MODELING OF SERIES OF DATA ON WIND VELOCITY VECTOR AND AEROSOL CONCENTRATION WITH A PRESET DISTRIBUTION LAW AND CORRELATIONS TYPICAL FOR THE REAL TURBULENT ATMOSPHERE

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The procedures are developed for obtaining the series of instantaneous values of the wind velocity components and concentrations of aerosol with a preset distribution law and correlations typical for real turbulent atmosphere. The correlations are represented by the Reynolds tensor of viscous stress and turbulent fluxes of admixtures.

Solution of some problems in atmospheric physics and ecology means a calculation of functions of instantaneous values of the wind velocity components and aerosol admixture concentration averaged over a statistical ensemble. For example, such a necessity appears when estimating the efficiency of aspiration of aerosol particles into samplers in a turbulent medium. One of the possible ways of solving such problems assumes statistical modeling of a series of instantaneous values of the wind velocity components and aerosol concentration with the subsequent averaging of these functions over the statistical ensemble obtained.

The aim of this work was to develop the procedure for obtaining the series of instantaneous values of the wind velocity components and concentration of aerosol with a preset distribution law and correlations typical for real turbulent atmosphere.

Let us present the instantaneous values of the wind velocity and aerosol concentration in the form of a sum of the mean values and pulsations[1]

$$U_i = \bar{U}_i + \hat{U}_i; \qquad C = \bar{C} + \hat{C}; \qquad (i = x,\, y,\, z), \quad (1)$$

where U_i is the ith component of the instantaneous value of the wind velocity; $\bar{U}_i$ is the mean value of this value (the upper bar means averaging over the statistical ensemble); $\hat{U}_i$ is the pulsation. Similar designations are accepted for the concentration C.

It is known that the distribution function of the value $\hat{U}_i$ can be represented by normal distribution quite accurately

$$F_i\,(\hat{U}_i) = \frac{1}{2}\left[1 + \mathrm{erf}\left(\frac{\hat{U}_i}{2^{1/2}\,\sigma_i}\right)\right], \quad (2)$$

where F_i is the distribution function of pulsations of the ith component of the wind velocity; *erf* is the designation of the probability integral;[3] σ_i is the rms deviation of pulsation of the ith component of the wind velocity. According to Ref. 4, the distribution function of the instantaneous value of concentration has the form

$$F_c(C) = 1 + \frac{1}{2}\left[\mathrm{erf}\left(\frac{C - \bar{C}}{\nu}\right) - \mathrm{erf}\left(\frac{C + \bar{C}}{\nu}\right)\right], \quad (3)$$

where F_c is the distribution function of the concentration instantaneous value, $\bar{C}$ is the mean value of the admixture concentration, and ν is the second parameter of the distribution function.

The distribution law (3) was obtained in our previous papers and is experimentally confirmed on the basis of a number of laboratory and independent field experiments.[4]

Then, without any loss of generality of the subsequent consideration, let us consider the near-ground layer of the atmosphere. Let the mean wind velocity vector be along the x-axis, then y-axis is perpendicular to it in the horizontal plane, and z-axis is directed upward. Let us suppose the horizontal homogeneity and stationarity of the problem to be solved, that will allow us to use the theory of similarity of the near-ground atmospheric layer.[1]

Then we obtain

$$U_x = \bar{U}_x + \hat{U}_x; \qquad U_y = \hat{U}_y; \qquad U_z = \hat{U}_z. \quad (4)$$

Correlations between the pulsations of the wind velocity components are determined by the Reynolds tensor[1] of viscous stress $\tau_{ij} = \overline{\hat{U}_i \cdot \hat{U}_j}$ In the ideal near-ground atmospheric layer it has the form

$$\tau_{ij} = \left\Vert \begin{array}{ccc} \sigma_x^2 & 0 & -U_*^2 \\ 0 & \sigma_y^2 & 0 \\ -U_*^2 & 0 & \sigma_z^2 \end{array} \right\Vert, \quad (5)$$

where σ_x^2, σ_y^2, σ_z^2 are the variances of the pulsations of x, y and z-components of the wind velocity, respectively; and U_* is the friction velocity.[1]

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Correlations between the pulsations of the aerosol concentration and the pulsations of the wind velocity components are determined by the turbulent fluxes[1] of the admixture φ_i:

$$\varphi_x = \overline{\hat{U}_x \cdot \hat{C}}, \quad \varphi_y = \overline{\hat{U}_y \cdot \hat{C}}, \quad \varphi_z = \overline{\hat{U}_z \cdot \hat{C}}, \quad (6)$$

