane by reconstructing it by small-aperture (≈ 2 mm) laser beam. It characteristic spherical aberration in paraxial focus of controlled optical system for $\alpha = 28' \pm 10''$.

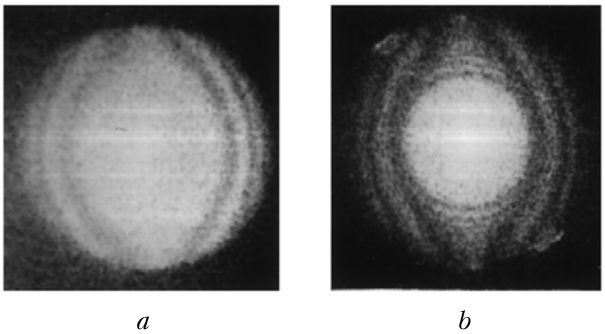

FIG. 2. *Lateral shear interferograms corresponding to two-exposure hologram recording: by Leit-Upatnieks method (a), by Gabor method (b).*

Then in the object plane the amplitude scatterer was set in place of the mat screen, and two-exposure recording of hologram by Gabor scheme was performed for $\alpha = 28' \pm 10''$. The view of interference pattern recorded in the focal plane of camera's lens when performing spatial filtration of diffraction field at the optical axis in the hologram plane by reconstructing it by small-aperture laser beam is shown in Fig. 2*b*.

Interference patterns shown in Fig. 3 correspond to the case when before performing two-exposure recording of the hologram the photographic plate was shifted from the plane of paraxial image formation by 2.3 mm, at $\alpha = 28' \pm 10''$. They characterize spherical aberration of the controlled optical system with defocusing behind the focal plane. In this case, as in Fig. 2*b* where concentric system of interference fringes characterizes spherical aberration of the telescope optical system, concentric system of interference fringes in Fig. 3*b* characterizes spherical aberration and additional defocusing. It should be noted that when shifting a two-exposure Gabor hologram against the laser beam reconstructing it, the view of interference pattern is distorted due to mismatch, in direction, of the diffracting waves in (–1) and (+1) orders of diffraction.[7]

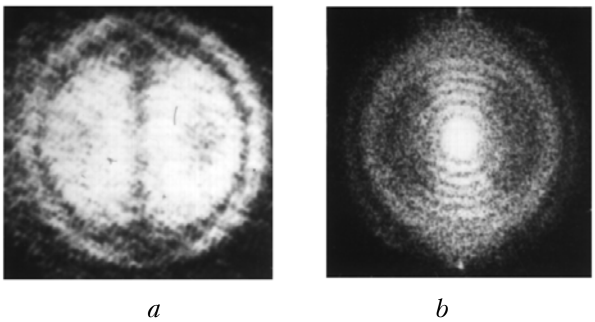

FIG. 3. *Lateral shear interferograms for the case of defocusing behind the focal plane. The interferograms correspond to two-exposure hologram recording: by Leit-Upatnieks method (a) and by Gabor method (b).*

Two-exposure recording of specklograms of a focused image of a mat screen was performed with the use of autocollimator of VU-200 type as an object under control.[1]

Figure 4*a* shows holographic lateral shear interferogram in infinite bands that was recorded in the camera's lens focal plane when performing spatial filtration of the diffraction field on the optical axis in the plane of the hologram by reconstructing it using a small-aperture laser beam. It characterizes spherical aberration of controlled object with defocusing behind the focal plane. Before the second exposure of the photographic plate, tilt angle α was changed by $30' \pm 10''$.

The lateral shear interferogram in infinite bands shown in Fig. 4*b* corresponds to the case of reconstruction of the two-exposure specklogram of a focused mat screen image for the same value of the angle change before the second exposure. It characterizes spherical aberration at defocusing, behind the focal plane, of the autocollimator optical system with a doubled sensitivity of the interferometer at a fixed lateral shear. The lateral shear speckle-interferogram in Fig. 4*b* looks like a holographic lateral shear interferogram for $\alpha = 1°$.

Speckle interferogram was recorded in the camera's lens focal plane when reconstructing the two-exposure specklogram using a small-aperture laser beam. In this case, its view remains unchanged when shifting the specklogram against the laser beam, that is indicative of insensitivity of the lateral shear speckle interferogram to the off-axis wave aberration of the autocollimator optical system. In addition, the known property of the Kepler telescopes consisting in the absence of vignetting (see, for example, Ref. 10) causes constant contrast of speckle interference pattern at specklogram reconstruction at a point lying both at optical axis and out of it. As a result, it becomes clear that recording of the lateral shear speckle-interferogram is possible without performing spatial filtration of the diffraction field in the plane of specklogram of a mat screen image focused with a telescopic optical system.

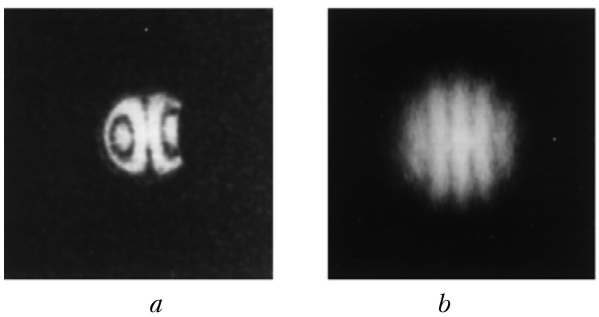

FIG. 4. *Holographic (a) and speckle (b) lateral shear interferograms characterizing axial wave aberrations of the autocollimator optical system.*

Thus, based on the research conducted, we can conclude that, at two-exposure recording of a hologram of an amplitude scatterer image, by Gabor method, focused with a telescopic optical system of Kepler type, the lateral shear interferogram, in infinite bands, characterizing wave aberrations of controlled object is formed at the stage of the hologram reconstruction in the far diffraction zone. At the same time, for its recording it is necessary to perform spatial filtration of the diffraction field on the optical axis in the hologram plane. In contrast to the case of the two-exposure hologram recording with the use of an off-axis reference wave,[11] rather low sensitivity of process of the hologram recording to vibrations is characteristic of the method considered, i.e. hologram can be recorded without meeting stringent requirements to mechanic stability of the setup.

At two-exposure recording of a specklogram of a mat screen image focused with a telescopic optical system of Kepler type, at the stage of its reconstruction in the far diffraction zone the lateral shear speckle-interferogram in infinite bands is formed that characterizes axial wave aberrations of controlled object with a doubly increased sensitivity of the speckle-interferometer for a fixed lateral shear. In this case for speckle-interferogram recording there is no need to perform spatial filtration of the diffraction field in the specklogram plane.

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