One can model the series of the instantaneous values $\hat{U}_i$ and $\hat{C}$, independent of each other, with the use of sequences of random values r_i and r uniformly distributed over the range from 0 to 1. To do this, it is necessary and sufficient to solve Eqs. (2) and (3) with the values r_i and r (see Ref. 5)

$$F_i(\hat{U}_i) = r_i, \qquad F_c(C) = r. \quad (7)$$

Then, let us consider the procedure for obtaining the random sequences $\hat{U}_i$ and C, obeying the relationships (5) and (6). Let us introduce a normally distributed series of the random values α_m and β_n with the following properties:

$$\overline{\alpha_m} = \overline{\beta_n} = 0, \qquad \overline{\alpha_m\, \alpha_n} = \delta_{mn}, \quad (8)$$

$$\overline{\beta_m\, \beta_n} = \delta_{mn}, \quad \overline{\alpha_m\, \beta_n} = 0, \quad (m, n = 1, 2, 3),$$

where δ_{mn} is the Kronecker simbol.[3] Let us simulate the pulsations of the wind velocity components by the following relationships

$$\begin{aligned} \hat{U}_x &= \alpha_1\, a_1 + \beta_1\, b_1, \\ \hat{U}_y &= \alpha_2\, a_2 + \beta_2\, b_2, \\ \hat{U}_z &= \alpha_3\, a_3 + \beta_3\, b_3 + \beta_1\, b_4. \end{aligned} \quad (9)$$

where a_1, a_2, a_3, b_1, b_2, b_3 and b_4 are the constants to be determined. The pulsations of the wind velocity components specified in such a form are distributed according to the normal law[6] and have zero mean values.

To obtain the instantaneous values of concentration with the distribution law (2) and (3), the terms with coefficients a_m where introduced. Let us introduce the random value $\alpha_0 = 3^{-1/3}(\alpha_1 + \alpha_2 + \alpha_3)$. It is normally distributed with zero mean value and unit variance. According to Eq. (7), α_0 is related with the random value r_0 uniformly distributed over the range (0, 1), which we use for generating the instantaneous values of the concentration pulsations using also Eq. (3).

Let us find seven unknown coefficients of Eq. (9) from the system of equations following from Eqs. (5) and (6)

$$\sigma_x^2 = a_1^2 + b_1^2;\ \sigma_y^2 = a_2^2 + b_2^2;\ \sigma_z^2 = a_3^2 + b_3^2 + b_4^2;\ -U_*^2 = b_1\, b_4,$$

$$\varphi_x = (\overline{\alpha_1\, \hat{C}})\, a_1,\ \varphi_y = (\overline{\alpha_2\, \hat{C}})\, a_2,\ \varphi_z = (\overline{\alpha_3\, \hat{C}})\, a_3. \quad (10)$$

The solutions of the aforementioned equations are as follows:

$$a_1 = \varphi_x = (\overline{\alpha_1\, \hat{C}})^{-1}, \quad a_2 = \varphi_y = (\overline{\alpha_2\, \hat{C}})^{-1},$$

$$a_3 = \varphi_z = (\overline{\alpha_3\, \hat{C}})^{-1};$$

$$b_1 = (\sigma_x^2 - a_1^2)^{1/2}, \qquad b_2 = (\sigma_y^2 - a_2^2)^{1/2}, \quad (11)$$

$$b_4 = -\,U_*^2\, b_1^{-1},\ \ b_3 = (\sigma_z^2 - a_3^2 - b_4^2)^{1/2}.$$

It follows from Eq. (11), that the limitations imposed on the coefficients a_1, a_2 and a_3 are:

$$|a_1| < \sigma_x\,[1 - U_*^4/(\sigma_x^2\, \sigma_z^2)]^{1/2}, \qquad |a_2| < \sigma_y, \quad (12)$$

$$|a_3| < \sigma_z\,\{1 - U_*^4/[\sigma_x^2\, \sigma_z^2\,(1 - a_1^2\, \sigma_x^{-2})]\}^{1/2},$$

Because of statistical independence of the random values α_m and the technique for generating the concentration pulsations described above, the following relationships are valid:

$$(\overline{\alpha_1\, \hat{C}}) = (\overline{\alpha_2\, \hat{C}}) = (\overline{\alpha_3\, \hat{C}}) = a_0.$$

Let us discuss the procedure for obtaining the series of pulsations of instantaneous values of the wind velocity components and aerosol concentrations. At the first stage one should obtain unknown value a_0 according to Eq. (7) by means of quite long sequences of the values α_m. Then one should make sure that the limitations (12) are fulfilled. And, finally having find the coefficients a_1, a_2, a_3, b_1, b_2, b_3 and b_4 from Eq. (11), to start simulation of the series of pulsations of the wind velocity components and concentration.

As an example, let us consider the process of the spread of aerosol particles from a linear stationary source situated at the altitude $z_0 = 60$ m and perpendicular to the X- and Z-axes. The calculations were performed by the method described in Ref. 7, and corresponded to real conditions of the spread of aerosol at an unstable stratification of the atmosphere. The values of the stress tensor components presented in Table I (the row "initial") are obtained for the point with the coordinate $z = 20$ m and $x = 100$ m far from the vertical plane where the source is situated. The parameters of the distribution function are the following: $\overline{C} = 0.85$ conventional units, $\nu = 1.02$ conv. units. The components of the vector of the aerosol turbulent flux and rms errors of its concentration found according to Ref. 7 are presented in Table II in the row "initial". For this example, one can not consider the points at $z = 20$ m and $x < 100$ m due to the limitations (12). By moving the point from the source one gradually comes to situation of practically full loss of the statistical relations between the pulsation fields of wind velocity and concentration.

TABLE I. The aerosol initial and simulated components of the stress tensor (m^2/sec^2).

Components	σ_x^2	σ_y^2	σ_z^2	$-U_*^2$
Initial	0.9200	0.8400	0.8400	0.530
Simulated	0.8945	0.8422	0.8398	0.498

TABLE II. The initial and simulated components of the vector of the turbulent flux of aerosol and rms error of its concentrations (conventional units).

Components	φ_x	φ_y	φ_z	σ_c
Initial	0.060	0	−0.032	0.684
Simulated	0.073	0.041	−0.060	0.679

An example of the fragments of the obtained pulsation series is shown in the figure, and the tables contain the characteristics calculated from the series obtained (rows "simulated"). Each pulsation series contains one thousand of readouts. Obviously, apart from the sample volume, the error is mainly related to the technique for obtaining the uniformly distributed random values and to the accuracy of solution of the transcendent equations (7). The estimate of the absolute values of the errors appearing when modeling the stress tensor and the components of the vector of the turbulent flux of aerosol, is of the order of 0.03 m^2/sec^2 and 0.04 conv. units. Thus, we are sure in the satisfactory agreement between the results obtained and the initial data.

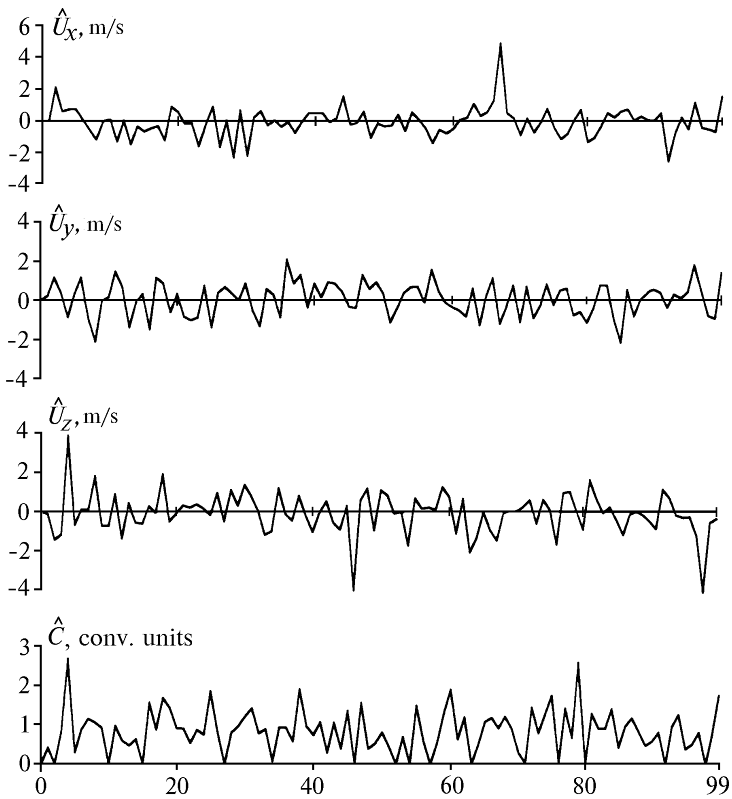

FIG.1. An example of the fragments of the obtained series of pulsations of the wind velocity components and the aerosol concentration.

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